SNS COLLEGE OF TECHNOLOGY

Coimbatore-35

An Autonomous Institution

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A++' Grade

Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF INFORMATION TECHNOLOGY

19CSE303 - ARTIFICIAL INTELLIGENCE

III YEAR IV SEM

UNIT IV – **UNCERTAIN KOWLEDGE AND REASONING**

TOPIC – **PROBABLISTIC AND UNCERTAINTY**

Uncertainty


NEXT 36 HOURS
HOURLY
10 DAYS
TONIGHT
CLEAR
LOW
20°
0%
THU
HIGH
36°
0%
THU NIGHT
LOW
25°
0%
FRI
HIGH
46°
0%
FRI NIGHT
LOW
32°
20%

Probability

Possible Worlds

ω

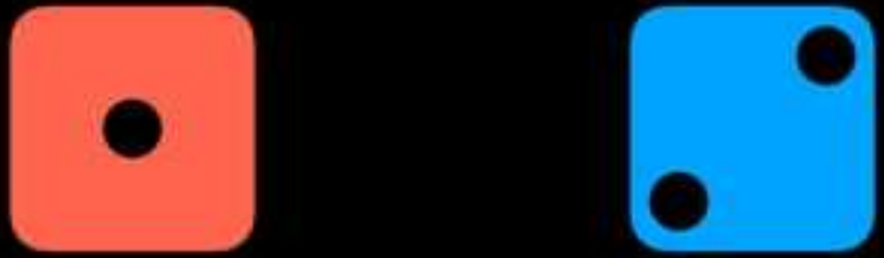
$$P(\omega)$$

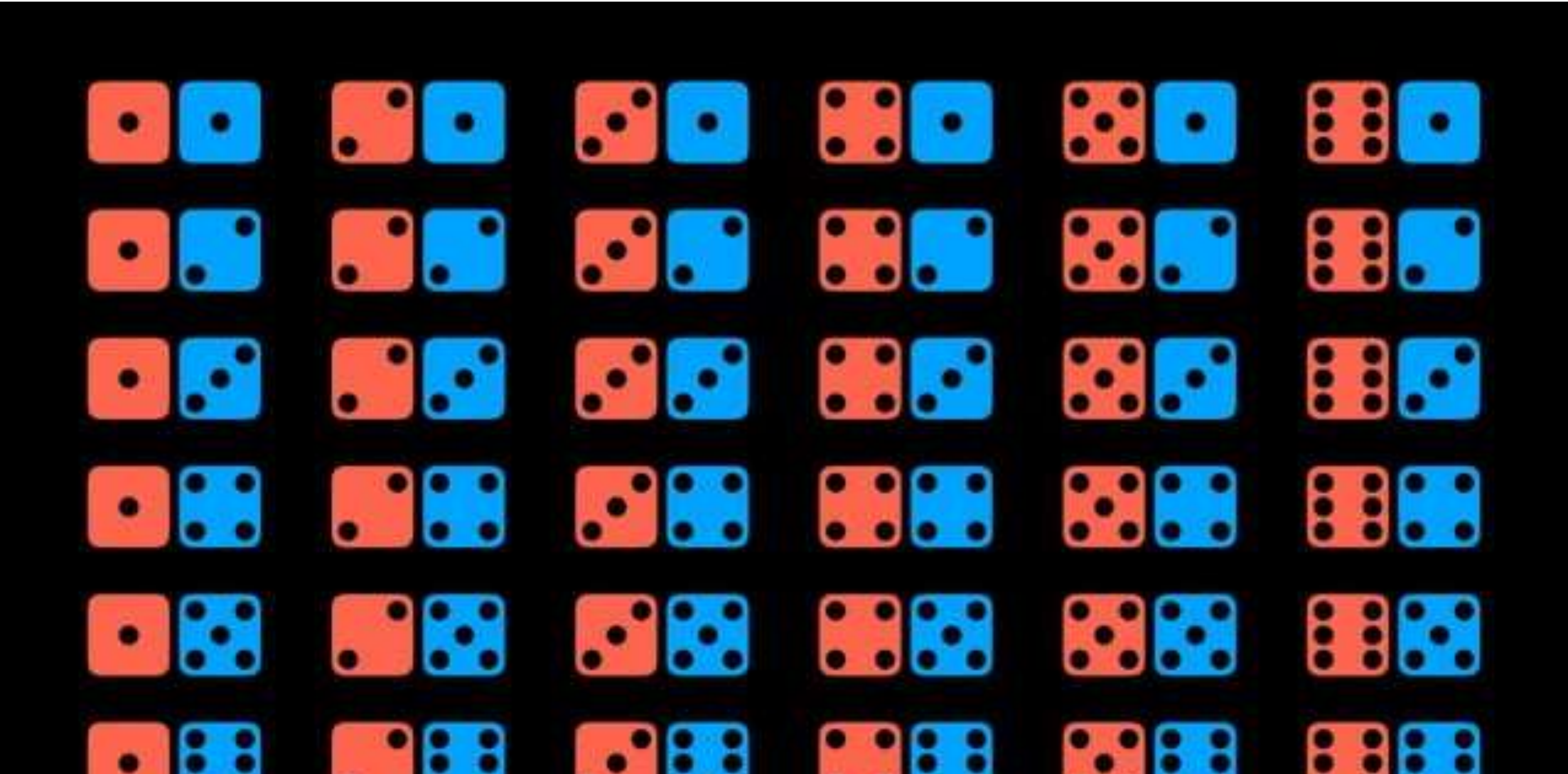
$$0 \le P(\omega) \le 1$$

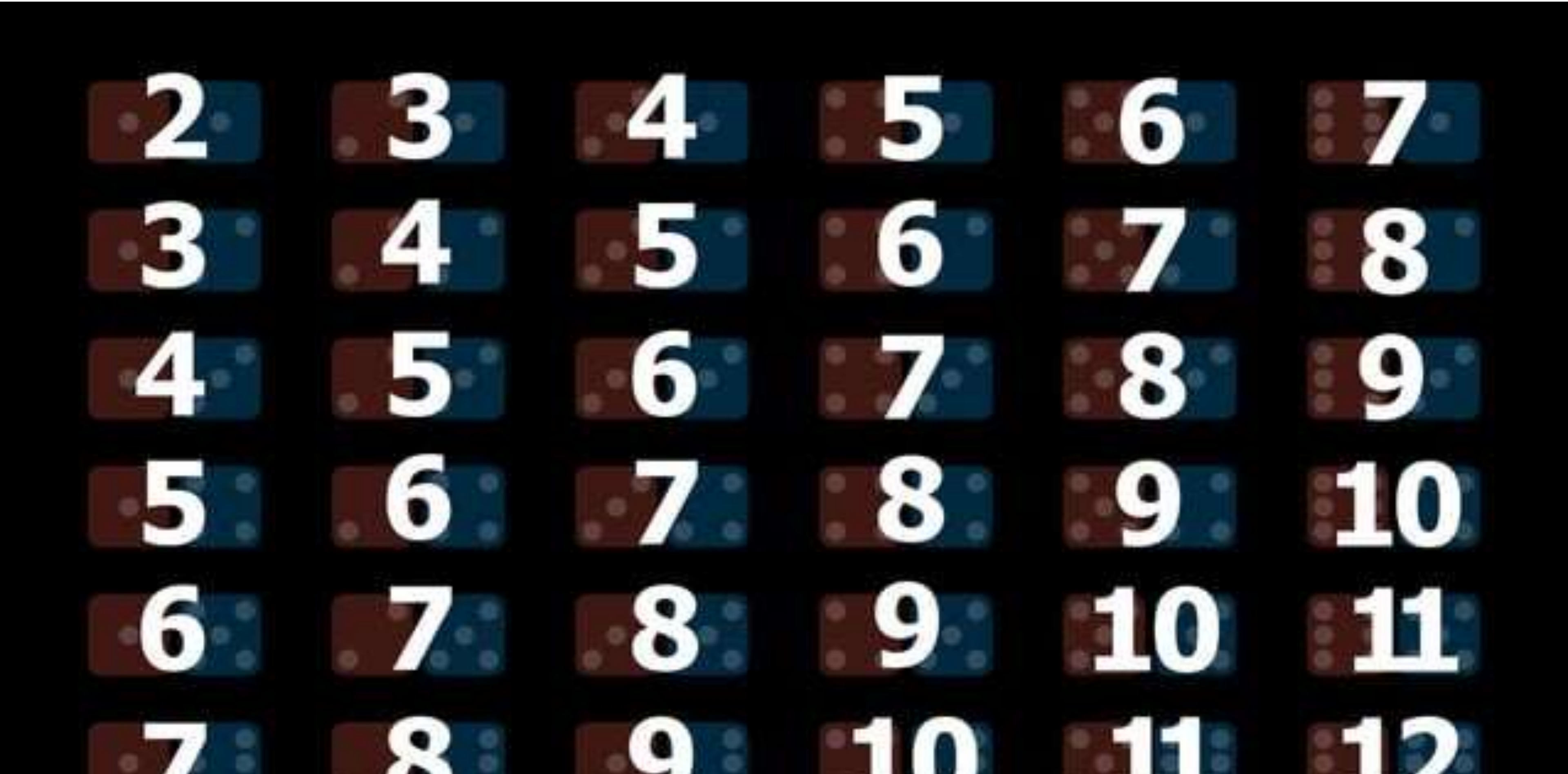
$$\sum_{\omega \in \Omega} P(\omega) = 1$$

$\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$

$P($  $) = \frac{1}{6}$






2 3 4 5 6 7
3 4 5 6 7 8
4 5 6 7 8 9
5 6 7 8 9 10
6 7 8 9 10 11
7 8 9 10 11 12

2	3	4	5	6	7
3	4	5	6	7	8
4	5	6	7	8	9
5	6	7	8	9	10
6	7	8	9	10	11
7	8	9	10	11	12

2	3	4	5	6	7
3	4	5	6	7	8
4	5	6	7	8	9
5	6	7	8	9	10
6	7	8	9	10	11
7	8	9	10	11	12

$$P(\textit{sum to } 12) = \frac{1}{36}$$

$$P(\textit{sum to } 7) = \frac{6}{36} = \frac{1}{6}$$

unconditional probability

degree of belief in a proposition
in the absence of any other evidence

conditional probability

degree of belief in a proposition given some evidence that has already been revealed conditional probability

