

# **SNS COLLEGE OF TECHNOLOGY**



# An Autonomous Institution Coimbatore-35

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A++' Grade Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

# DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING

19GET276 - VQAR II

II YEAR/ IV SEMESTER

UNIT 2 – QUANTITATIVE ABILITIES
TOPIC – PERMUTATION AND COMBINATION

18/03/2024 1/18





#### 1. Factorial Notation:

Let n be a positive integer. Then, factorial n, denoted n! is defined as:

$$n! = n(n - 1)(n - 2) \dots 3.2.1.$$

#### Examples:

We define 0! = 1.

ii. 
$$4! = (4 \times 3 \times 2 \times 1) = 24$$
.

iii. 
$$5! = (5 \times 4 \times 3 \times 2 \times 1) = 120$$
.

#### 2. Permutations:

The different arrangements of a given number of things by taking some or all at a time, are called permutations.

#### Examples:

- i. All permutations (or arrangements) made with the letters a, b, c by taking two at a time are (ab, ba, ac, ca, bc, cb).
- ii. All permutations made with the letters a, b, c taking all at a time are: (abc, acb, bac, bca, cab, cba)





#### 3. Number of Permutations:

Number of all permutations of n things, taken r at a time, is given by:

$$^{n}P_{r} = n(n-1)(n-2) \dots (n-r+1) = \frac{n!}{(n-r)!}$$

#### Examples:

i. 
$$^{6}P_{2} = (6 \times 5) = 30$$
.

ii. 
$$^{7}P_{3} = (7 \times 6 \times 5) = 210$$
.

iii. Cor. number of all permutations of n things, taken all at a time = n!.

#### 4. An Important Result:

If there are n subjects of which  $p_1$  are alike of one kind;  $p_2$  are alike of another kind;  $p_3$  are alike of third kind and so on and  $p_r$  are alike of  $r^{th}$  kind, such that  $(p_1 + p_2 + ... p_r) = n$ .

Then, number of permutations of these *n* objects is = 
$$\frac{n!}{(p_1!).(p_2)!....(p_r!)}$$





#### 5. Combinations:

Each of the different groups or selections which can be formed by taking some or all of a number of objects is called a combination.

#### Examples:

- Suppose we want to select two out of three boys A, B, C. Then, possible selections are AB, BC and CA.
  - Note: AB and BA represent the same selection.
- 2. All the combinations formed by a, b, c taking ab, bc, ca.
- 3. The only combination that can be formed of three letters a, b, c taken all at a time is abc.
- 4. Various groups of 2 out of four persons A, B, C, D are:
  - AB, AC, AD, BC, BD, CD.
- Note that ab ba are two different permutations but they represent the same combination.





#### 6. Number of Combinations:

The number of all combinations of *n* things, taken *r* at a time is:

$${}^{n}C_{r} = \frac{n!}{(r!)(n-r)!} = \frac{n(n-1)(n-2) \dots \text{ to } r \text{ factors}}{r!}.$$

#### Note:

ii. 
$${}^{n}C_{r} = {}^{n}C_{(n-r)}$$

#### **Examples:**

i. 
$${}^{11}C_4 = \frac{(11 \times 10 \times 9 \times 8)}{(4 \times 3 \times 2 \times 1)} = 330.$$

ii. 
$${}^{16}C_{13} = {}^{16}C_{(16-13)} = {}^{16}C_3 = \frac{16 \times 15 \times 14}{3!} = \frac{16 \times 15 \times 14}{3 \times 2 \times 1} = 560.$$





From a group of 7 men and 6 women, five persons are to be selected to form a committee so that at least 3 men are there on the committee. In how many ways can it be done?

- A. 564
- B. 645
- C. 735
- D. 756
- E. None of these

Answer: Option D

#### Explanation:

We may have (3 men and 2 women) or (4 men and 1 woman) or (5 men only).

