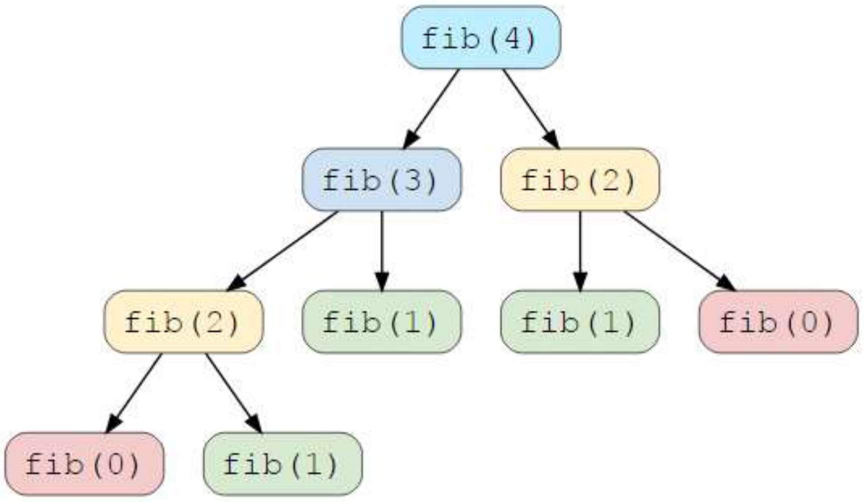
Unit III
Dynamic Programming and Greedy Technique

- Dynamic Programming
  - ***Computing a Binomial Coefficient***
  - Warshall's algorithm
  - Floyd's algorithm
  - Optimal Binary Search Trees
  - Knapsack Problem and Memory functions

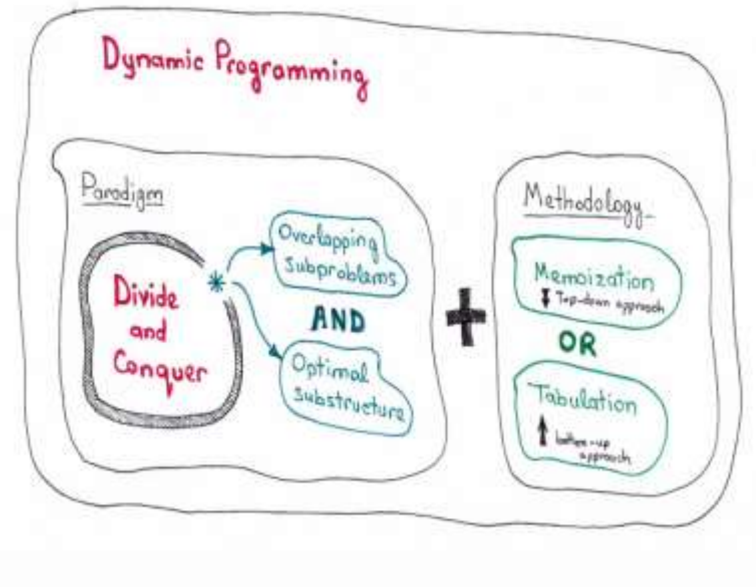
Dynamic Programming

- Dynamic programming – pblm →similar sub problems →reuse the solution
- **Characteristics**
  - Overlapping sub problems – solving same sub problems
  - Optimal substructure property – optimal solution can be built from sub problem
  - Example : Fibonacci series



Dynamic Programming

- **Methodology**
  - Top-down with memoization
    - Storing the result of already solved sub-problem is called memoization
  - Bottom-up with Tabulation
    - Sub-problems (bottom – up)



