



Type

$$\text{RHS} = e^{ax+by} \phi(x, y)$$

Replace  $D$  by  $D+a$

$D'$  by  $D'+b$

J. So find the PI of  $(D^2 - 2DD' + D'^2)z = x^2 y^2 e^{x+y}$

Soln.:

$$\text{PI} = \frac{1}{D^2 - 2DD' + D'^2} x^2 y^2 e^{x+y}$$

$$= \frac{1}{(D-D')^2} e^{x+y} x^2 y^2$$



# SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore-641035.



UNIT-III PARTIAL DIFFERENTIAL EQUATIONS Solution of Second Order Partial Differential Equations

$$= e^{x+y} \frac{1}{(D+1-(D'+1))^2} x^2 y^2 \quad \begin{matrix} D \rightarrow D+a=D+1 \\ D' \rightarrow D'+b=D'+1 \end{matrix}$$

$$= e^{x+y} \frac{1}{(D-D')^2} x^2 y^2$$

$$= e^{x+y} \frac{1}{D^2 - 2DD' + D'^2} x^2 y^2$$

$$= e^{x+y} \frac{1}{D^2 \left[ 1 - \frac{2D'}{D} + \frac{D'^2}{D^2} \right]} x^2 y^2$$

$$= e^{x+y} \frac{1}{D^2} \left[ 1 - \left( \frac{2D'}{D} - \frac{D'^2}{D^2} \right) \right]^{-1} x^2 y^2$$

$$= e^{x+y} \frac{1}{D^2} \left[ 1 + \frac{2D'}{D} - \frac{D'^2}{D^2} + \frac{4D'^2}{D^2} \right] x^2 y^2$$

$$= e^{x+y} \frac{1}{D^2} \left[ x^2 y^2 + \frac{2D'}{D} x^2 y^2 + \frac{3D'^2}{D^2} x^2 y^2 \right]$$

$$= e^{x+y} \left[ \frac{1}{D^2} x^2 y^2 + \frac{2}{D^3} 2x^2 y + \frac{3}{D^4} (2x^2) \right]$$

$$= e^{x+y} \left[ \frac{1}{D^2} x^2 y^2 + \frac{2}{D^3} 2x^2 y + \frac{3}{D^4} (2x^2) \right]$$

$$PI = e^{x+y} \left[ \frac{x^4 y^2}{12} + \frac{x^5 y}{15} + \frac{x^7}{70} \right]$$

$$\begin{aligned} \frac{1}{D^2} x^2 y^2 &\rightarrow \frac{x^3}{3} y^2 \quad \text{1st} \\ &\quad \text{2nd} \rightarrow \frac{x^4}{12} y^2 \\ \frac{1}{D^3} 4x^2 y &\rightarrow 4 \frac{x^3}{3} y \quad \text{1st} \\ &\quad \text{2nd} \quad \text{3rd} \\ \frac{4x^4}{12} y &\rightarrow \frac{4x^5}{60} y \\ \frac{1}{D^4} 6x^2 &\rightarrow \frac{6x^3}{3} \rightarrow \frac{6x^4}{12} \\ &\rightarrow \frac{6x^5}{60} \rightarrow \frac{6x^7}{420} \end{aligned}$$

Scanned with CamScanner