



SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
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DEPARTMENT OF INFORMATION TECHNOLOGY

19CSE303 - ARTIFICIAL INTELLIGENCE
III YEAR IV SEM

UNIT II – LOGICAL REASONING

TOPIC – First-order logic





OUTLINE

- First-order logic
 - Properties, relations, functions, quantifiers, ...
 - Terms, sentences, axioms, theories, proofs, ...
- Extensions to first-order logic
- Logical agents
 - Reflex agents
 - Representing change: situation calculus, frame problem
 - Preferences on actions
 - Goal-based agents





FIRST-ORDER LOGIC

- First-order logic (FOL) models the world in terms of
 - **Objects**, which are things with individual identities
 - **Properties** of objects that distinguish them from other objects
 - **Relations** that hold among sets of objects
 - **Functions**, which are a subset of relations where there is only one “value” for any given “input”
- Examples:
 - Objects: Students, lectures, companies, cars ...
 - Relations: Brother-of, bigger-than, outside, part-of, has-color, occurs-after, owns, visits, precedes, ...
 - Properties: blue, oval, even, large, ...
 - Functions: father-of, best-friend, second-half, one-more-than ...





USER PROVIDES

- **Constant symbols**, which represent individuals in the world
 - Mary
 - 3
 - Green
- **Function symbols**, which map individuals to individuals
 - father-of(Mary) = John
 - color-of(Sky) = Blue
- **Predicate symbols**, which map individuals to truth values
 - greater(5,3)
 - green(Grass)
 - color(Grass, Green)





FOL PROVIDES

○ Variable symbols

- E.g., x , y , foo

○ Connectives

- Same as in PL: not (\neg), and (\wedge), or (\vee), implies (\rightarrow), if and only if (biconditional \leftrightarrow)

○ Quantifiers

- Universal $\forall \mathbf{x}$ or (\mathbf{Ax})
- Existential $\exists \mathbf{x}$ or (\mathbf{Ex})





SENTENCES ARE BUILT FROM TERMS AND ATOMS



- A **term** (denoting a real-world individual) is a constant symbol, a variable symbol, or an n-place function of n terms.
x and $f(x_1, \dots, x_n)$ are terms, where each x_i is a term.
A term with no variables is a **ground term**
- An **atomic sentence** (which has value true or false) is an n-place predicate of n terms
- A **complex sentence** is formed from atomic sentences connected by the logical connectives:
 $\neg P, P \vee Q, P \wedge Q, P \rightarrow Q, P \leftrightarrow Q$ where P and Q are sentences
- A **quantified sentence** adds quantifiers \forall and \exists
- A **well-formed formula (wff)** is a sentence containing no “free” variables. That is, all variables are “bound” by universal or existential quantifiers.
 $(\forall x)P(x,y)$ has x bound as a universally quantified variable, but y is free.





QUANTIFIERS

○ **Universal quantification**

- $(\forall x)P(x)$ means that P holds for **all** values of x in the domain associated with that variable
- E.g., $(\forall x) \text{dolphin}(x) \rightarrow \text{mammal}(x)$

○ **Existential quantification**

- $(\exists x)P(x)$ means that P holds for **some** value of x in the domain associated with that variable
- E.g., $(\exists x) \text{mammal}(x) \wedge \text{lays-eggs}(x)$
- Permits one to make a statement about some object without naming it





QUANTIFIERS

- Universal quantifiers are often used with “implies” to form “rules”:
 $(\forall x) \text{ student}(x) \rightarrow \text{smart}(x)$ means “All students are smart”
- Universal quantification is *rarely* used to make blanket statements about every individual in the world:
 $(\forall x) \text{ student}(x) \wedge \text{smart}(x)$ means “Everyone in the world is a student and is smart”
- Existential quantifiers are usually used with “and” to specify a list of properties about an individual:
 $(\exists x) \text{ student}(x) \wedge \text{smart}(x)$ means “There is a student who is smart”
- A common mistake is to represent this English sentence as the FOL sentence:
 $(\exists x) \text{ student}(x) \rightarrow \text{smart}(x)$
 - But what happens when there is a person who is *not* a student?