$$P(a \mid b)$$

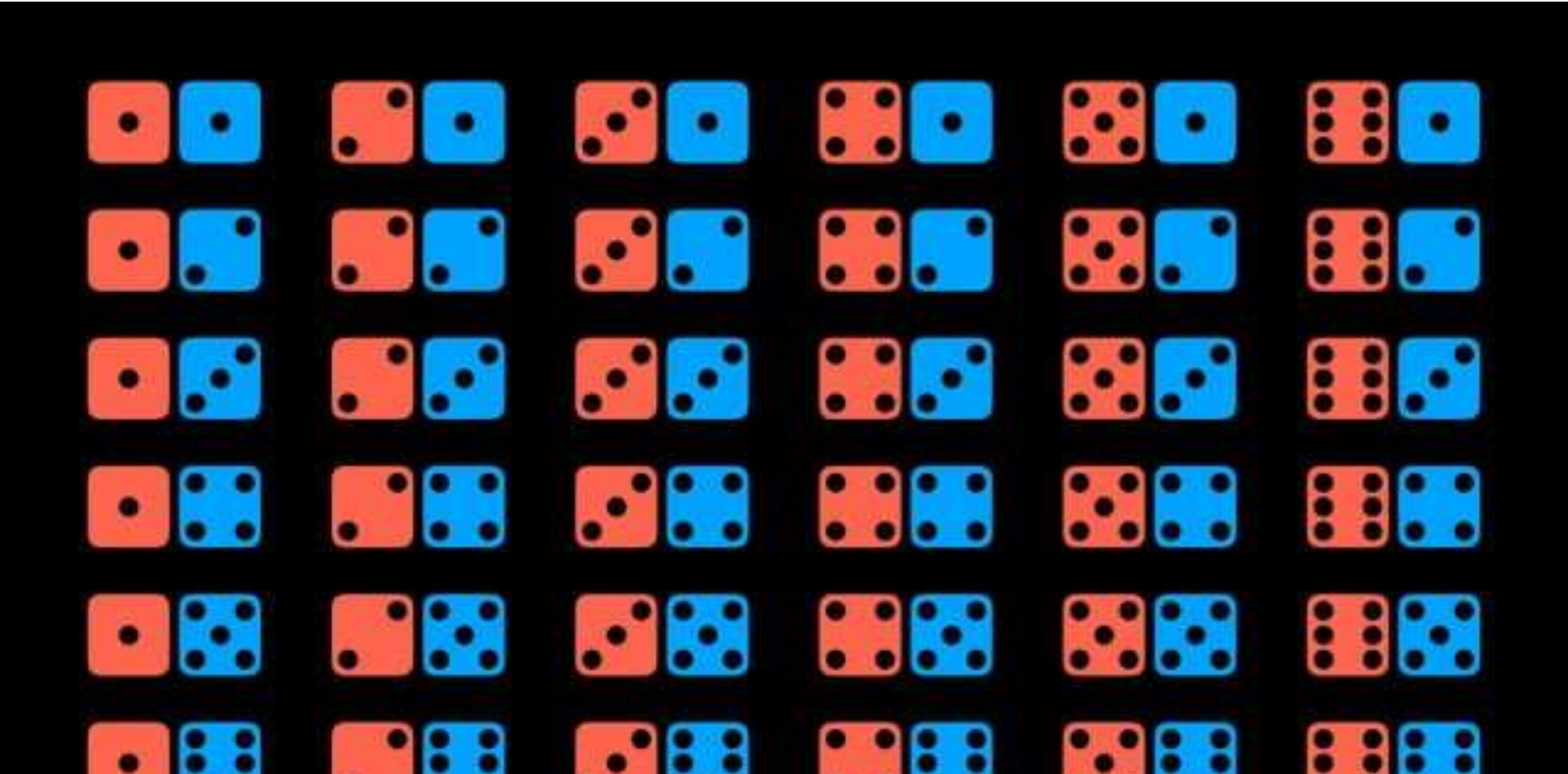
$$P(rain\ today \mid rain\ yesterday)$$

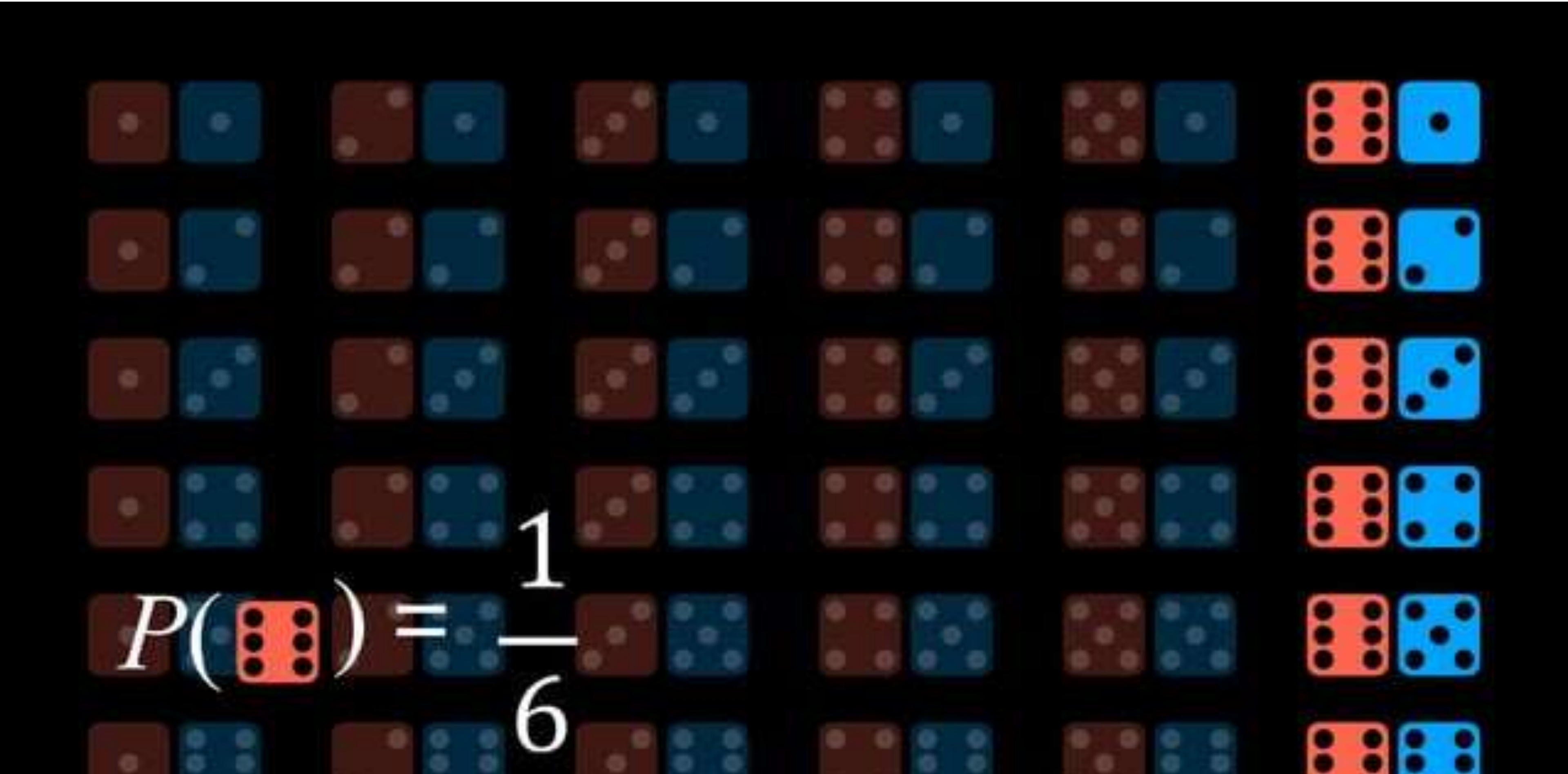
$$P(route\ change \mid traffic\ conditions)$$

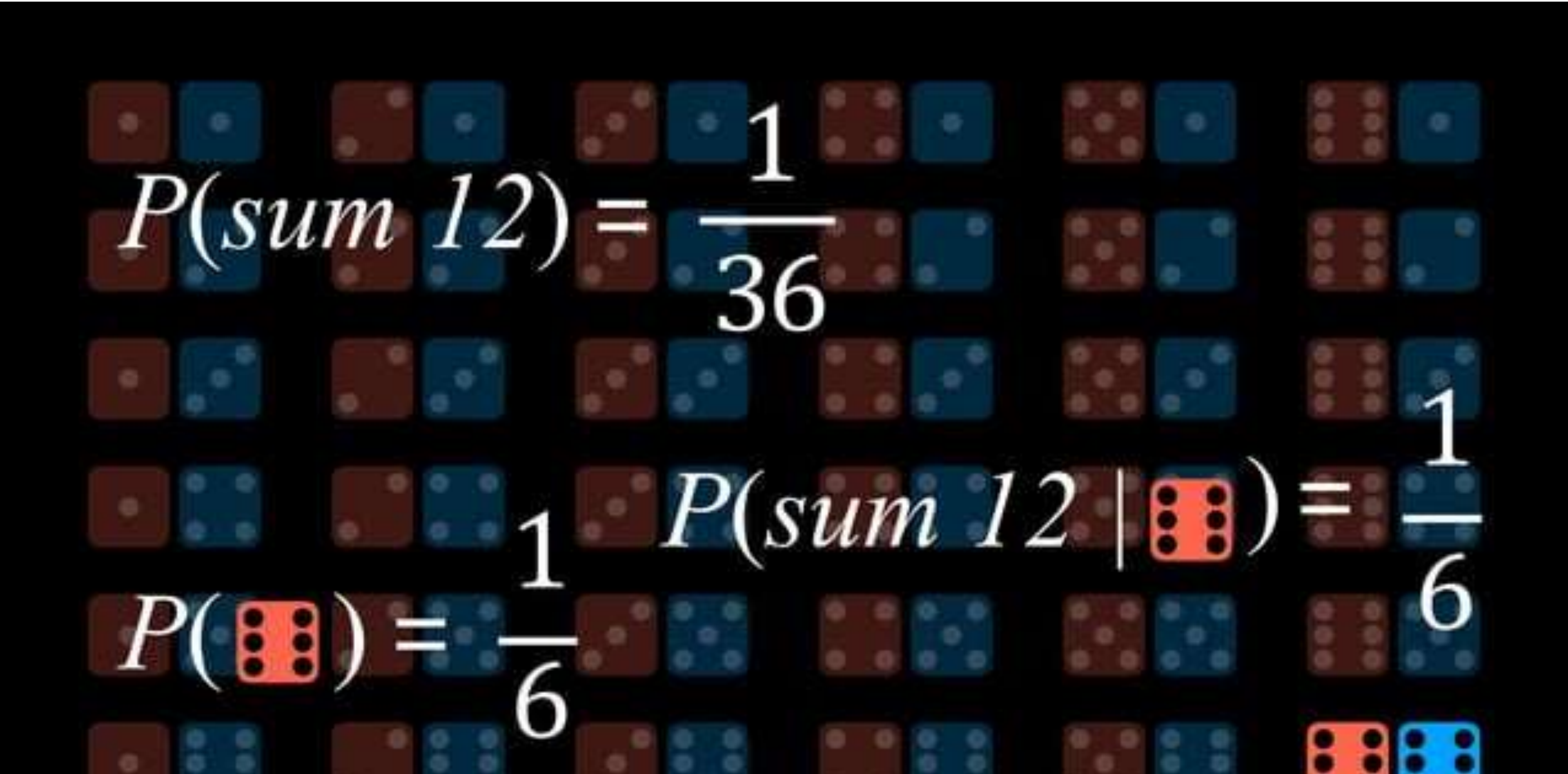
$$P(disease \mid test\ results)$$

$$P(a \mid b) = \frac{P(a \wedge b)}{P(b)}$$

$P(sum\ 12 \mid$  $)$




P( ) = 1/6


P(sum 12) = 1/36
P(sum 12 |) = 1/6
P( ) = 1/6

$$P(a \mid b) = \frac{P(a \wedge b)}{P(b)}$$

$$P(a \wedge b) = P(b)P(a \mid b)$$

$$P(a \wedge b) = P(a)P(b \mid a)$$

random variable

a variable in probability theory with a domain of possible values it can take on

random variable

$Roll$

$\{1, 2, 3, 4, 5, 6\}$

random variable

Weather

$\{sun, cloud, rain, wind, snow\}$

random variable

Traffic

$\{none, light, heavy\}$

random variable

Flight

$\{on\ time,\ delayed,\ cancelled\}$

probability distribution

$$P(Flight = on\ time) = 0.6$$
$$P(Flight = delayed) = 0.3$$
$$P(Flight = cancelled) = 0.1$$

