Type

RHS $= e^{ax+by}\,\phi(x, y)$

Replace $D$ by $D+a$

$D'$ by $D'+b$

1. To find the PI of $(D^2 - 2DD' + D'^2)z = x^2y^2e^{x+y}$

Soln. :

$$PI = \frac{1}{D^2 - 2DD' + D'^2}\, x^2y^2e^{x+y}$$

$$= \frac{1}{(D-D')^2}\, e^{x+y}x^2y^2$$

$$= e^{x+y} \frac{1}{(D+1-(D'+1))^2} x^2 y^2$$

$D \to D+a = D+1$

$D' \to D'+b = D'+1$

$$= e^{x+y} \frac{1}{(D-D')^2} x^2 y^2$$

$$= e^{x+y} \frac{1}{D^2 - 2DD' + D'^2} x^2 y^2$$

$$= e^{x+y} \frac{1}{D^2\left[1 - \frac{2D'}{D} + \frac{D'^2}{D^2}\right]} x^2 y^2$$

$$= e^{x+y} \frac{1}{D^2}\left[1 - \left(\frac{2D'}{D} - \frac{D'^2}{D^2}\right)\right]^{-1} x^2 y^2$$

$$= e^{x+y} \frac{1}{D^2}\left[1 + \frac{2D'}{D} - \frac{D'^2}{D^2} + \frac{4D'^2}{D^2}\right] x^2 y^2$$

$$= e^{x+y} \frac{1}{D^2}\left[x^2y^2 + \frac{2D'}{D} x^2y^2 + \frac{3D'^2}{D^2} x^2 y^2\right]$$

$$= e^{x+y}\left[\frac{1}{D^2} x^2 y^2 + \frac{2}{D^3} 2x^2 y + \frac{3}{D^4}(2x^2)\right]$$

$$= e^{x+y}\Big[$$

$\frac{1}{D^2} x^2 y^2 \xrightarrow{1st} \frac{x^3}{3} y^2$

$\xrightarrow{2nd} \frac{x^4}{12} y^2$

$\frac{1}{D^3} 4x^2 y \to 4\frac{x^3}{3} y$

$\xrightarrow{2nd} \frac{4x^4}{12} y \xrightarrow{3rd} \frac{4x^5}{60} y$

$\frac{1}{D^4} 6x^2 \to \frac{6x^3}{3} \to \frac{6x^4}{12}$

$\to \frac{6x^5}{60} \to \frac{6x^7}{420}$

$$PI = e^{x+y}\left[\frac{x^4y^2}{12} + \frac{x^5 y}{15} + \frac{x^7}{70}\right]$$

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