QUANTIFIER SCOPE

- Switching the order of universal quantifiers *does not* change the meaning:
 - $(\forall x)(\forall y)P(x,y) \leftrightarrow (\forall y)(\forall x) P(x,y)$
- Similarly, you can switch the order of existential quantifiers:
 - $(\exists x)(\exists y)P(x,y) \leftrightarrow (\exists y)(\exists x) P(x,y)$
- Switching the order of universals and existentials *does* change meaning:
 - Everyone likes someone: $(\forall x)(\exists y) \text{ likes}(x,y)$
 - Someone is liked by everyone: $(\exists y)(\forall x) \text{ likes}(x,y)$





CONNECTIONS BETWEEN ALL AND EXISTS

We can relate sentences involving \forall and \exists using De Morgan's laws:

$$(\forall x) \neg P(x) \leftrightarrow \neg(\exists x) P(x)$$

$$\neg(\forall x) P \leftrightarrow (\exists x) \neg P(x)$$

$$(\forall x) P(x) \leftrightarrow \neg (\exists x) \neg P(x)$$

$$(\exists x) P(x) \leftrightarrow \neg(\forall x) \neg P(x)$$





QUANTIFIED INFERENCE RULES

- Universal instantiation
 - $\forall x P(x) \therefore P(A)$
- Universal generalization
 - $P(A) \wedge P(B) \dots \therefore \forall x P(x)$
- Existential instantiation
 - $\exists x P(x) \therefore P(F)$ ← skolem constant F
- Existential generalization
 - $P(A) \therefore \exists x P(x)$





UNIVERSAL INSTANTIATION

(A.K.A. UNIVERSAL ELIMINATION)


- If $(\forall x) P(x)$ is true, then $P(C)$ is true, where C is *any* constant in the domain of x
- Example:
 $(\forall x) \text{ eats}(\text{Ziggy}, x) \Rightarrow \text{ eats}(\text{Ziggy}, \text{IceCream})$
- The variable symbol can be replaced by any ground term, i.e., any constant symbol or function symbol applied to ground terms only





EXISTENTIAL INSTANTIATION

(A.K.A. EXISTENTIAL ELIMINATION)

- From $(\exists x) P(x)$ infer $P(c)$
 - Example:
 - $(\exists x) \text{ eats}(\text{Ziggy}, x) \rightarrow \text{eats}(\text{Ziggy}, \text{Stuff})$
 - Note that the variable is replaced by a **brand-new constant** not occurring in this or any other sentence in the KB
 - Also known as skolemization; constant is a **skolem constant**
 - In other words, we don't want to accidentally draw other inferences about it by introducing the constant
 - Convenient to use this to reason about the unknown object, rather than constantly manipulating the existential quantifier
- 



EXISTENTIAL GENERALIZATION

(A.K.A. EXISTENTIAL INTRODUCTION)

- If $P(c)$ is true, then $(\exists x) P(x)$ is inferred.
- Example
$$\text{eats}(\text{Ziggy}, \text{IceCream}) \Rightarrow (\exists x) \text{eats}(\text{Ziggy}, x)$$
- All instances of the given constant symbol are replaced by the new variable symbol
- Note that the variable symbol cannot already exist anywhere in the expression



TRANSLATING ENGLISH TO FOL

Every gardener likes the sun.

$\forall x \text{ gardener}(x) \rightarrow \text{likes}(x, \text{Sun})$

You can fool some of the people all of the time.

$\exists x \forall t \text{ person}(x) \wedge \text{time}(t) \rightarrow \text{can-fool}(x, t)$

You can fool all of the people some of the time.

$\forall x \exists t (\text{person}(x) \rightarrow \text{time}(t) \wedge \text{can-fool}(x, t))$

$\forall x (\text{person}(x) \rightarrow \exists t (\text{time}(t) \wedge \text{can-fool}(x, t)))$

←
← **Equivalent**

All purple mushrooms are poisonous.

$\forall x (\text{mushroom}(x) \wedge \text{purple}(x)) \rightarrow \text{poisonous}(x)$

No purple mushroom is poisonous.

$\neg \exists x \text{ purple}(x) \wedge \text{mushroom}(x) \wedge \text{poisonous}(x)$

$\forall x (\text{mushroom}(x) \wedge \text{purple}(x)) \rightarrow \neg \text{poisonous}(x)$

←
← **Equivalent**

There are exactly two purple mushrooms.

$\exists x \exists y \text{ mushroom}(x) \wedge \text{purple}(x) \wedge \text{mushroom}(y) \wedge \text{purple}(y) \wedge \neg(x=y) \wedge \forall z (\text{mushroom}(z) \wedge \text{purple}(z)) \rightarrow ((x=z) \vee (y=z))$

Clinton is not tall.

$\neg \text{tall}(\text{Clinton})$

X is above Y iff X is on directly on top of Y or there is a pile of one or more other objects directly on top of one another starting with X and ending with Y.

$\forall x \forall y \text{ above}(x, y) \leftrightarrow (\text{on}(x, y) \vee \exists z (\text{on}(x, z) \wedge \text{above}(z, y)))$





MONTY PYTHON AND THE ART OF FALLACY

Cast

- Sir Bedevere the Wise, master of (odd) logic
- King Arthur
- Villager 1, witch-hunter
- Villager 2, ex-newt
- Villager 3, one-line wonder
- All, the rest of you scoundrels, mongrels, and nere-do-wells.