probability distribution

$$\mathbf{P}(Flight) = \langle 0.6, 0.3, 0.1 \rangle$$

independence

the knowledge that one event occurs does not affect the probability of the other event

independence

$$P(a \wedge b) = P(a)P(b \mid a)$$

independence

$$P(a \wedge b) = P(a)P(b)$$

independence

$P($$) = P($$)P($$)$

$$= \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$$

independence

$$P(⚅⚃) \neq P(⚅)P(⚃)$$

$$= \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$$

independence

$$P(⚅⚃) \neq P(⚅)P(⚃ \mid ⚅)$$

$$= \frac{1}{6} \cdot 0 = 0$$

Bayes' Rule

$$P(a \wedge b) = P(b)\, P(a \mid b)$$

$$P(a \wedge b) \qquad P(a)\, P(b \mid a)$$

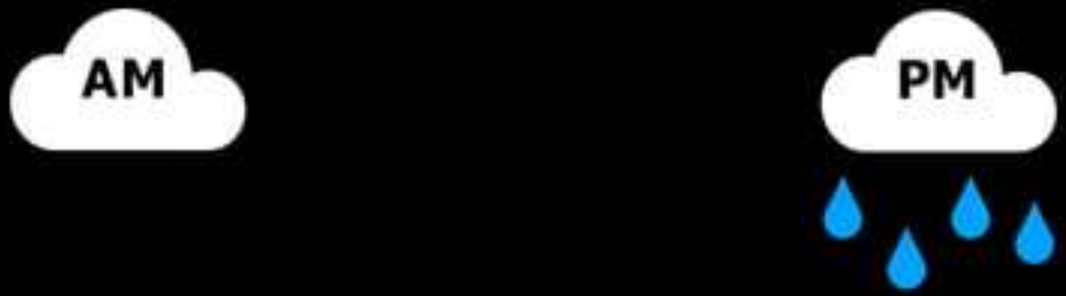
$$P(a)\,P(b \mid a) = P(b)\,P(a \mid b)$$

Bayes' Rule

$$P(b \mid a) = \frac{P(b)\, P(a \mid b)}{P(a)}$$

Bayes' Rule

$$P(b|a) = \frac{P(a|b)\,P(b)}{P(a)}$$



Given clouds in the morning,
what's the probability of rain in the afternoon?

- 80% of rainy afternoons start with cloudy mornings.
- 40% of days have cloudy mornings.
- 10% of days have rainy afternoons

$$P(rain \mid clouds) = \frac{P(clouds \mid rain)P(rain)}{P(clouds)}$$

$$= \frac{(.8)(.1)}{.4}$$

$$= 0.2$$

Knowing

$$P(cloudy\ morning \mid rainy\ afternoon)$$

we can calculate

$$P(rainy\ afternoon \mid cloudy\ morning)$$

Knowing

$$P(\textit{visible effect} \mid \textit{unknown cause})$$

we can calculate

$$P(\textit{unknown cause} \mid \textit{visible effect})$$

Knowing

$$P(medical\ test\ result \mid disease)$$

we can calculate

$$P(disease \mid medical\ test\ result)$$

Knowing

$$P(blurry\ text \mid counterfeit\ bill)$$

we can calculate

$$P(counterfeit\ bill \mid blurry\ text)$$

Joint Probability

C = *cloud*	C = ¬*cloud*
0.4	0.6

R = *rain*	R = ¬*rain*
0.1	0.9

	R = *rain*	R = ¬*rain*
C = *cloud*	0.08	0.32

$$\mathbf{P}(C \mid \text{rain})$$

$$\mathbf{P}(C \mid \text{rain}) = \frac{\mathbf{P}(C, \text{rain})}{P(\text{rain})} = \alpha \mathbf{P}(C, \text{rain})$$

$$= \alpha \langle 0.08, 0.02 \rangle = \langle 0.8, 0.2 \rangle$$

	$R = rain$	$R = \neg rain$
$C = cloud$	0.08	0.32

Probability Rules

Negation

$$P(\neg a) = 1 - P(a)$$

Inclusion-Exclusion

$$P(a \vee b) = P(a) + P(b) - P(a \wedge b)$$

Marginalization

$$P(a) = P(a, b) + P(a, \neg b)$$

Marginalization

$$P(X = x_i) = \sum_j P(X = x_i, Y = y_j)$$

Marginalization

	R = *rain*	R = ¬*rain*
C = *cloud*	0.08	0.32
C = ¬*cloud*	0.02	0.58

$$P(\mathrm{C} = cloud)$$
$$= P(\mathrm{C} = cloud, R = rain) + P(\mathrm{C} = cloud, R = \neg rain)$$
$$= 0.08 + 0.32$$
$$= 0.40$$

Conditioning

$$P(a) = P(a \mid b)P(b) + P(a \mid \neg b)P(\neg b)$$

Conditioning

$$P(X = x_i) = \sum_j P(X = x_i \mid Y = y_j) P(Y = y_j)$$

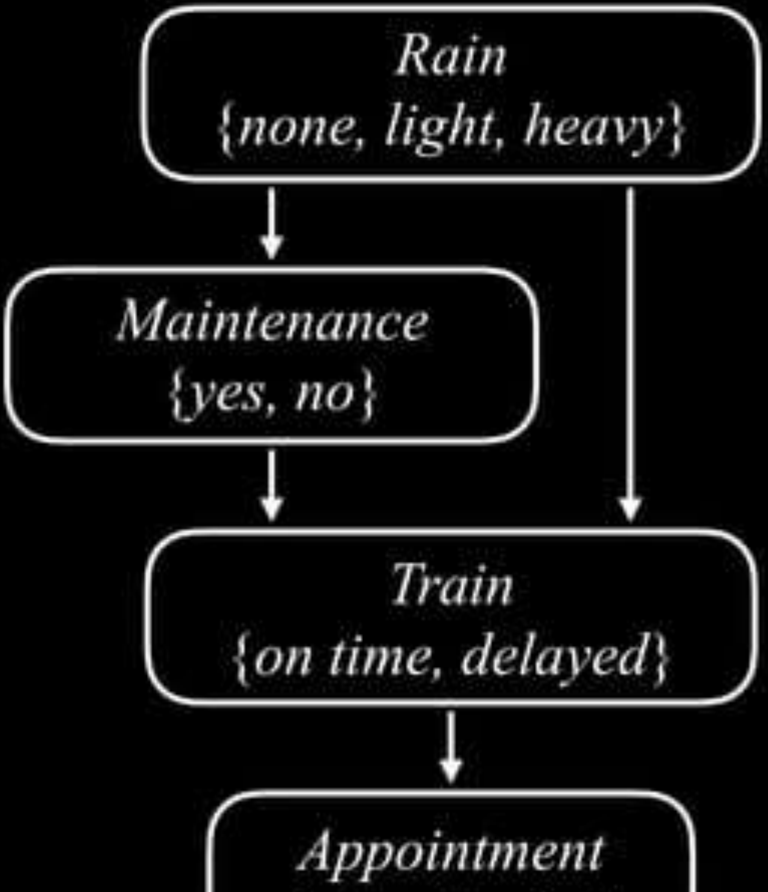
Bayesian Networks

Bayesian network

data structure that represents the dependencies among random variables

Bayesian network

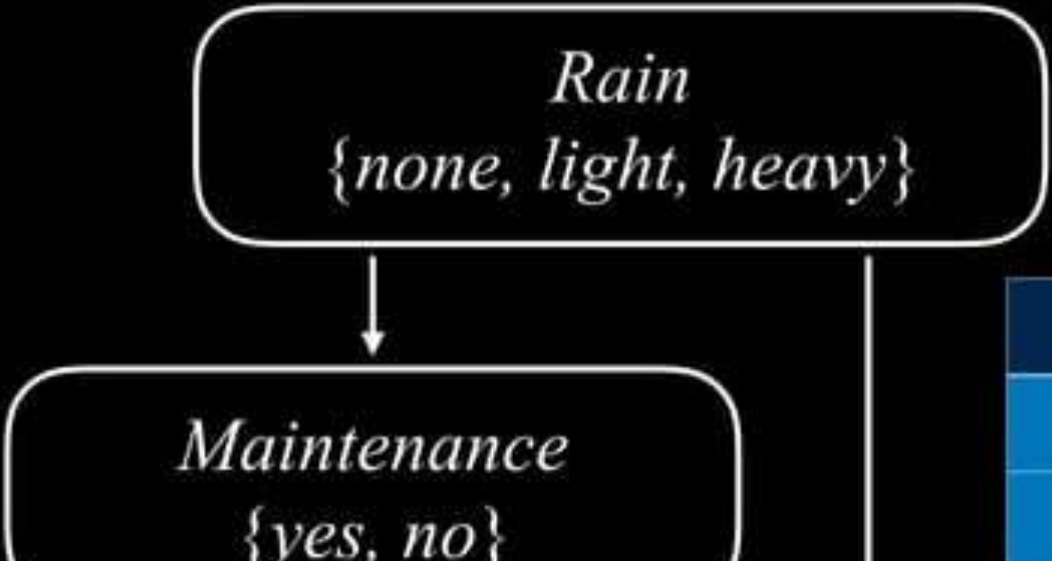
- directed graph
- each node represents a random variable
- arrow from X to Y means X is a parent of Y
- each node X has probability distribution $\mathbf{P}(X \mid Parents(X))$


Rain
{none, light, heavy}
Maintenance
{yes, no}
Train
{on time, delayed}
Appointment

