



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore-641035.



UNIT-III PARTIAL DIFFERENTIAL EQUATIONS

Solution of First Order Partial Differential Equations

Type - IV $f_1(x, p) = f_2(y, q)$
 For this type, there is no singular integral.

1. Solve $q^2 - p = y - x$

Soln.:

Given. $q^2 - y = p - x = k$ (a constant)

Now $q^2 - y = k$

$$q^2 = k + y$$

$$q = \sqrt{k + y}$$

$$p - x = k$$

$$p = k + x$$

We know that $z = \int p dx + \int q dy$

$$z = \int (k + x) dx + \int \sqrt{k + y} dy$$

$$= kx + \frac{x^2}{2} + \frac{(k + y)^{3/2}}{3/2} + c$$

$$= kx + \frac{x^2}{2} + \frac{2}{3} (k + y)^{3/2} + c, \text{ which is the complete integral.}$$

2. Solve $\sqrt{p} + \sqrt{q} = x + y$

Soln.:

Given. $\sqrt{p} - x = y - \sqrt{q} = k$

Now $\sqrt{p} - x = k$

$$\sqrt{p} = k + x$$

$$p = (k + x)^2$$

$y - \sqrt{q} = k$

$$\sqrt{q} = y - k$$

$$q = (y - k)^2$$

We know that

$$z = \int p dx + \int q dy$$



$$z = \int (k+x)^2 dx + \int (y-k)^2 dy$$

$$= \frac{(k+x)^3}{3} + \frac{(y-k)^3}{3} + c, \text{ which is the Complete Integral.}$$

6. Find the Complete Integral of $xp - yq = y^2 - x^2$

Soln:

Given: $xp + x^2 = y^2 + yq = k$ (a constant)

Now $xp + x^2 = k$

$$xp = k - x^2$$

$$p = \frac{k - x^2}{x}$$

$$p = \frac{k}{x} - x$$

$y^2 + yq = k$

$$yq = k - y^2$$

$$q = \frac{k - y^2}{y}$$

$$q = \frac{k}{y} - y$$

We know that $z = \int p dx + \int q dy$

$$z = \int \left(\frac{k}{x} - x \right) dx + \int \left(\frac{k}{y} - y \right) dy$$

$$= k \log x - \frac{x^2}{2} + k \log y - \frac{y^2}{2} + c$$

$$= k \log xy - \left(\frac{x^2 + y^2}{2} \right) + c \text{ which is the CI.}$$

How

7. Solve $\sqrt{p} + \sqrt{q} = \sqrt{x}$