AN EXAMPLE FROM MONTY PYTHON BY WAY OF RUSSELL & NORVIG



- **FIRST VILLAGER:** We have found a witch. May we burn her?
- **ALL:** A witch! Burn her!
- **BEDEVERE:** Why do you think she is a witch?
- **SECOND VILLAGER:** She turned *me* into a newt.
- **B:** A newt?
- **V2** (*after looking at himself for some time*): I got better.
- **ALL:** Burn her anyway.
- **B:** Quiet! Quiet! There are ways of telling whether she is a witch.





MONTY PYTHON CONT.

- **B:** Tell me... what do you do with witches?
- **ALL:** Burn them!
- **B:** And what do you burn, apart from witches?
- **Third Villager:** ...wood?
- **B:** So **why do witches burn?**
- **V2** (*after a beat*): **because they're made of wood?**
- **B:** Good.
- **ALL:** I see. Yes, of course.





MONTY PYTHON CONT.

- **B:** So how can we tell if she is made of wood?
- **V1:** Make a bridge out of her.
- **B:** Ah... but can you not also make bridges out of stone?
- **ALL:** Yes, of course... um... er...
- **B:** Does wood sink in water?
- **ALL:** No, no, it floats. Throw her in the pond.
- **B:** Wait. Wait... tell me, what also floats on water?
- **ALL:** Bread? No, no no. Apples... gravy... very small rocks...
- **B:** No, no, no,





MONTY PYTHON CONT.

- **KING ARTHUR:** A duck!
- *(They all turn and look at Arthur. Bedevere looks up, very impressed.)*
- **B:** Exactly. So... logically...
- **V1** *(beginning to pick up the thread):* **If she... weighs the same as a duck... she's made of wood.**
- **B:** And therefore?
- **ALL:** **A witch!**





MONTY PYTHON FALLACY #1

- $\forall x \text{ witch}(x) \rightarrow \text{burns}(x)$
- $\forall x \text{ wood}(x) \rightarrow \text{burns}(x)$
- -----
- $\therefore \forall z \text{ witch}(x) \rightarrow \text{wood}(x)$

- $p \rightarrow q$
- $r \rightarrow q$
- -----
- $p \rightarrow r$
conclusion

Fallacy: Affirming the





MONTY PYTHON NEAR-FALLACY #2

- $\text{wood}(x) \rightarrow \text{can-build-bridge}(x)$
 - -----
 - $\therefore \text{can-build-bridge}(x) \rightarrow \text{wood}(x)$
-
- B: Ah... but can you not also make bridges out of stone?





MONTY PYTHON FALLACY #3

- $\forall x \text{ wood}(x) \rightarrow \text{floats}(x)$
- $\forall x \text{ duck-weight}(x) \rightarrow \text{floats}(x)$
- -----
- $\therefore \forall x \text{ duck-weight}(x) \rightarrow \text{wood}(x)$

- $p \rightarrow q$
- $r \rightarrow q$
- -----
- $\therefore r \rightarrow p$





MONTY PYTHON FALLACY #4

○ $\forall z \text{ light}(z) \rightarrow \text{wood}(z)$

○ $\text{light}(W)$

○ -----

○ $\therefore \text{wood}(W)$

ok.....

○ $\text{witch}(W) \rightarrow \text{wood}(W)$
instan.

applying universal
to fallacious

conclusion #1

○ $\text{wood}(W)$

○ -----

○ $\therefore \text{witch}(z)$





EXAMPLE: A SIMPLE GENEALOGY KB BY FOL



- **Build a small genealogy knowledge base using FOL that**
 - contains facts of immediate family relations (spouses, parents, etc.)
 - contains definitions of more complex relations (ancestors, relatives)
 - is able to answer queries about relationships between people
- **Predicates:**
 - $\text{parent}(x, y)$, $\text{child}(x, y)$, $\text{father}(x, y)$, $\text{daughter}(x, y)$, etc.
 - $\text{spouse}(x, y)$, $\text{husband}(x, y)$, $\text{wife}(x, y)$
 - $\text{ancestor}(x, y)$, $\text{descendant}(x, y)$
 - $\text{male}(x)$, $\text{female}(y)$
 - $\text{relative}(x, y)$
- **Facts:**
 - $\text{husband}(\text{Joe}, \text{Mary})$, $\text{son}(\text{Fred}, \text{Joe})$
 - $\text{spouse}(\text{John}, \text{Nancy})$, $\text{male}(\text{John})$, $\text{son}(\text{Mark}, \text{Nancy})$
 - $\text{father}(\text{Jack}, \text{Nancy})$, $\text{daughter}(\text{Linda}, \text{Jack})$
 - $\text{daughter}(\text{Liz}, \text{Linda})$
 - etc.