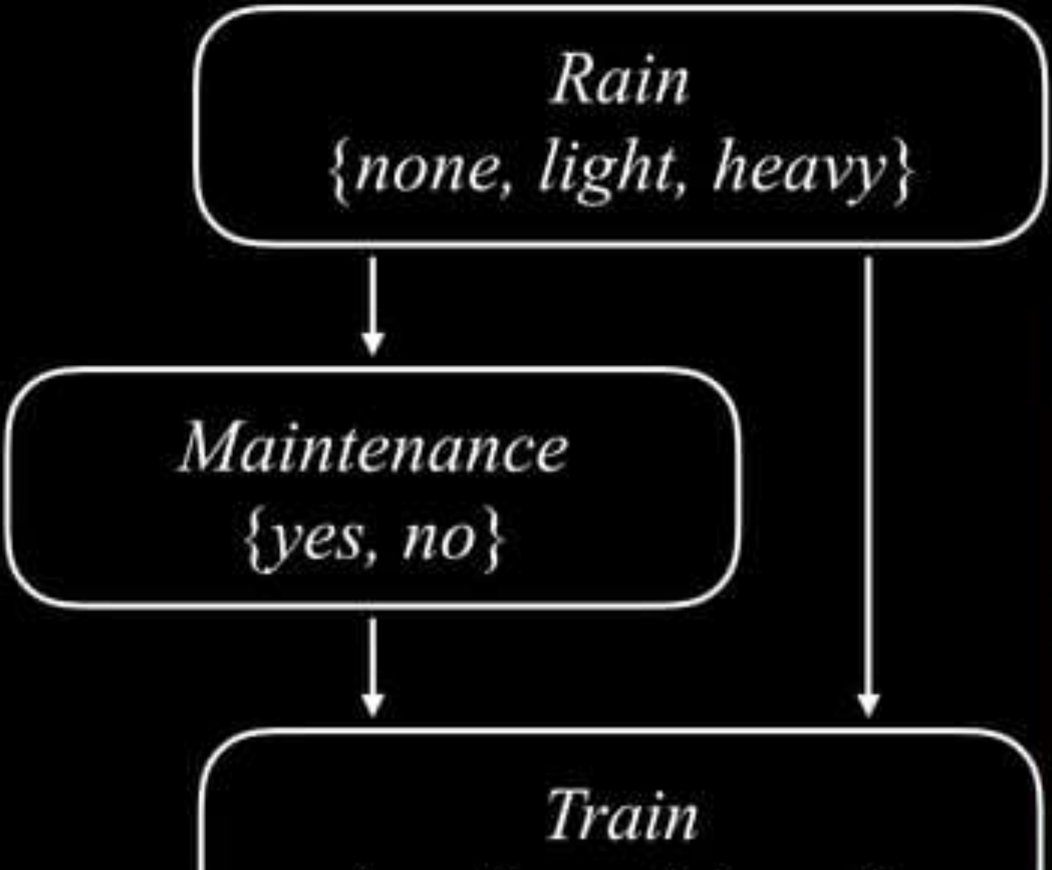
Rain

{none, light, heavy}

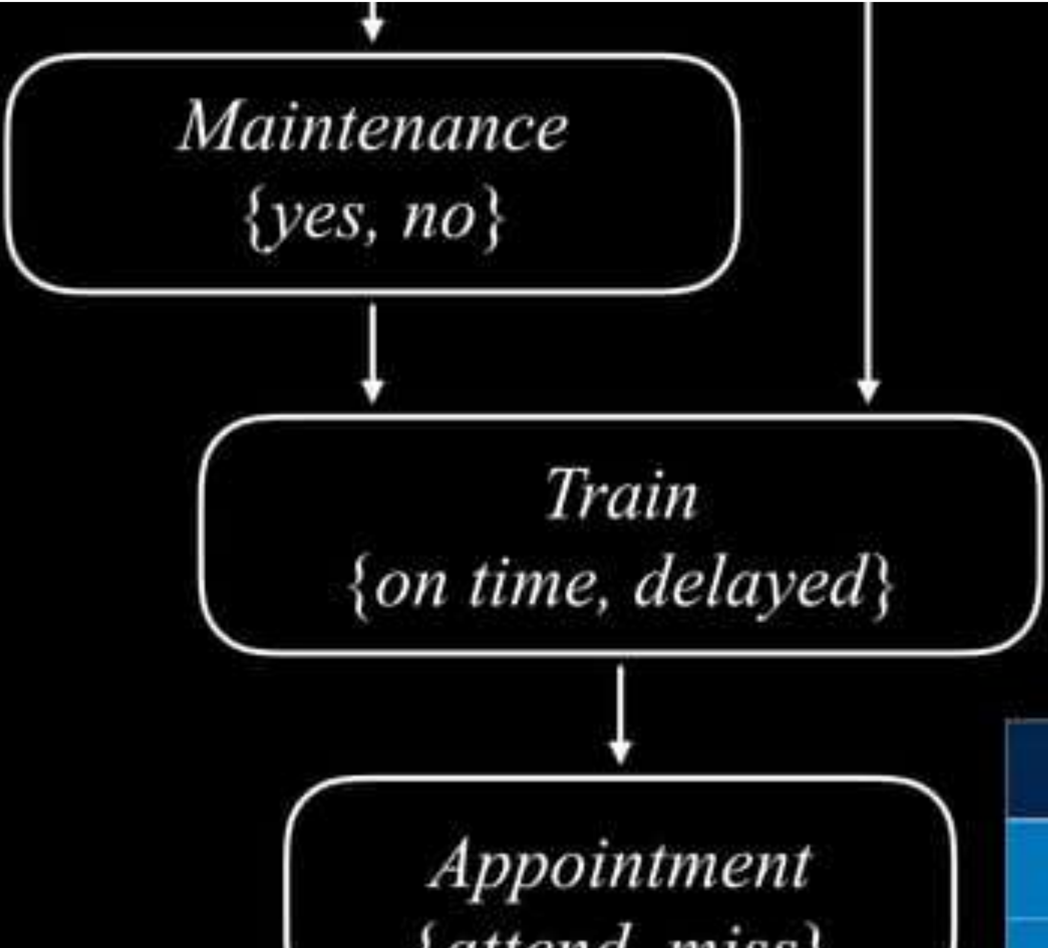
none	light	heavy
0.7	0.2	0.1



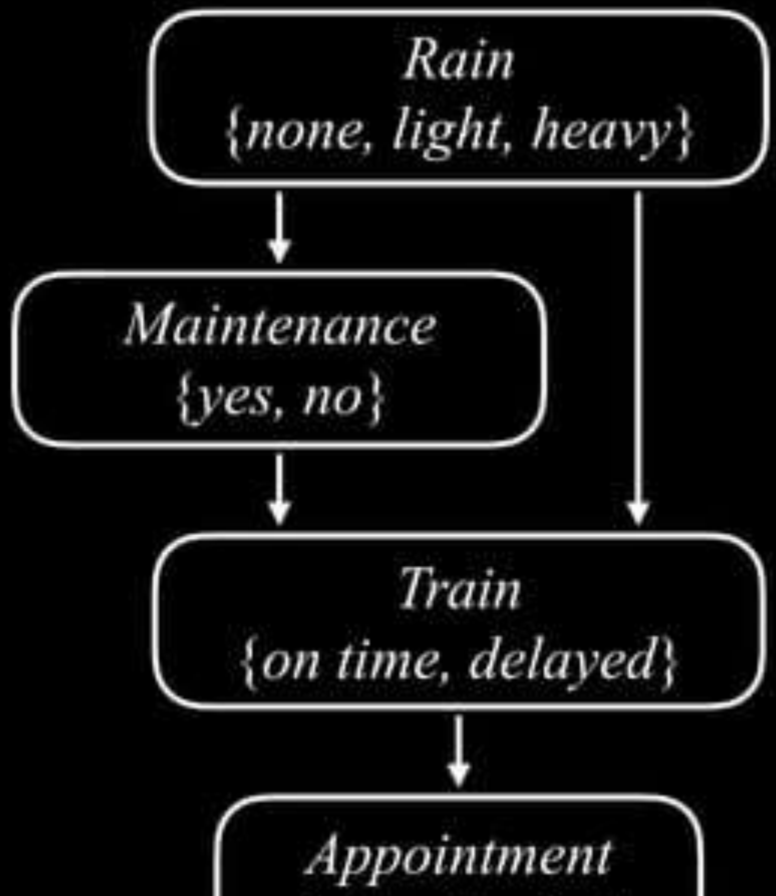
R	yes	no
none	0.4	0.6
light	0.2	0.8



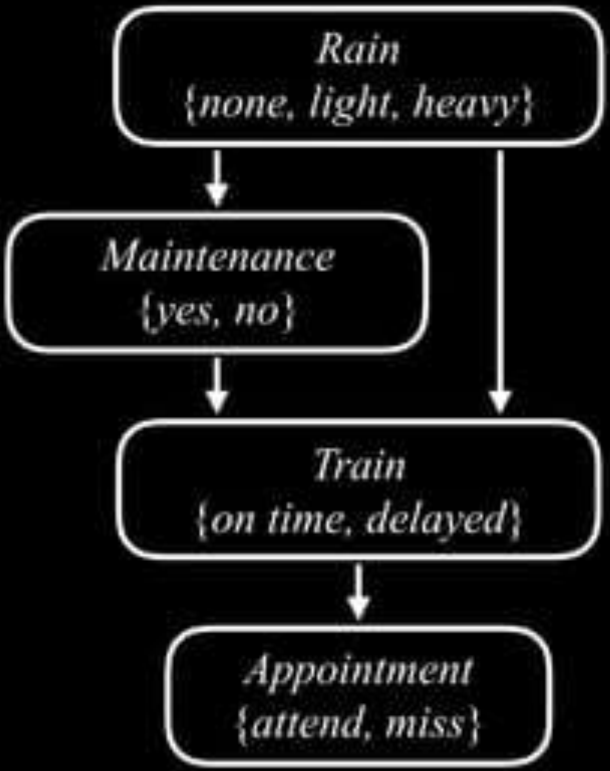
R	*M*	*on time*	*delayed*
none	*yes*	0.8	0.2
none	*no*	0.9	0.1
light	*yes*	0.6	0.4
light	*no*	0.7	0.3
heavy	*yes*	0.4	0.6



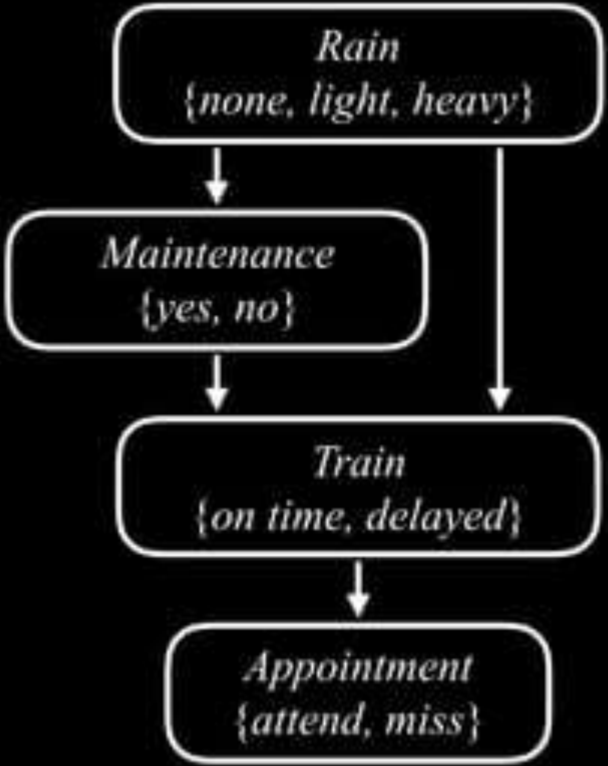
T	attend	miss
on time	0.9	0.1


Rain
{none, light, heavy}
Maintenance
{yes, no}
Train
{on time, delayed}
Appointment