∴ Required number of ways = 
$$(^{7}C_{3} \times ^{6}C_{2}) + (^{7}C_{4} \times ^{6}C_{1}) + (^{7}C_{5})$$
  
=  $\left(\frac{7 \times 6 \times 5}{3 \times 2 \times 1} \times \frac{6 \times 5}{2 \times 1}\right) + (^{7}C_{3} \times ^{6}C_{1}) + (^{7}C_{2})$   
=  $525 + \left(\frac{7 \times 6 \times 5}{3 \times 2 \times 1} \times 6\right) + \left(\frac{7 \times 6}{2 \times 1}\right)$   
=  $(525 + 210 + 21)$ 





In how many different ways can the letters of the word 'LEADING' be arranged in such a way that the vowels always come together?

- A. 360
- B. 480
- C. 720
- D. 5040
- E. None of these

Answer: Option C

#### Explanation:

The word 'LEADING' has 7 different letters.

When the vowels EAI are always together, they can be supposed to form one letter.

Then, we have to arrange the letters LNDG (EAI).

Now, 5(4 + 1 = 5) letters can be arranged in 5! = 120 ways.

The vowels (EAI) can be arranged among themselves in 3! = 6 ways.

∴ Required number of ways = (120 x 6) = 720.





In how many different ways can the letters of the word 'CORPORATION' be arranged so that the vowels always come together?

- A. 810
- B. 1440
- C. 2880
- D. 50400
- E. 5760

Answer: Option D

#### Explanation:

In the word 'CORPORATION', we treat the vowels OOAIO as one letter.

Thus, we have CRPRTN (OOAIO).

This has 7 (6 + 1) letters of which R occurs 2 times and the rest are different.

Number of ways arranging these letters =  $\frac{7!}{2!}$  = 2520.

Now, 5 vowels in which O occurs 3 times and the rest are different, can be arranged

$$\frac{5!}{3!} = 20 \text{ ways.}$$

→ Required number of ways = (2520 x 20) = 50400.





Out of 7 consonants and 4 vowels, how many words of 3 consonants and 2 vowels can be formed?

- A. 210
- B. 1050
- C. 25200
- D. 21400
- E. None of these

Answer: Option C

#### Explanation:

Number of ways of selecting (3 consonants out of 7) and (2 vowels out of 4)

$$= (^{7}C_{3} \times {}^{4}C_{2})$$

$$= \left(\frac{7 \times 6 \times 5}{3 \times 2 \times 1} \times \frac{4 \times 3}{2 \times 1}\right)$$

= 210.

Number of groups, each having 3 consonants and 2 vowels = 210.

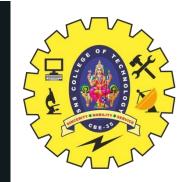
Each group contains 5 letters.

Number of ways of arranging = 5! 5 letters among themselves

$$= 5 \times 4 \times 3 \times 2 \times 1$$

= 120.

→ Required number of ways = (210 x 120) = 25200.





In how many ways can the letters of the word 'LEADER' be arranged?

- A. 72
- B. 144
- C. 360
- D. 720
- E. None of these

Answer: Option C

## Explanation:

The word 'LEADER' contains 6 letters, namely 1L, 2E, 1A, 1D and 1R.

Required number of ways =  $\frac{6!}{(1!)(2!)(1!)(1!)(1!)}$  = 360.





In a group of 6 boys and 4 girls, four children are to be selected. In how many different ways can they be selected such that at least one boy should be there?

- A. 159
- B. 194
- C. 205
- D. 209
- E. None of these

Answer: Option D

#### Explanation:

We may have (1 boy and 3 girls) or (2 boys and 2 girls) or (3 boys and 1 girl) or (4 boys).

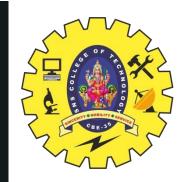
$$\therefore$$
 Required number =  $(^{6}C_{1} \times ^{4}C_{3}) + (^{6}C_{2} \times ^{4}C_{2}) + (^{6}C_{3} \times ^{4}C_{1}) + (^{6}C_{4})$  of ways

$$= ({}^{6}C_{1} \times {}^{4}C_{1}) + ({}^{6}C_{2} \times {}^{4}C_{2}) + ({}^{6}C_{3} \times {}^{4}C_{1}) + ({}^{6}C_{2})$$

$$= (6 \times 4) + \left(\frac{6 \times 5}{2 \times 1} \times \frac{4 \times 3}{2 \times 1}\right) + \left(\frac{6 \times 5 \times 4}{3 \times 2 \times 1} \times 4\right) + \left(\frac{6 \times 5}{2 \times 1}\right)$$

$$= (24 + 90 + 80 + 15)$$

= 209.