○ Rules for genealogical relations

- $(\forall x,y)$ $\text{parent}(x, y) \leftrightarrow \text{child}(y, x)$
- $(\forall x,y)$ $\text{father}(x, y) \leftrightarrow \text{parent}(x, y) \wedge \text{male}(x)$ (similarly for $\text{mother}(x, y)$)
- $(\forall x,y)$ $\text{daughter}(x, y) \leftrightarrow \text{child}(x, y) \wedge \text{female}(x)$ (similarly for $\text{son}(x, y)$)
- $(\forall x,y)$ $\text{husband}(x, y) \leftrightarrow \text{spouse}(x, y) \wedge \text{male}(x)$ (similarly for $\text{wife}(x, y)$)
- $(\forall x,y)$ $\text{spouse}(x, y) \leftrightarrow \text{spouse}(y, x)$ (**spouse relation is symmetric**)
- $(\forall x,y)$ $\text{parent}(x, y) \rightarrow \text{ancestor}(x, y)$
- $(\forall x,y)(\exists z)$ $\text{parent}(x, z) \wedge \text{ancestor}(z, y) \rightarrow \text{ancestor}(x, y)$
- $(\forall x,y)$ $\text{descendant}(x, y) \leftrightarrow \text{ancestor}(y, x)$
- $(\forall x,y)(\exists z)$ $\text{ancestor}(z, x) \wedge \text{ancestor}(z, y) \rightarrow \text{relative}(x, y)$
(related by common ancestry)
- $(\forall x,y)$ $\text{spouse}(x, y) \rightarrow \text{relative}(x, y)$ (related by marriage)
- $(\forall x,y)(\exists z)$ $\text{relative}(z, x) \wedge \text{relative}(z, y) \rightarrow \text{relative}(x, y)$ (**transitive**)
- $(\forall x,y)$ $\text{relative}(x, y) \leftrightarrow \text{relative}(y, x)$ (**symmetric**)

○ Queries

- $\text{ancestor}(\text{Jack}, \text{Fred})$ /* the answer is yes */
- $\text{relative}(\text{Liz}, \text{Joe})$ /* the answer is yes */
- $\text{relative}(\text{Nancy}, \text{Matthew})$
/* no answer in general, no if under closed world assumption */
- $(\exists z)$ $\text{ancestor}(z, \text{Fred}) \wedge \text{ancestor}(z, \text{Liz})$





SEMANTICS OF FOL



- **Domain M:** the set of all objects in the world (of interest)
- **Interpretation I:** includes
 - Assign each constant to an object in M
 - Define each function of n arguments as a mapping $M^n \Rightarrow M$
 - Define each predicate of n arguments as a mapping $M^n \Rightarrow \{T, F\}$
 - Therefore, every ground predicate with any instantiation will have a truth value
 - In general there is an infinite number of interpretations because $|M|$ is infinite
- **Define logical connectives:** $\sim, \wedge, \vee, \Rightarrow, \Leftrightarrow$ as in PL
- **Define semantics of $(\forall x)$ and $(\exists x)$**
 - $(\forall x) P(x)$ is true iff P(x) is true under all interpretations
 - $(\exists x) P(x)$ is true iff P(x) is true under some interpretation





- **Model:** an interpretation of a set of sentences such that every sentence is *True*
- **A sentence is**
 - **satisfiable** if it is true under some interpretation
 - **valid** if it is true under all possible interpretations
 - **inconsistent** if there does not exist any interpretation under which the sentence is true
- **Logical consequence:** $S \models X$ if all models of S are also models of X





AXIOMS, DEFINITIONS AND THEOREMS

- **Axioms** are facts and rules that attempt to capture all of the (important) facts and concepts about a domain; axioms can be used to prove **theorems**
 - Mathematicians don't want any unnecessary (dependent) axioms –ones that can be derived from other axioms
 - Dependent axioms can make reasoning faster, however
 - Choosing a good set of axioms for a domain is a kind of design problem
- A **definition** of a predicate is of the form “ $p(X) \leftrightarrow \dots$ ” and can be decomposed into two parts
 - **Necessary** description: “ $p(x) \rightarrow \dots$ ”
 - **Sufficient** description “ $p(x) \leftarrow \dots$ ”
 - Some concepts don't have complete definitions (e.g., $\text{person}(x)$)