Computing Joint Probabilities



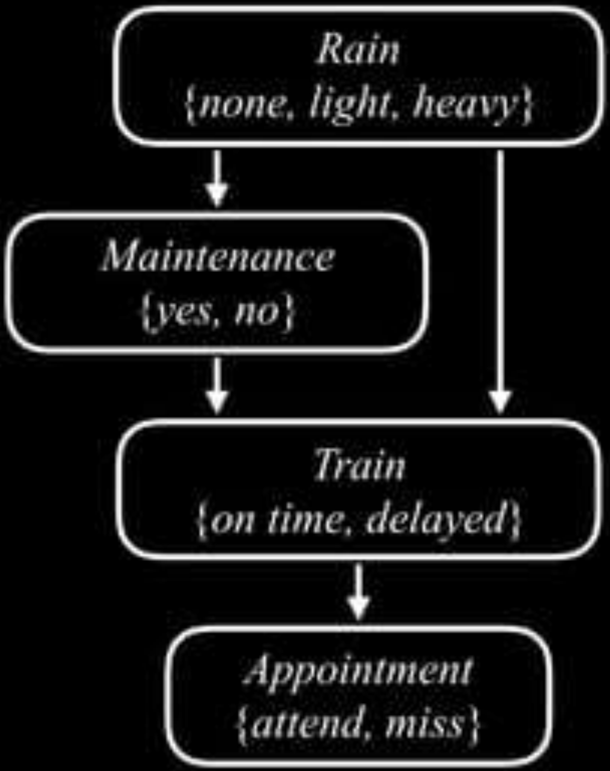
$$P(light)$$



Computing Joint Probabilities

$$P(light, no)$$

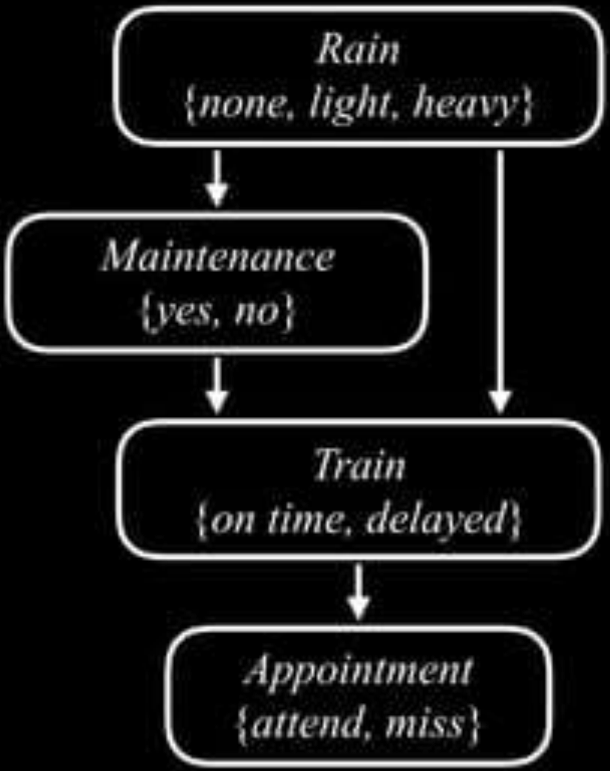
Computing Joint Probabilities



$$P(light, no, delayed)$$

$$P(light)\,P(no \mid light)\,P(delayed \mid light, no)$$

Computing Joint Probabilities



$$P(light, no, delayed, miss)$$

$$P(light)\, P(no \mid light)\, P(delayed \mid light, no)\, P(miss \mid delayed)$$

Inference

Inference

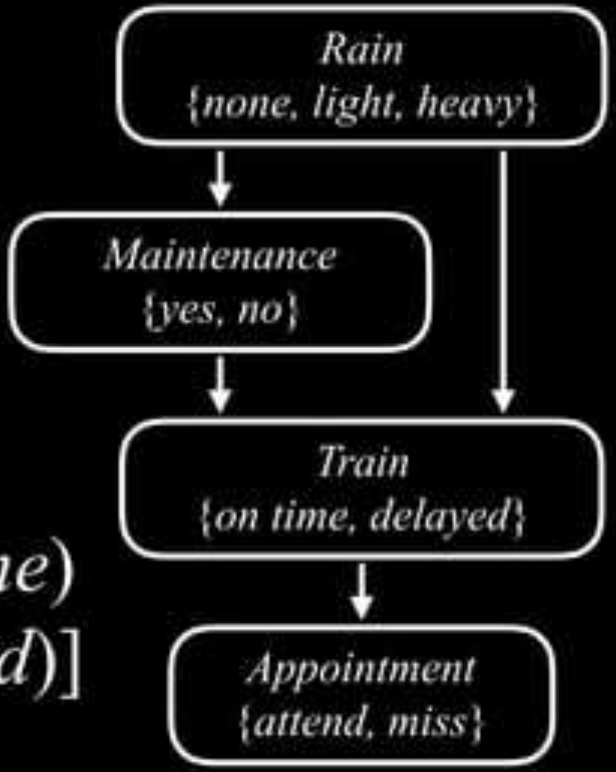
- Query $\mathbf{X}$: variable for which to compute distribution
- Evidence variables $\mathbf{E}$: observed variables for event $\mathbf{e}$
- Hidden variables $\mathbf{Y}$: non-evidence, non-query variable.

- Goal: Calculate $\mathbf{P}(X \mid \mathbf{e})$

$$P(\text{Appointment} \mid \textit{light}, \textit{no})$$

$$= \alpha\, P(\text{Appointment}, \textit{light}, \textit{no})$$

$$= \alpha\, [P(\text{Appointment}, \textit{light}, \textit{no}, \textit{on time}) + P(\text{Appointment}, \textit{light}, \textit{no}, \textit{delayed})]$$



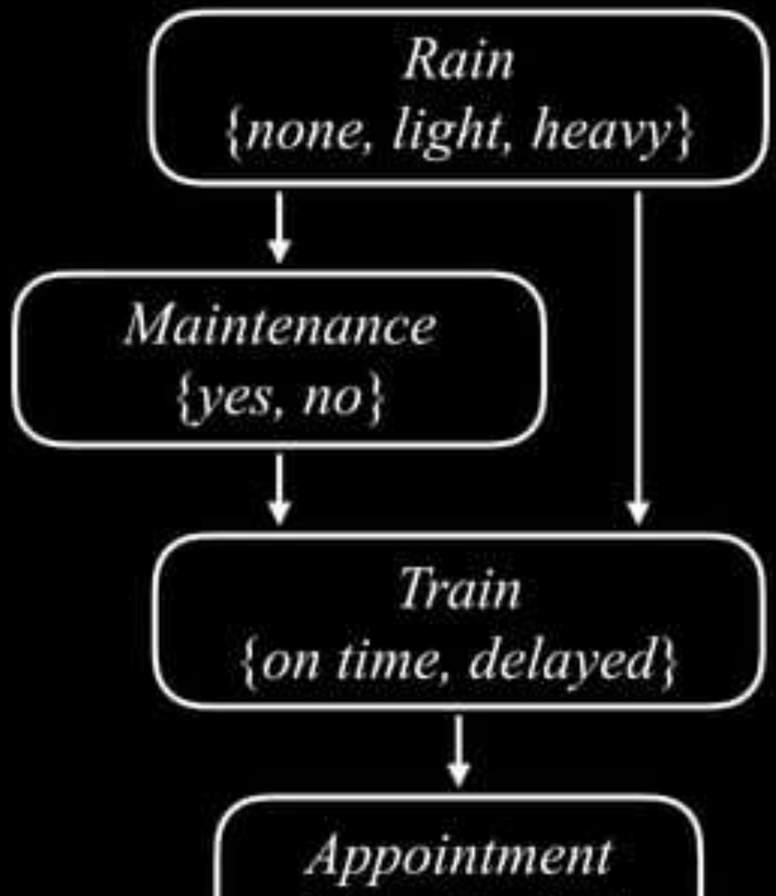
Inference by Enumeration

$$\mathbf{P}(X \mid \mathbf{e}) = \alpha\, \mathbf{P}(X, \mathbf{e}) = \alpha \sum_{\mathbf{y}} \mathbf{P}(X, \mathbf{e}, \mathbf{y})$$

X is the query variable.
e is the evidence.
y ranges over values of hidden variables.

Approximate Inference

Sampling

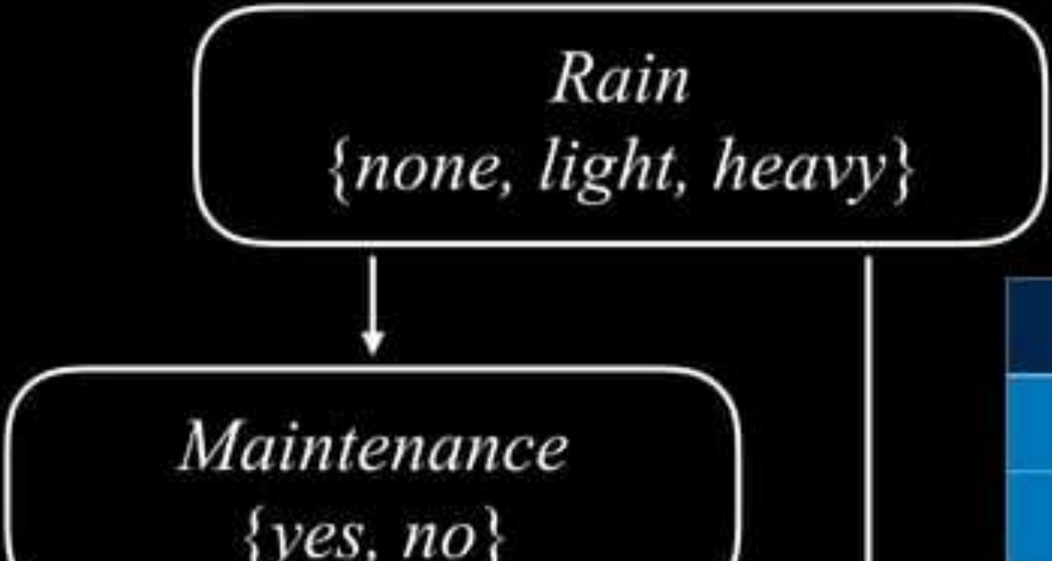

Rain
{none, light, heavy}
Maintenance
{yes, no}
Train
{on time, delayed}
Appointment

