# SNS COLLEGE OF TECHNOLOGY 

Coimbatore-35
An Autonomous Institution
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## DEPARTMENT OF INFORMATION TECHNOLOGY <br> 19CSE303 - ARTIFICIAL INTELLIGENCE III YEAR IV SEM

UNIT IV - UNCERTAIN KOWLEDGE AND REASONING

## TOPIC - Probability Uncertainty

## Reasoning and Decision Making Under Uncertainty

1. Quick Review Probability Theory
2. Bayes' Theorem and Naïve Bayesian Systems
3. Bayesian Belief Networks

- Structure and Concepts
- D-Separation
- How do they compute probabilities?
- How to design BBN using simple examples
- Other capabilities of Belief Network short!
- Netica Demo
- Develop a BBN using Netica likely Task6

4. Hidden Markov Models (HMM)

## Causes of not knowing things

 precisely

Fuzzy Sets and Fuzzy Logic
Reasoning with concepts that do not have a clearly defined boundary; e.g. old, long street, very old..."

## Random Variable

Definition: A variable that can take on several values, each value having a probability of occurrence.

There are two types of random variables:
$>$ Discrete. Take on a countable number of values.
$>$ Continuous. Take on a range of values.

## The Sample Space

> The space of all possible outcomes of a given process or situation is called the sample space S .
red \& small
blue \& small
red \& Jarge
blue \& large

## An Event

>An event A is a subset of the sample space.


## Atomic Event

An atomic event is a single point in S .

Properties:
$\square$ Atomic events are mutually exclusive
$\square$ The set of all atomic events is exhaustive
$\square$ A proposition is the disjunction of the atomic events it covers.

## The Laws of Probability

$>$ The probability of the sample space S is 1 , $\mathrm{P}(\mathrm{S})=1$
$>$ The probability of any event A is such that

$$
0<=P(A)<=1 \text {. }
$$

$>$ Law of Addition
If $A$ and $B$ are mutually exclusive events, then the probability that either one of them will occur is the sum of the individual probabilities:

$$
\mathrm{P}(\mathrm{~A} \text { or } \mathrm{B})=\mathrm{P}(\mathrm{~A})+\mathrm{P}(\mathrm{~B})
$$

## The Laws of Probability

## If $A$ and $B$ are not mutually exclusive:

## $P(A$ or $B)-P(A)+P(B)-P(A$ and $B)$

## Statistical Independence Example Discussion

- Population of 1000 students
- 600 students know how to swim (S)
- 700 students know how to bike (B)
- 420 students know how to swim and bike ( $\mathrm{S}, \mathrm{B}$ )
- In general, between ... and ... can swim and bike
- $\mathrm{P}(\mathrm{S} \wedge \mathrm{B})=420 / 1000=0.42$
- $P(S) \times P(B)=0.6 \times 0.7=0.42$
- In general: $\mathrm{P}(\mathrm{S} \wedge \mathrm{B})=\mathrm{P}(\mathrm{S}) * \mathrm{P}(\mathrm{B} \mid \mathrm{S})=\mathrm{P}(\mathrm{B}) * \mathrm{P}(\mathrm{S} \mid \mathrm{B})$
- $P(S \wedge B)=P(S) \times P(B)=>$ Statistical independence
- $P(S \wedge B)>P(S) \times P(B)=>$ Positively correlated
- $P(S \wedge B)<P(S) \times P(B)=>$ Negatively correlated
- $\quad \max (0, P(S)+P(B)-1) \leq P(S \wedge B) \leq \min (P(S), P(B))$


## Conditional Probabilities and $\mathrm{P}(\mathrm{A}, \mathrm{B})$

$>$ Given that A and B are events in sample space S , and $\mathrm{P}(\mathrm{B})$ is different of 0 , then the conditional probability of $A$ given $B$ is

$$
\mathrm{P}(\mathrm{~A} \mid \mathrm{B})=\mathrm{P}(\mathrm{~A}, \mathrm{~B}) / \mathrm{P}(\mathrm{~B})
$$