MORE ON DEFINITIONS

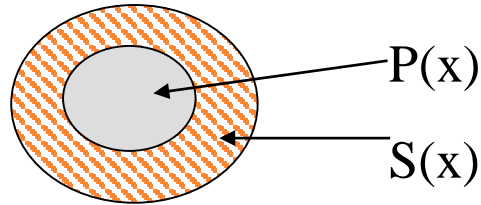
- A **necessary** condition must be satisfied for a statement to be true.
- A **sufficient** condition, if satisfied, assures the statement's truth.
- Duality: "P is sufficient for Q" is the same as "Q is necessary for P."
- Examples: define $\text{father}(x, y)$ by $\text{parent}(x, y)$ and $\text{male}(x)$
 - $\text{parent}(x, y)$ is a necessary (**but not sufficient**) description of $\text{father}(x, y)$
 - $\text{father}(x, y) \rightarrow \text{parent}(x, y)$
 - $\text{parent}(x, y) \wedge \text{male}(x) \wedge \text{age}(x, 35)$ is a **sufficient (but not necessary)** description of $\text{father}(x, y)$:
$$\text{father}(x, y) \leftarrow \text{parent}(x, y) \wedge \text{male}(x) \wedge \text{age}(x, 35)$$
 - $\text{parent}(x, y) \wedge \text{male}(x)$ is a **necessary and sufficient** description of $\text{father}(x, y)$
$$\text{parent}(x, y) \wedge \text{male}(x) \leftrightarrow \text{father}(x, y)$$





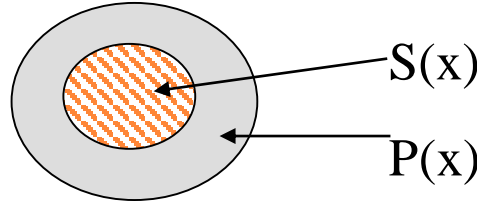
MORE ON DEFINITIONS

$S(x)$ is a
necessary
condition of $P(x)$



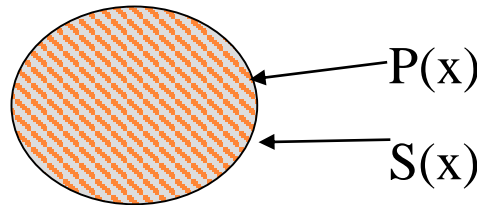
$$(\forall x) P(x) \Rightarrow S(x)$$

$S(x)$ is a
sufficient
condition of $P(x)$



$$(\forall x) P(x) \Leftarrow S(x)$$

$S(x)$ is a
necessary and
sufficient
condition of $P(x)$



$$(\forall x) P(x) \Leftrightarrow S(x)$$





HIGHER-ORDER LOGIC

- FOL only allows to quantify over variables, and variables can only range over objects.
- HOL allows us to quantify over relations
- Example: (quantify over functions)
“two functions are equal iff they produce the same value for all arguments”
$$\forall f \forall g (f = g) \leftrightarrow (\forall x f(x) = g(x))$$
- Example: (quantify over predicates)
$$\forall r \text{ transitive}(r) \rightarrow (\forall xyz) r(x,y) \wedge r(y,z) \rightarrow r(x,z))$$
- More expressive, but **undecidable**. (there isn't an effective algorithm to decide whether all sentences are valid)
 - First-order logic is decidable only when it uses predicates with only one argument.






EXPRESSING UNIQUENESS

- Sometimes we want to say that there is a single, unique object that satisfies a certain condition
- “There exists a unique x such that $\text{king}(x)$ is true”
 - $\exists x \text{ king}(x) \wedge \forall y (\text{king}(y) \rightarrow x=y)$
 - $\exists x \text{ king}(x) \wedge \neg \exists y (\text{king}(y) \wedge x \neq y)$
 - $\exists! x \text{ king}(x)$
- “Every country has exactly one ruler”
 - $\forall c \text{ country}(c) \rightarrow \exists! r \text{ ruler}(c,r)$
- Iota operator: “ $\iota x P(x)$ ” means “the unique x such that $p(x)$ is true”
 - “The unique ruler of Freedonia is dead”
 - $\text{dead}(\iota x \text{ ruler}(\text{freedonia},x))$





NOTATIONAL DIFFERENCES

- **Different symbols for *and*, *or*, *not*, *implies*, ...**
 - $\forall \exists \Rightarrow \Leftrightarrow \wedge \vee \neg \bullet \supset$
 - $p \vee (q \wedge r)$
 - $p + (q * r)$
 - etc
 - **Prolog**
cat(X) :- furry(X), meows (X), has(X, claws)
 - **Lispy notations**
(forall ?x (implies (and (furry ?x)
 (meows ?x)
 (has ?x claws))
 (cat ?x))))
- 

LOGICAL AGENTS FOR THE WUMPUS WORLD

Three (non-exclusive) agent architectures:

- Reflex agents
 - Have rules that classify situations, specifying how to react to each possible situation
- Model-based agents
 - Construct an internal model of their world
- Goal-based agents
 - Form goals and try to achieve them