R = *none*

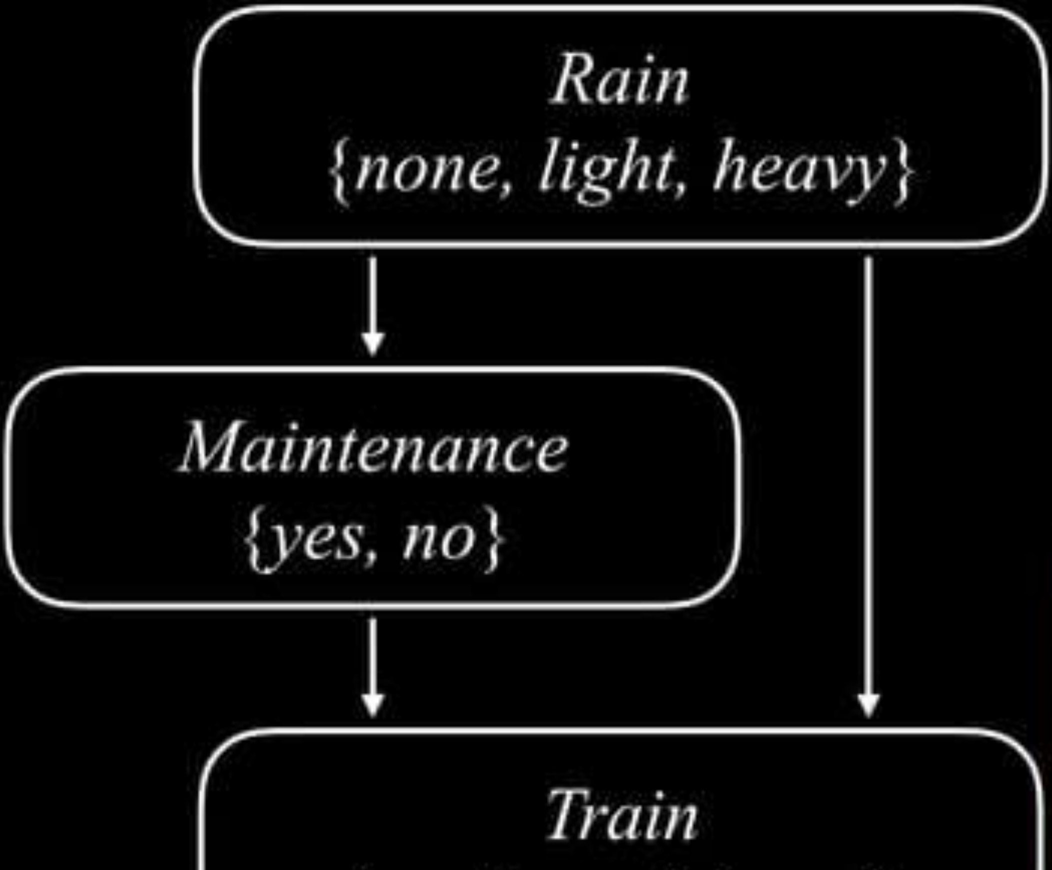
Rain
{*none*, *light*, *heavy*}

none	*light*	*heavy*
0.7	0.2	0.1

R = *none*
M = *yes*

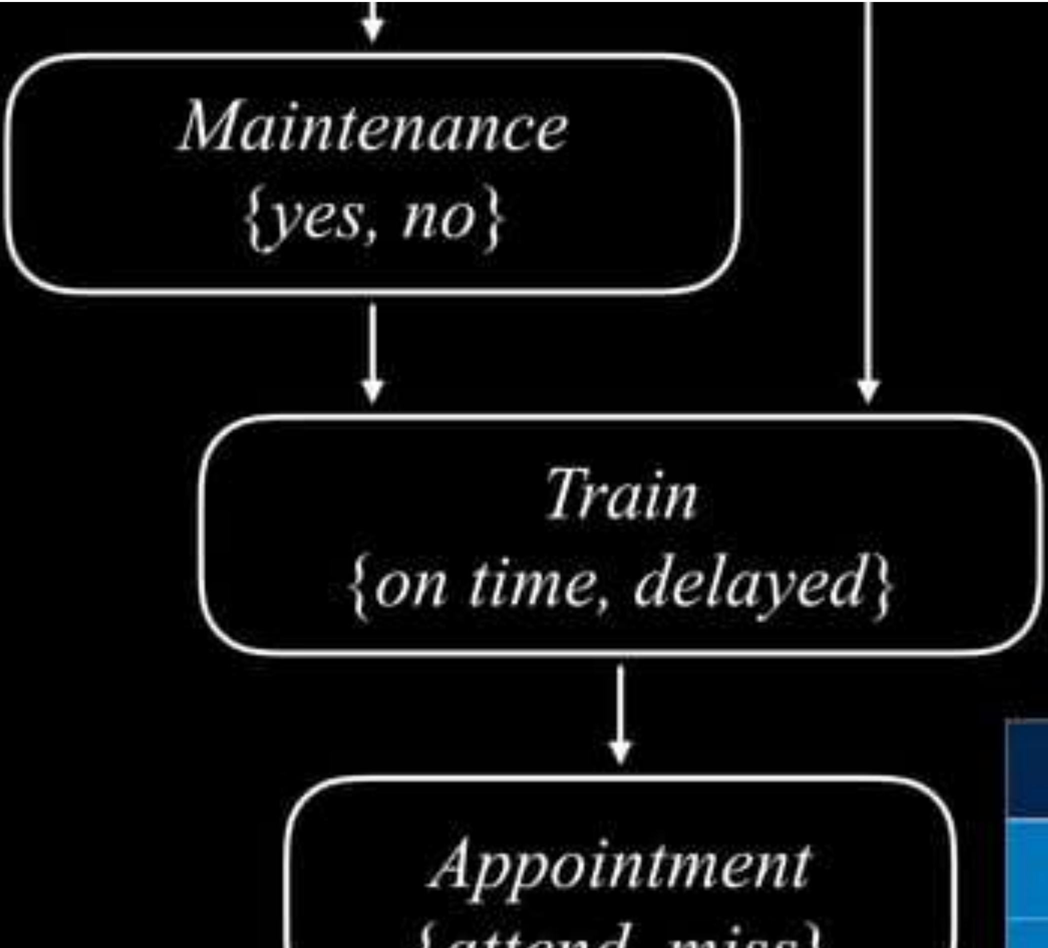


R	*yes*	*no*
none	0.4	0.6
light	0.2	0.8



R = *none*
M = *yes*
T = *on time*

R	*M*	*on time*	*delayed*
none	*yes*	0.8	0.2
none	*no*	0.9	0.1
light	*yes*	0.6	0.4
light	*no*	0.7	0.3
heavy	*yes*	0.4	0.6



R = *none*	
M = *yes*	
T = *on time*	
A = *attend*	

T	*attend*	*miss*
on time	0.9	0.1
[illegible]	[illegible]	[illegible]

R = *none*
M = *yes*
T = *on time*
A = *attend*

R = *light*	R = *light*	R = *none*	R = *none*
M = *no*	M = *yes*	M = *no*	M = *yes*
T = *on time*	T = *delayed*	T = *on time*	T = *on time*
A = *miss*	A = *attend*	A = *attend*	A = *attend*

R = *none*	R = *none*	R = *heavy*	R = *light*
M = *yes*	M = *yes*	M = *no*	M = *no*
T = *on time*	T = *on time*	T = *delayed*	T = *on time*
A = *attend*	A = *attend*	A = *miss*	A = *attend*

$$P(Train = on\ time)\ ?$$

R = *light*	R = *light*	R = *none*	R = *none*
M = *no*	M = *yes*	M = *no*	M = *yes*
T = *on time*	T = *delayed*	T = *on time*	T = *on time*
A = *miss*	A = *attend*	A = *attend*	A = *attend*

R = *none*	R = *none*	R = *heavy*	R = *light*
M = *yes*	M = *yes*	M = *no*	M = *no*
T = *on time*	T = *on time*	T = *delayed*	T = *on time*
A = *attend*	A = *attend*	A = *miss*	A = *attend*

R = *light*	R = *light*	R = *none*	R = *none*
M = *no*	M = *yes*	M = *no*	M = *yes*
T = *on time*	T = *delayed*	T = *on time*	T = *on time*
A = *miss*	A = *attend*	A = *attend*	A = *attend*

R = *none*	R = *none*	R = *heavy*	R = *light*
M = *yes*	M = *yes*	M = *no*	M = *no*
T = *on time*	T = *on time*	T = *delayed*	T = *on time*
A = *attend*	A = *attend*	A = *miss*	A = *attend*

$$P(\text{Rain} = \textit{light} \mid \text{Train} = \textit{on time})\ ?$$

R = *light*	R = *light*	R = *none*	R = *none*
M = *no*	M = *yes*	M = *no*	M = *yes*
T = *on time*	T = *delayed*	T = *on time*	T = *on time*
A = *miss*	A = *attend*	A = *attend*	A = *attend*

R = *none*	R = *none*	R = *heavy*	R = *light*
M = *yes*	M = *yes*	M = *no*	M = *no*
T = *on time*	T = *on time*	T = *delayed*	T = *on time*
A = *attend*	A = *attend*	A = *miss*	A = *attend*

R = *light*	R = *light*	R = *none*	R = *none*
M = *no*	M = *yes*	M = *no*	M = *yes*
T = *on time*	T = *delayed*	T = *on time*	T = *on time*
A = *miss*	A = *attend*	A = *attend*	A = *attend*

R = *none*	R = *none*	R = *heavy*	R = *light*
M = *yes*	M = *yes*	M = *no*	M = *no*
T = *on time*	T = *on time*	T = *delayed*	T = *on time*
A = *attend*	A = *attend*	A = *miss*	A = *attend*

R = *light* M = *no* T = *on time* A = *miss*	R = *light* M = *yes* T = *delayed* A = *attend*	R = *none* M = *no* T = *on time* A = *attend*	R = *none* M = *yes* T = *on time* A = *attend*

R = *none* M = *yes* T = *on time* A = *attend*	R = *none* M = *yes* T = *on time* A = *attend*	R = *heavy* M = *no* T = *delayed* A = *miss*	R = *light* M = *no* T = *on time* A = *attend*

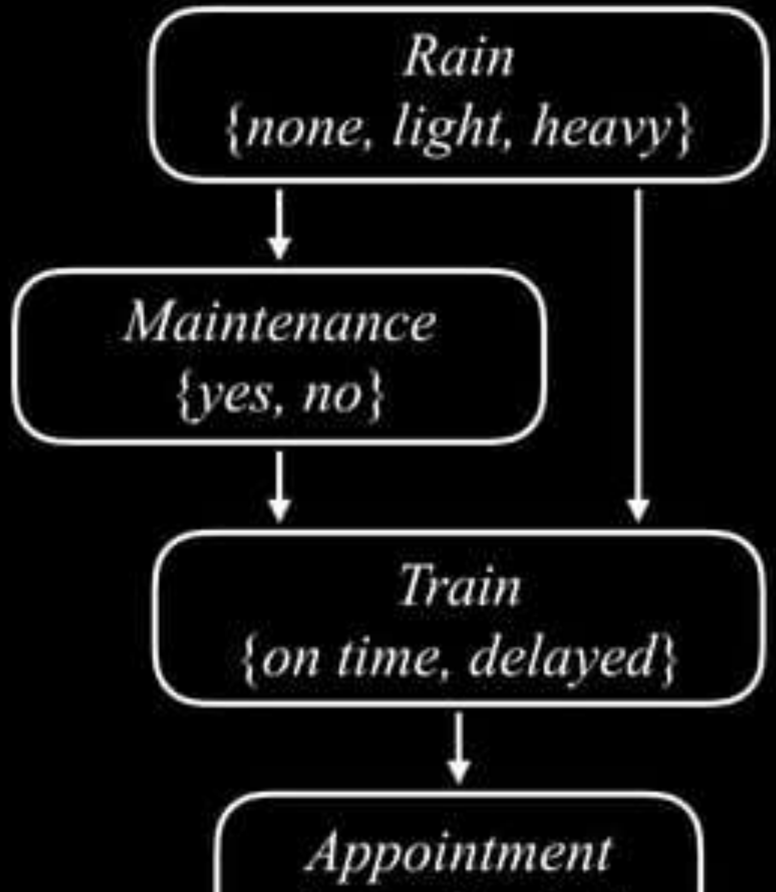
Rejection Sampling

Likelihood Weighting

Likelihood Weighting

- Start by fixing the values for evidence variables.
- Sample the non-evidence variables using conditional probabilities in the Bayesian Network.
- Weight each sample by its likelihood: the probability of all of the evidence.

$$P(\text{Rain} = \textit{light} \mid \text{Train} = \textit{on time})\ ?$$



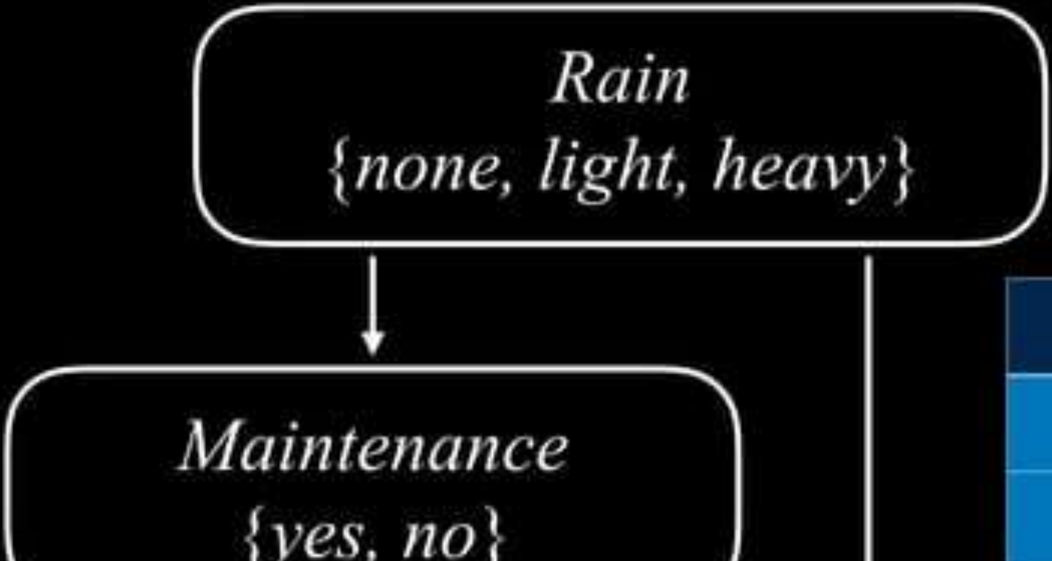
Rain
{none, light, heavy}
Maintenance
{yes, no}
Train
{on time, delayed}
Appointment

