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Coimbatore-641035.
UNIT-III
PARTIAL DIFFERENTIAL EQUATIONS Solution of First Order Partial Differential Equations Type-II dailaut's form $\quad z=p x+q y+f(p, q)$ working Rule: complete Integral:
Replace $b \rightarrow a$ and $q \rightarrow b$
Singular Integral:

$$
\frac{\partial z}{\partial a}=0 \quad \text { and } \quad \frac{\partial \pi}{\partial b}=0
$$

conceal Integral:
pat $b=\phi(a)$ in Complete Integral.
I. Solve $z=p x+q y+p q$

Soln.:

$$
\text { Goon } x=p x+q y+p q \rightarrow(1)
$$

Complete Integral:

$$
\begin{array}{ll}
z=a x+b y+a b \quad \text { Replace } p \rightarrow a \\
\text { integral: } & q>b
\end{array}
$$

Singular Integral:

$$
\frac{\partial z}{\partial a}=0 \quad \text { and } \quad \frac{\partial z}{\partial b}=0
$$

$$
\begin{array}{rlrl}
x+b=0 & y+a & =0 \\
b=-x & a & =-y
\end{array}
$$

Subs a of b an (2)

$$
\begin{aligned}
& z=-y x-x y-y(-x) \\
& z=-x y
\end{aligned}
$$

General Integral:
Subs. $b=\phi(a)$ an (2),
$z=a x+\phi(a) y+a \phi(a) \longrightarrow(3)$
WKT $\frac{\partial z}{\partial a}=0$
$\Rightarrow x+\phi^{\prime}(a) y+a \phi^{\prime}(a)+\phi(a)=0 \rightarrow(4)$
Eliminate ta, bl (4)\& (3), we get
the general soln.

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UNIT-III PARTIAL DIFFERENTIAL EQUATIONS
Solution of First Order Partial Differential Equations
solve $\quad z=p x+q y+p^{2}-q^{2}$
(5).
8012.:

Give $z=p x+q y+p^{2}-q^{2} \rightarrow(1)$
Complete Integral :

$$
\begin{array}{ll}
\text { plete Integral } \\
z=a x+b y+a^{2}-b^{2} & \text { Replace } \\
\qquad(2) & q \rightarrow a \\
&
\end{array}
$$

Songutal Intogral:

$$
\begin{aligned}
\frac{\partial z}{\partial a} & =0 \\
x+2 a & =0 \\
2 a & =-x \\
a & =\frac{-x}{2}
\end{aligned}
$$

$$
\begin{array}{r}
\frac{\partial z}{\partial b}=0 \\
y-2 b=0 \\
y=2 b \\
\Rightarrow b=\frac{y}{2}
\end{array}
$$

Subs. \&a \& 8 in (2),

$$
\begin{aligned}
& \text { Subs. } \\
& z=-\frac{x}{2} x+\frac{y}{2} y+\left(\frac{-x}{2}\right)^{2}-\left(\frac{y}{2}\right)^{2} \\
&=-\frac{x^{2}}{2}+\frac{y^{2}}{2}+\frac{x^{2}}{4}-\frac{y^{2}}{4} \\
& 4 z=-2 x^{2}+2 y^{2}+x^{2}-y^{2} \\
&=-x^{2}+y^{2} \\
& y^{2}-x^{2}=4 z
\end{aligned}
$$

ceneral Integral:

$$
\begin{aligned}
& z=a x+\phi(a) y-[\phi(a)]^{2}+a^{2} \rightarrow(3)
\end{aligned}
$$

get the general sols,
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UNIT-III PARTIAL DIFFERENTIAL EQUATIONS Solution of First Order Partial Differential Equations 3]. Solve $z=\beta x+q y+\sqrt{1+p^{2}+q^{2}}$
Soln. :

$$
\text { Gro. } z=p x+q y+\sqrt{1+p^{2}+q^{2}}
$$

Complete Integral:

$$
z=a x+b y+\sqrt{1+a^{2}+b^{2}} \rightarrow(A)
$$

Sisogulas Integral:

$$
\begin{gathered}
\frac{\partial z}{\partial a}=0 \\
x+\frac{1(2 a)}{2 \sqrt{1+a^{2}+b^{2}}}=0 \\
x=\frac{-a}{\sqrt{1+a^{2}+b^{2}}} \rightarrow(1)
\end{gathered}\left\{\begin{array}{l}
\frac{a z}{\partial b}=0 \\
y+\frac{1(2 b)}{2 \sqrt{1+a^{2}+b^{2}}}=0 \\
y=\frac{-b}{\sqrt{1+a^{2}+b^{2}}} \rightarrow(2)
\end{array}\right.
$$

squaring os botssides,

$$
x^{2}=\frac{a^{2}}{1+a^{2}+b^{2}} \quad y^{2}=\frac{b^{2}}{1+a^{2}+b 2}
$$

Now,

$$
x^{2}+y^{2}=\frac{a^{2}+b^{2}}{1+a^{2}+b^{2}}
$$

$$
1-\left(x^{2}+y^{2}\right)=1-\frac{a^{2}+b^{2}}{1+a^{2}+b^{2}}
$$

$$
1-x^{2}-y^{2}=\frac{1+a^{2}+b^{2}-a^{2}-b^{2}}{1+a^{2}+b^{2}}=\frac{1}{1+a^{2}+b^{2}}
$$

Taring square root,


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UNIT-III PARTIAL DIFFERENTIAL EQUATIONS
Solution of First Order Partial Differential Equations

$$
\begin{aligned}
(1) \Rightarrow x & =-a \sqrt{1-x^{2}-y^{2}} \Rightarrow a=\frac{-x}{\sqrt{1-x^{2}-y^{2}}} \\
(2) \Rightarrow y & =-b \sqrt{1-x^{2}-y^{2}} \Rightarrow b=\frac{-y}{\sqrt{1-x^{2}-y^{2}}} \\
(A) \Rightarrow z & =\frac{-x^{2}}{\sqrt{1-x^{2}-y^{2}}}-\frac{y^{2}}{\sqrt{1-x^{2}-y^{2}}}+\frac{1}{\sqrt{1-x^{2}-y^{2}}} \\
& =\frac{1-x^{2}-y^{2}}{\sqrt{1-x^{2}-y^{2}}} \\
& =\sqrt{1-x^{2}-y^{2}} \\
z^{2} & =1-x^{2}-y^{2}
\end{aligned}
$$

HO

$$
\begin{aligned}
& \text { ग. } z=p x+q y+(p q)^{3 / 2} \\
& \text { 2]. } z=p x+q y+p^{2} q^{2}
\end{aligned}
$$