A SIMPLE REFLEX AGENT

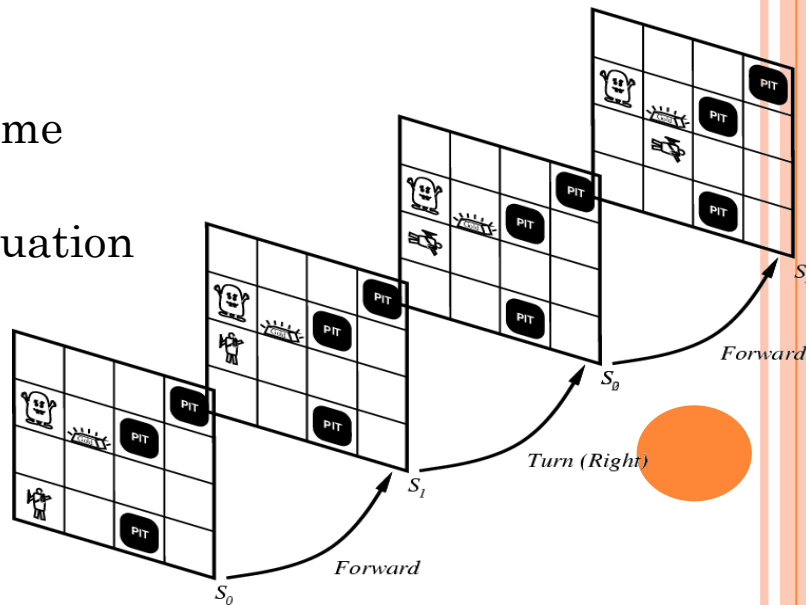
- Rules to **map percepts into observations**:
 - $\forall b, g, u, c, t \text{ Percept}([Stench, b, g, u, c], t) \rightarrow Stench(t)$
 - $\forall s, g, u, c, t \text{ Percept}([s, Breeze, g, u, c], t) \rightarrow Breeze(t)$
 - $\forall s, b, u, c, t \text{ Percept}([s, b, Glitter, u, c], t) \rightarrow AtGold(t)$
- Rules to **select an action given observations**:
 - $\forall t \text{ AtGold}(t) \rightarrow \text{Action}(\text{Grab}, t);$
- Some difficulties:
 - Consider Climb. There is no percept that indicates the agent should climb out – **position and holding gold are not part of the percept sequence**
 - Loops – the percept will be repeated when you return to a square, which should cause the same response (unless we maintain some **internal model of the world**)