R = *light*
T = *on time*

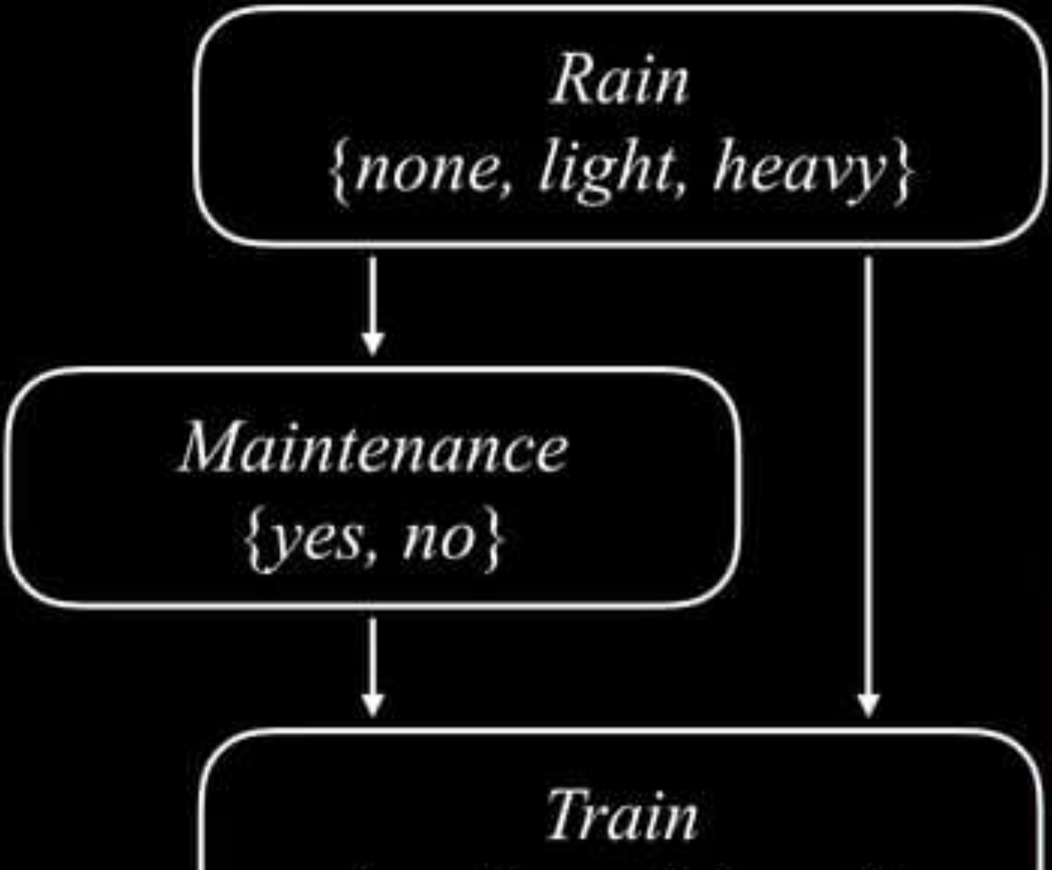
Rain
{*none*, *light*, *heavy*}

none	*light*	*heavy*
0.7	0.2	0.1

R = *light*
M = *yes*
T = *on time*

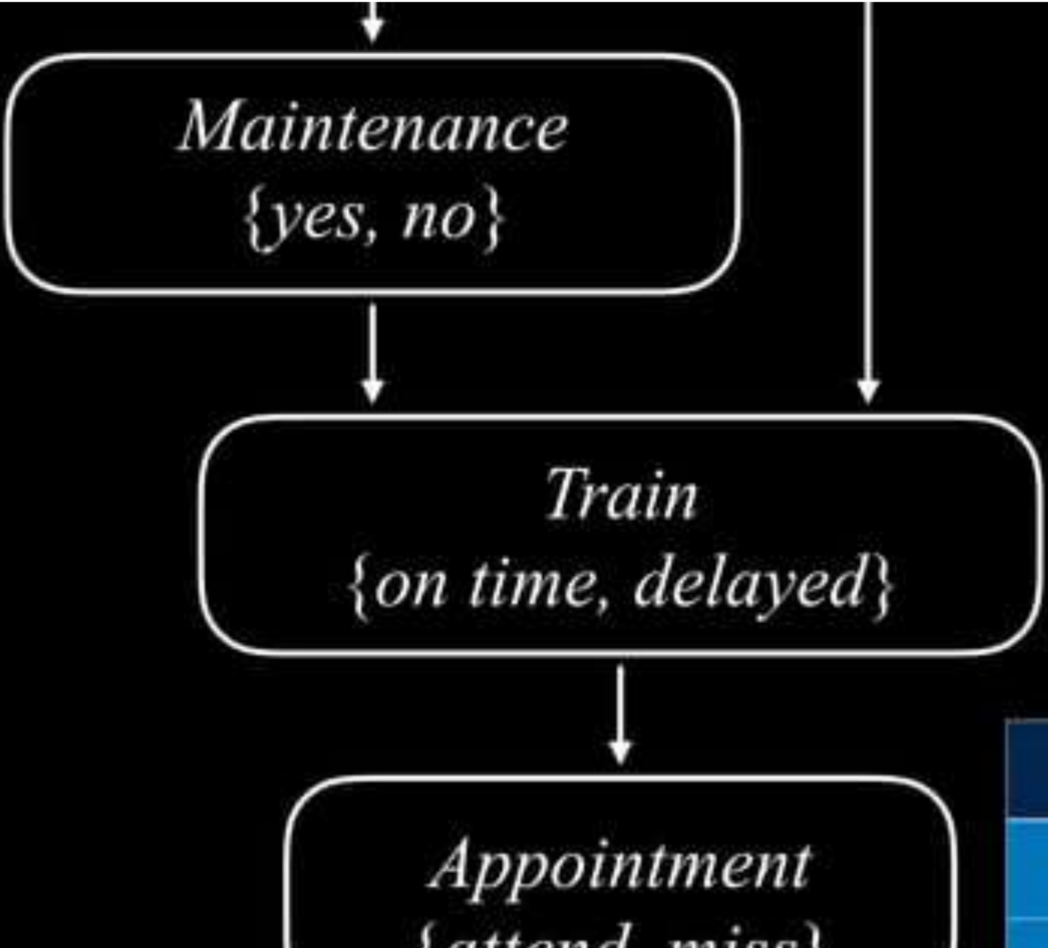


R	*yes*	*no*
none	0.4	0.6
light	0.2	0.8



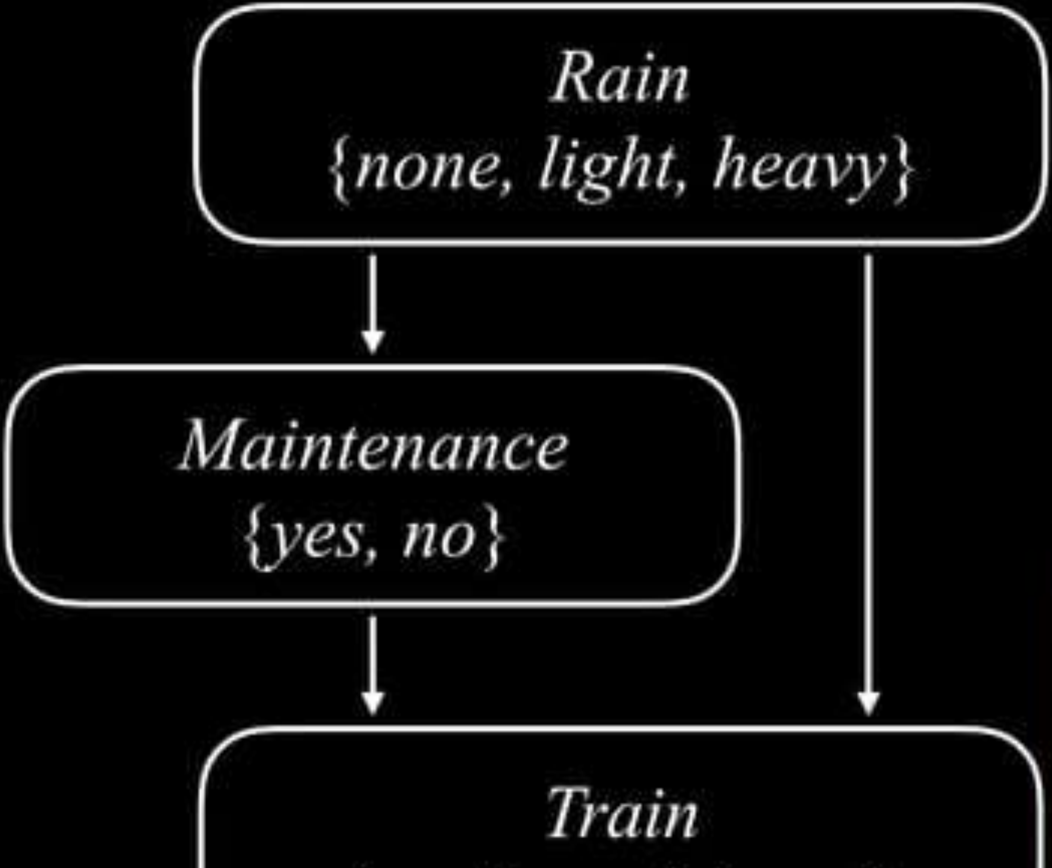
R = *light*
M = *yes*
T = *on time*

R	*M*	*on time*	*delayed*
none	*yes*	0.8	0.2
none	*no*	0.9	0.1
light	*yes*	0.6	0.4
light	*no*	0.7	0.3
heavy	*yes*	0.4	0.6