In how many ways a committee, consisting of 5 men and 6 women can be formed from 8 men and 10 women?

- A. 266
- B. 5040
- C. 11760
- D. 86400
- E. None of these

Answer: Option C

## Explanation:

Required number of ways = 
$$(^{8}C_{5} \times ^{10}C_{6})$$
  
=  $(^{8}C_{3} \times ^{10}C_{4})$   
=  $\left(\frac{8 \times 7 \times 6}{3 \times 2 \times 1} \times \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1}\right)$   
= 11760.





A box contains 2 white balls, 3 black balls and 4 red balls. In how many ways can 3 balls be drawn from the box, if at least one black ball is to be included in the draw?

- A. 32
- B. 48
- C. 64
- D. 96
- E. None of these

Answer: Option C

#### Explanation:

We may have(1 black and 2 non-black) or (2 black and 1 non-black) or (3 black).

.. Required number of ways =  $({}^{3}C_{1} \times {}^{6}C_{2}) + ({}^{3}C_{2} \times {}^{6}C_{1}) + ({}^{3}C_{3})$ 

$$= \left(3 \times \frac{6 \times 5}{2 \times 1}\right) + \left(\frac{3 \times 2}{2 \times 1} \times 6\right) + 1$$

$$= (45 + 18 + 1)$$

= 64.





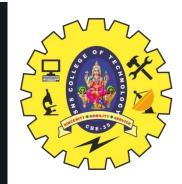
In how many ways can a group of 5 men and 2 women be made out of a total of 7 men and 3 women?

- A. 63
- B. 90
- C. 126
- D. 45
- E. 135

Answer: Option A

Explanation:

Required number of ways =  $({}^{7}C_{5} \times {}^{3}C_{2}) = ({}^{7}C_{2} \times {}^{3}C_{1}) = \left(\frac{7 \times 6}{2 \times 1} \times 3\right) = 63.$ 





How many 4-letter words with or without meaning, can be formed out of the letters of the word, 'LOGARITHMS', if repetition of letters is not allowed?

- A. 40
- B. 400
- C. 5040
- D. 2520

Answer: Option C

### Explanation:

'LOGARITHMS' contains 10 different letters.

Required number of words = Number of arrangements of 10 letters, taking 4 at a time.

$$= ^{10}P_4$$

 $= (10 \times 9 \times 8 \times 7)$ 

= 5040.





In how many different ways can the letters of the word 'MATHEMATICS' be arranged so that the vowels always come together?

- A. 10080
- B. 4989600
- C. 120960
- D. None of these

Answer: Option C

#### Explanation:

In the word 'MATHEMATICS', we treat the vowels AEAI as one letter.

Thus, we have MTHMTCS (AEAI).

Now, we have to arrange 8 letters, out of which M occurs twice, T occurs twice and the rest are different.

∴ Number of ways of arranging these letters =  $\frac{8!}{(2!)(2!)}$  = 10080.

Now, AEAI has 4 letters in which A occurs 2 times and the rest are different.

Number of ways of arranging these letters =  $\frac{4!}{2!}$  = 12.

∴ Required number of words = (10080 x 12) = 120960.





In how many different ways can the letters of the word 'OPTICAL' be arranged so that the vowels always come together?

- A. 120
- B. 720
- C. 4320
- D. 2160
- E. None of these

Answer: Option B

#### Explanation:

The word 'OPTICAL' contains 7 different letters.

When the vowels OIA are always together, they can be supposed to form one letter.

Then, we have to arrange the letters PTCL (OIA).

Now, 5 letters can be arranged in 5! = 120 ways.

The vowels (OIA) can be arranged among themselves in 3! = 6 ways.

Required number of ways = (120 x 6) = 720.





# THANK YOU