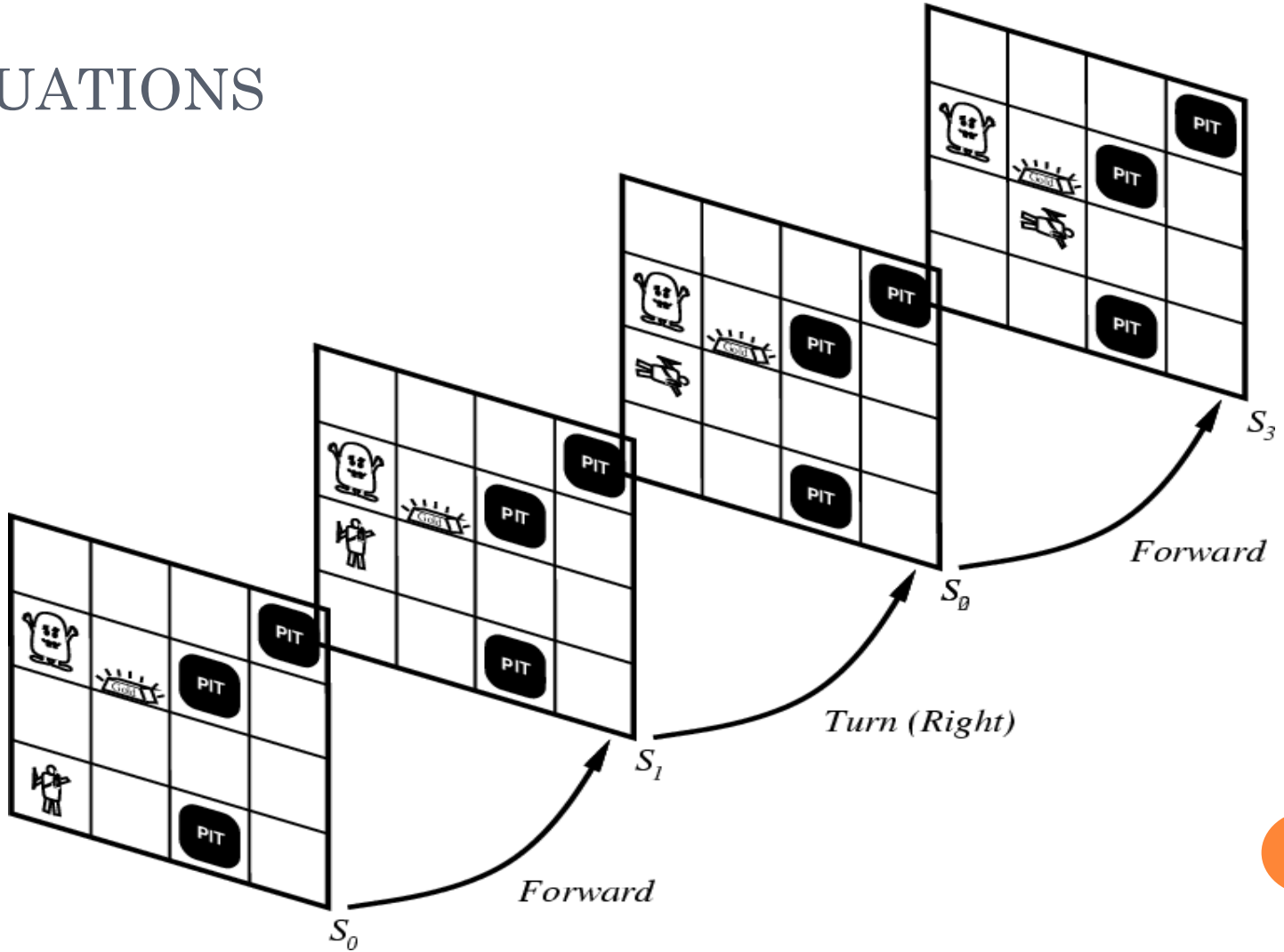
REPRESENTING CHANGE

- Representing change in the world in logic can be tricky.
- One way is just to change the KB
 - Add and delete sentences from the KB to reflect changes
 - How do we remember the past, or reason about changes?
- **Situation calculus** is another way
- A **situation** is a snapshot of the world at some instant in time
- When the agent performs an action A in situation S_1 , the result is a new situation S_2 .





SITUATIONS





SITUATION CALCULUS

- A **situation** is a snapshot of the world at an interval of time during which nothing changes
- Every true or false statement is made with respect to a particular situation.
 - Add **situation variables** to every predicate.
 - $\text{at}(\text{Agent}, 1, 1)$ becomes $\text{at}(\text{Agent}, 1, 1, s_0)$: $\text{at}(\text{Agent}, 1, 1)$ is true in situation (i.e., state) s_0 .
 - Alternatively, add a special 2nd-order predicate, **holds(f,s)**, that means “f is true in situation s.” E.g., $\text{holds}(\text{at}(\text{Agent}, 1, 1), s_0)$
- Add a new function, **result(a,s)**, that maps a situation s into a new situation as a result of performing action a. For example, $\text{result}(\text{forward}, s)$ is a function that returns the successor state (situation) to s
- Example: The action agent-walks-to-location-y could be represented by
 - $(\forall x)(\forall y)(\forall s) (\text{at}(\text{Agent}, x, s) \wedge \neg \text{onbox}(s)) \rightarrow \text{at}(\text{Agent}, y, \text{result}(\text{walk}(y), s))$





DEDUCING HIDDEN PROPERTIES

- From the perceptual information we obtain in situations, we can **infer properties of locations**
 - $\forall l, s \text{ at}(\text{Agent}, l, s) \wedge \text{Breeze}(s) \rightarrow \text{Breezy}(l)$
 - $\forall l, s \text{ at}(\text{Agent}, l, s) \wedge \text{Stench}(s) \rightarrow \text{Smelly}(l)$
- Neither Breezy nor Smelly need situation arguments because pits and Wumpuses do not move around





DEDUCING HIDDEN PROPERTIES II

- We need to write some rules that relate various aspects of a single world state (as opposed to across states)
- There are two main kinds of such rules:

- **Causal rules** reflect the assumed direction of causality in the world:

$$(\forall l1, l2, s) \text{ At}(\text{Wumpus}, l1, s) \wedge \text{ Adjacent}(l1, l2) \rightarrow \text{ Smelly}(l2)$$

$$(\forall l1, l2, s) \text{ At}(\text{Pit}, l1, s) \wedge \text{ Adjacent}(l1, l2) \rightarrow \text{ Breezy}(l2)$$

Systems that reason with causal rules are called **model-based reasoning systems**

- **Diagnostic rules** infer the presence of **hidden properties** directly from the percept-derived information. We have already seen two diagnostic rules:

$$(\forall l, s) \text{ At}(\text{Agent}, l, s) \wedge \text{ Breeze}(s) \rightarrow \text{ Breezy}(l)$$

$$(\forall l, s) \text{ At}(\text{Agent}, l, s) \wedge \text{ Stench}(s) \rightarrow \text{ Smelly}(l)$$



REPRESENTING CHANGE:

THE FRAME PROBLEM

- **Frame axioms:** If property x doesn't change as a result of applying action a in state s , then it stays the same.
 - $\text{On}(x, z, s) \wedge \text{Clear}(x, s) \rightarrow$
 $\text{On}(x, \text{table}, \text{Result}(\text{Move}(x, \text{table}), s)) \wedge$
 $\neg \text{On}(x, z, \text{Result}(\text{Move}(x, \text{table}), s))$
 - $\text{On}(y, z, s) \wedge y \neq x \rightarrow \text{On}(y, z, \text{Result}(\text{Move}(x, \text{table}), s))$
 - The proliferation of frame axioms becomes very cumbersome in complex domains