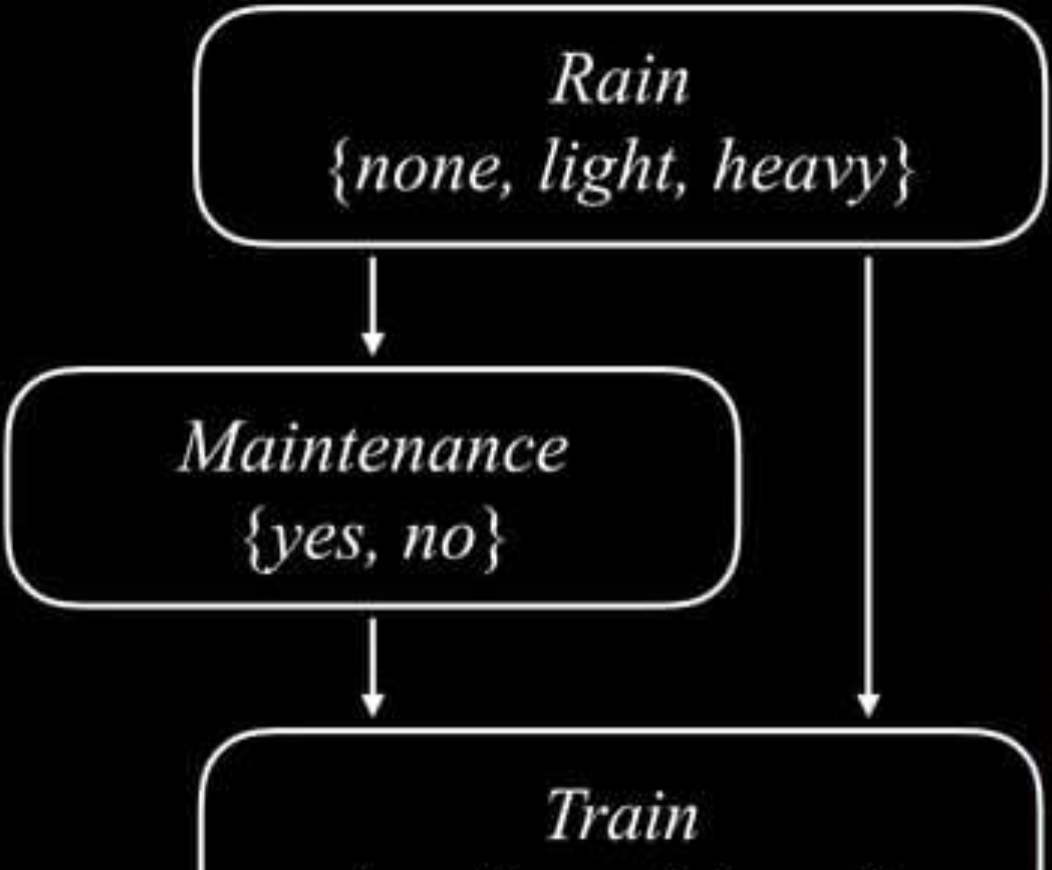
R = *light*
M = *yes*
T = *on time*
A = *attend*

T	*attend*	*miss*
on time	0.9	0.1



R = *light*
M = *yes*
T = *on time*
A = *attend*

R	***M***	*on time*	*delayed*
none	*yes*	0.8	0.2
none	*no*	0.9	0.1
light	*yes*	0.6	0.4
light	*no*	0.7	0.3
heavy	*yes*	0.4	0.6



R = *light*
M = *yes*
T = *on time*
A = *attend*

R	***M***	*on time*	*delayed*
none	*yes*	0.8	0.2
none	*no*	0.9	0.1
light	*yes*	0.6	0.4
light	*no*	0.7	0.3
heavy	*yes*	0.4	0.6

Uncertainty over Time

X_t: Weather at time t

Markov assumption

the assumption that the current state depends on only a finite fixed number of previous states

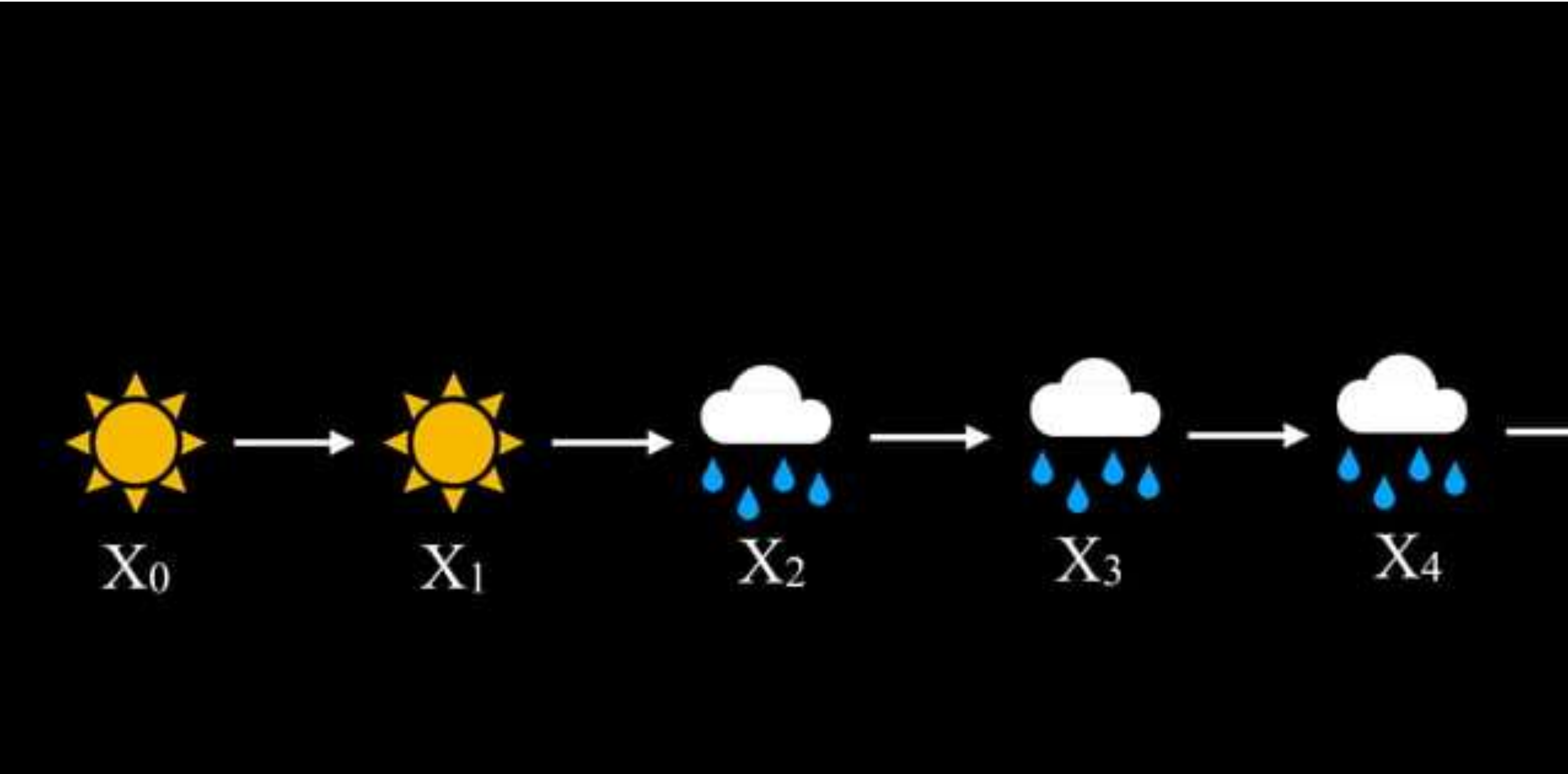
Markov Chain

Markov chain

a sequence of random variables where the distribution of each variable follows the Markov assumption

Transition Model

		Tomorrow (X_{t+1})	
			
Today (X_t)		0.8	0.2
		0.3	0.7



X_0
X_1
X_2
X_3
X_4

Sensor Models

Hidden State	Observation
robot's position	robot's sensor data
words spoken	audio waveforms
user engagement	website or app analytics
weather	umbrella

Hidden Markov Models

Hidden Markov Model

a Markov model for a system with hidden states that generate some observed event

Sensor Model

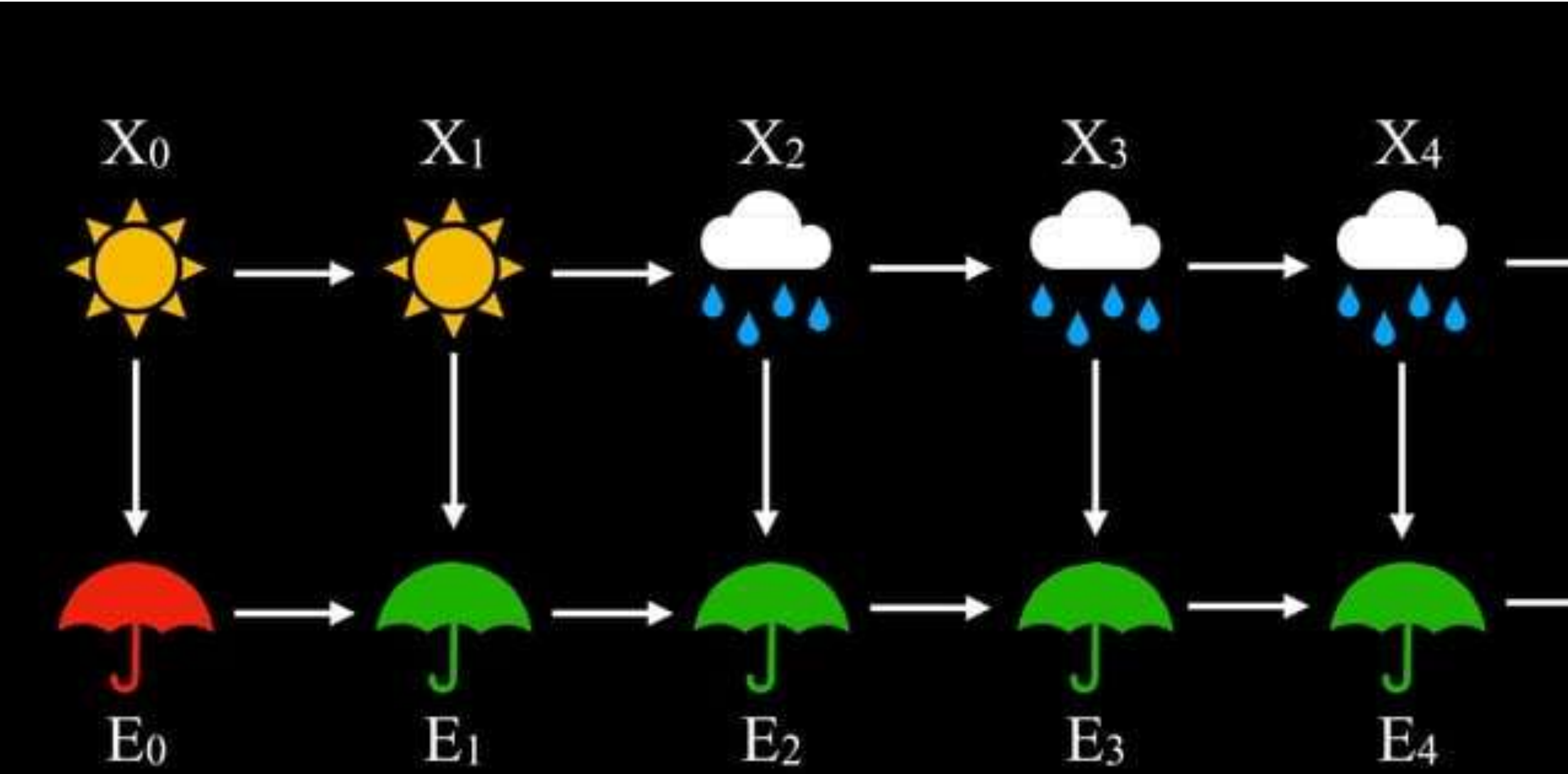
Observation (E_t)

State (X_t)

		
	0.2	0.8
	0.9	0.1

sensor Markov assumption

the assumption that the evidence variable depends only the corresponding state


X_0
X_1
X_2
X_3
X_4
E_0
E_1
E_2
E_3
E_4

Task	Definition
filtering	given observations from start until now, calculate distribution for **current** state
prediction	given observations from start until now, calculate distribution for a **future** state
smoothing	given observations from start until now, calculate distribution for **past** state
most likely explanation	given observations from start until now, calculate most likely **sequence** of states