Type 3

$$f(z, p, q) = 0$$

1. Solve $p(1+q) = qz$

Soln.

Given. $p(1+q) = qz \rightarrow (1)$

Let $u = x + ay$

Then. $p = \frac{\partial z}{\partial x} = \frac{dz}{du}$.

Type – III $\quad f(z, p, q) = 0$

1. Solve $p(1+q) = qz \rightarrow (1)$

Soln.:

Let $u = x + ay$

Then $p = \frac{dz}{du}$ and $q = a\frac{dz}{du}$

$$(1) \Rightarrow \frac{dz}{du}\left(1 + a\frac{dz}{du}\right) = a\frac{dz}{du}z$$

$$1 + a\frac{dz}{du} = az$$

$$a\frac{dz}{du} = az - 1$$

$$\frac{dz}{du} = \frac{az-1}{a}$$

$$\frac{du}{dz} = \frac{a}{az-1}$$

$$du = \frac{a}{az-1}dz$$

Integrating, $$u = \int \frac{a}{az-1}dz$$

$$u = \log(az-1) + \log c$$

$$x + ay = \log[c(az-1)]$$

2. Solve $z^2 = 1 + p^2 + q^2$

Soln.

$$z^2 = 1 + p^2 + q^2 \rightarrow (1)$$

Let $u = x + ay$

$$p = \frac{dz}{du}, \quad q = a\frac{dz}{du}$$

(1) ⇒ $z^2 = 1 + \left(\frac{dz}{du}\right)^2 + \left(a\frac{dz}{du}\right)^2$

$$z^2 = \left(\frac{dz}{du}\right)^2(1+a^2) + 1$$

$$z^2 - 1 = \left(\frac{dz}{du}\right)^2(1+a^2)$$

$$\left(\frac{dz}{du}\right)^2 = \frac{z^2-1}{1+a^2}$$

$$\frac{dz}{du} = \sqrt{\frac{z^2-1}{1+a^2}} = \frac{\sqrt{z^2-1}}{\sqrt{1+a^2}}$$

$$\frac{dz}{\sqrt{z^2-1}} = \frac{du}{\sqrt{1+a^2}}$$

Integrating on both sides,

$$\cosh^{-1} z = \frac{1}{\sqrt{1+a^2}}\, u + c$$

$$= \frac{1}{\sqrt{1+a^2}}(x+ay) + c$$

HW
II. Solve $p(1+q^2) = q(z-a)$