THE FRAME PROBLEM II

- **Successor-state axiom:** General statement that characterizes every way in which a particular predicate can become true:
 - Either it can be **made true**, or it can **already be true and not be changed**:
 - $\text{On}(x, \text{table}, \text{Result}(a, s)) \leftrightarrow$
 $[\text{On}(x, z, s) \wedge \text{Clear}(x, s) \wedge a = \text{Move}(x, \text{table})] \wedge$
 $[\text{On}(x, \text{table}, s) \wedge a \neq \text{Move}(x, z)]$
- In complex worlds, where you want to reason about longer chains of action, even these types of axioms are too cumbersome
 - Planning systems use special-purpose inference methods to reason about the expected state of the world at any point in time during a multi-step plan





QUALIFICATION PROBLEM

- Qualification problem:
 - How can you possibly characterize every single effect of an action, or every single exception that might occur?
 - When I put my bread into the toaster, and push the button, it will become toasted after two minutes, unless...
 - The toaster is broken, or...
 - The power is out, or...
 - I blow a fuse, or...
 - A neutron bomb explodes nearby and fries all electrical components, or...
 - A meteor strikes the earth, and the world we know it ceases to exist, or...





RAMIFICATION PROBLEM

- Similarly, it's just about impossible to characterize every side effect of every action, at every possible level of detail:
 - When I put my bread into the toaster, and push the button, the bread will become toasted after two minutes, and...
 - The crumbs that fall off the bread onto the bottom of the toaster over tray will also become toasted, and...
 - Some of the aforementioned crumbs will become burnt, and...
 - The outside molecules of the bread will become “toasted,” and...
 - The inside molecules of the bread will remain more “breadlike,” and...
 - The toasting process will release a small amount of humidity into the air because of evaporation, and...
 - The heating elements will become a tiny fraction more likely to burn out the next time I use the toaster, and...
 - The electricity meter in the house will move up slightly, and...





KNOWLEDGE ENGINEERING!

- Modeling the “right” conditions and the “right” effects at the “right” level of abstraction is very difficult
- Knowledge engineering (creating and maintaining knowledge bases for intelligent reasoning) is an entire field of investigation
- Many researchers hope that automated knowledge acquisition and machine learning tools can fill the gap:
 - Our intelligent systems should be able to **learn** about the conditions and effects, just like we do!
 - Our intelligent systems should be able to learn when to pay attention to, or reason about, certain aspects of processes, depending on the context!





PREFERENCES AMONG ACTIONS

- A problem with the Wumpus world knowledge base that we have built so far is that it is difficult to decide which action is best among a number of possibilities.
- For example, to decide between a forward and a grab, axioms describing when it is OK to move to a square would have to mention glitter.
- This is not modular!
- We can solve this problem by separating facts about actions from facts about goals. This way our agent can be reprogrammed just by asking it to achieve different goals.





PREFERENCES AMONG ACTIONS

- The first step is to describe the desirability of actions independent of each other.
- In doing this we will use a simple scale: actions can be Great, Good, Medium, Risky, or Deadly.
- Obviously, the agent should always do the best action it can find:

$$(\forall a,s) \text{Great}(a,s) \rightarrow \text{Action}(a,s)$$

$$(\forall a,s) \text{Good}(a,s) \wedge \neg(\exists b) \text{Great}(b,s) \rightarrow \text{Action}(a,s)$$

$$(\forall a,s) \text{Medium}(a,s) \wedge (\neg(\exists b) \text{Great}(b,s) \vee \text{Good}(b,s)) \rightarrow \text{Action}(a,s)$$

...





PREFERENCES AMONG ACTIONS

- We use this action quality scale in the following way.
- Until it finds the gold, the basic strategy for our agent is:
 - Great actions include picking up the gold when found and climbing out of the cave with the gold.
 - Good actions include moving to a square that's OK and hasn't been visited yet.
 - Medium actions include moving to a square that is OK and has already been visited.
 - Risky actions include moving to a square that is not known to be deadly or OK.
 - Deadly actions are moving into a square that is known to have a pit or a Wumpus.





GOAL-BASED AGENTS

- Once the gold is found, it is necessary to change strategies. So now we need a new set of action values.
- We could encode this as a rule:
 - $(\forall s) \text{ Holding}(\text{Gold},s) \rightarrow \text{GoalLocation}([1,1],s)$
- We must now decide how the agent will work out a sequence of actions to accomplish the goal.
- Three possible approaches are:
 - **Inference**: good versus wasteful solutions
 - **Search**: make a problem with operators and set of states
 - **Planning**: to be discussed later



THANK YOU

