Type – IV $\quad f_1(x, p) = f_2(y, q)$

For this type, there is no Singular Integral.

1. Solve $q^2 - p = y - x$

Soln. :

Given. $q^2 - y = p - x = k$ (a constant)

Now $q^2 - y = k$ | $p - x = k$

$q^2 = k + y$ | $p = k + x$

$q = \sqrt{k+y}$

we know that $z = \int p\,dx + \int q\,dy$

$$z = \int (k+x)\,dx + \int \sqrt{k+y}\,dy$$

$$= kx + \frac{x^2}{2} + \frac{(k+y)^{3/2}}{3/2} + c$$

$$= kx + \frac{x^2}{2} + \frac{2}{3}(k+y)^{3/2} + c,$$ which is the complete Integral.

2. Solve $\sqrt{p} + \sqrt{q} = x + y$

Soln.

Given. $\sqrt{p} - x = y - \sqrt{q} = k$

Now $\sqrt{p} - x = k$ | $y - \sqrt{q} = k$

$\sqrt{p} = k + x$ | $\sqrt{q} = y - k$

$p = (k+x)^2$ | $q = (y-k)^2$

we know that

$$z = \int p\,dx + \int q\,dy$$

$$z = \int (k+x)^2 dx + \int (y-k)^2 dy$$

$$= \frac{(k+x)^3}{3} + \frac{(y-k)^3}{3} + c,$$ which is the Complete Integral.

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3. Find the Complete Integral of $xp - yq = y^2 - x^2$

Soln.

Gvn. $xp + x^2 = y^2 + yq = k$ (a constant)

Now $xp + x^2 = k$

$xp = k - x^2$

$p = \frac{k - x^2}{x}$

$p = \frac{k}{x} - x$

$y^2 + yq = k$

$yq = k - y^2$

$q = \frac{k - y^2}{y}$

$q = \frac{k}{y} - y$

we know that $z = \int p\,dx + \int q\,dy$

$$z = \int \left(\frac{k}{x} - x\right) dx + \int \left(\frac{k}{y} - y\right) dy$$

$$= k \log x - \frac{x^2}{2} + k \log y - \frac{y^2}{2} + c$$

$$= k \log xy - \left(\frac{x^2 + y^2}{2}\right) + c$$ which is the CI.

HW 1. Solve $\sqrt{p} + \sqrt{q} = \sqrt{x}$