$>$ If A and B are independent then

$$
\mathrm{P}(\mathrm{~A}, \mathrm{~B})=\mathrm{P}(\mathrm{~A}) * \mathrm{P}(\mathrm{~B}) \rightarrow \mathrm{P}(\mathrm{~A} \mid \mathrm{B})=\mathrm{P}(\mathrm{~A})
$$

$>$ In general:
$\min (\mathrm{P}(\mathrm{A}), \mathrm{P}(\mathrm{B}) \geq \mathrm{P}(\mathrm{A}) * \mathrm{P}(\mathrm{B}) \geq \max (0,1-\mathrm{P}(\mathrm{A})-\mathrm{P}(\mathrm{B}))$
For example, if $\mathrm{P}(\mathrm{A})=0.7$ and $\mathrm{P}(\mathrm{B})=0.6$ then $\mathrm{P}(\mathrm{A}, \mathrm{B})$
has to be between 0.3 and 0.6 , but not necessarily be 0.42 !!

## The Laws of Probability

> Law of Multiplication
What is the probability that both A and B occur together?
$\mathrm{P}(\mathrm{A}$ and B$)=\mathrm{P}(\mathrm{A}) \mathrm{P}(\mathrm{B} \mid \mathrm{A})$
where $P(B \mid A)$ is the probability of $B$ conditioned on A.

## The Laws of Probability

If $A$ and $B$ are statistically independent:
$\mathrm{P}(\mathrm{B} \mid \mathrm{A})=\mathrm{P}(\mathrm{B})$ and then
$\mathrm{P}(\mathrm{A}$ and B$)=\mathrm{P}(\mathrm{A}) \mathrm{P}(\mathrm{B})$

## Independence on Two Variables

$$
\mathrm{P}(\mathrm{~A}, \mathrm{~B} \mid \mathrm{C})=\mathrm{P}(\mathrm{~A} \mid \mathrm{C}) \mathrm{P}(\mathrm{~B} \mid \mathrm{A}, \mathrm{C})
$$

If $A$ and $B$ are conditionally independent:

$$
\mathrm{P}(\mathrm{~A} \mid \mathrm{B}, \mathrm{C})=\mathrm{P}(\mathrm{~A} \mid \mathrm{C}) \text { and }
$$

$$
\mathrm{P}(\mathrm{~B} \mid \mathrm{A}, \mathrm{C})=\mathrm{P}(\mathrm{~B} \mid \mathrm{C})
$$

## Multivariate Joint Distributions

$$
P(x, y)=P(X=x \text { and } Y=y) .
$$

$>\mathrm{P}^{\prime}(\mathrm{x})=\operatorname{Prob}(\mathrm{X}=\mathrm{x})=\sum_{\mathrm{y}} \mathrm{P}(\mathrm{x}, \mathrm{y})$
It is called the marginal distribution of $X$
The same can be done on $Y$ to define the marginal distribution of $\mathrm{Y}, \mathrm{P}$ " $(\mathrm{y})$.
$>$ If X and Y are independent then $P(x, y)=P^{\prime}(x) P^{\prime \prime}(y)$

## Bayes' Theorem

$$
\begin{aligned}
& \mathrm{P}(\mathrm{~A}, \mathrm{~B})=\mathrm{P}(\mathrm{~A} \mid \mathrm{B}) \mathrm{P}(\mathrm{~B}) \\
& \mathrm{P}(\mathrm{~B}, \mathrm{~A})=\mathrm{P}(\mathrm{~B} \mid \mathrm{A}) \mathrm{P}(\mathrm{~A})
\end{aligned}
$$

The theorem:

$$
\mathrm{P}(\mathrm{~B} \mid \mathrm{A})=\mathrm{P}(\mathrm{~A} \mid \mathrm{B}) * \mathrm{P}(\mathrm{~B}) / \quad \mathrm{P}(\mathrm{~A})
$$

Example: P(Disease|Symptom)= $\mathrm{P}($ Symptom $\mid$ Disease $) * \mathrm{P}($ Disease $) / \mathrm{P}($ Symptom $)$

## THANK YOU