Difference between Divide and conquer and Dynamic Programming

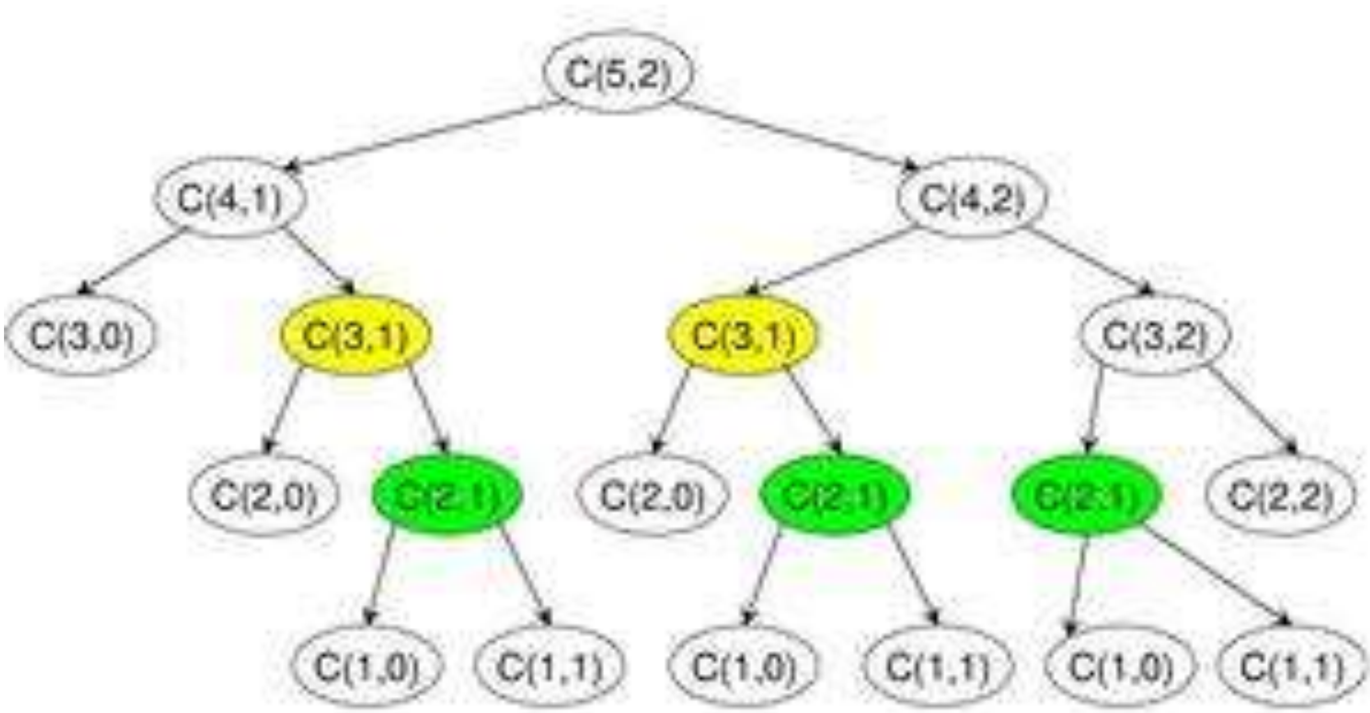
Divide and conquer	**Dynamic Programming**
Sub problems are not dependent on each other	Sub problems are dependent on each other
Doesn’t store the solution of sub-problem	Stores the solution of sub problem

Computing a Binomial Coefficient

- Binomial coefficient – computation of no of ways r items that can be chosen from n elements $C(^{n}_{r})$
- C(n, k) = n! / (n-k)! * k!
- C(n, k) = C (n-1, k-1) + C (n-1, k) , n>k, k>0
- C (n,0) = 1, C (n,n) = 1
- *Example:*
- 1^{st} formula : C (4,2) → 4! /(2!) * 2! → 24 / 4 →6
- 2^{nd} formula : C(4,2) → C(3,1) + C(3,2) → ….. → 6
- C (4,2) → how many two combinations of elements can be picked from set of 4 elements
- Example: possibilities of 1,2,3,4 → (1,2) (1,3) (1,4) (2,3) (2,4) (3,4)

Computing a Binomial Coefficient

- Example : C (5,2)
- C(n, k) = C (n-1, k-1) + C (n-1, k) , n>k, k>0
- C (n,0) = 1, C (n,n) = 1



Computing a Binomial Coefficient - Tabulation

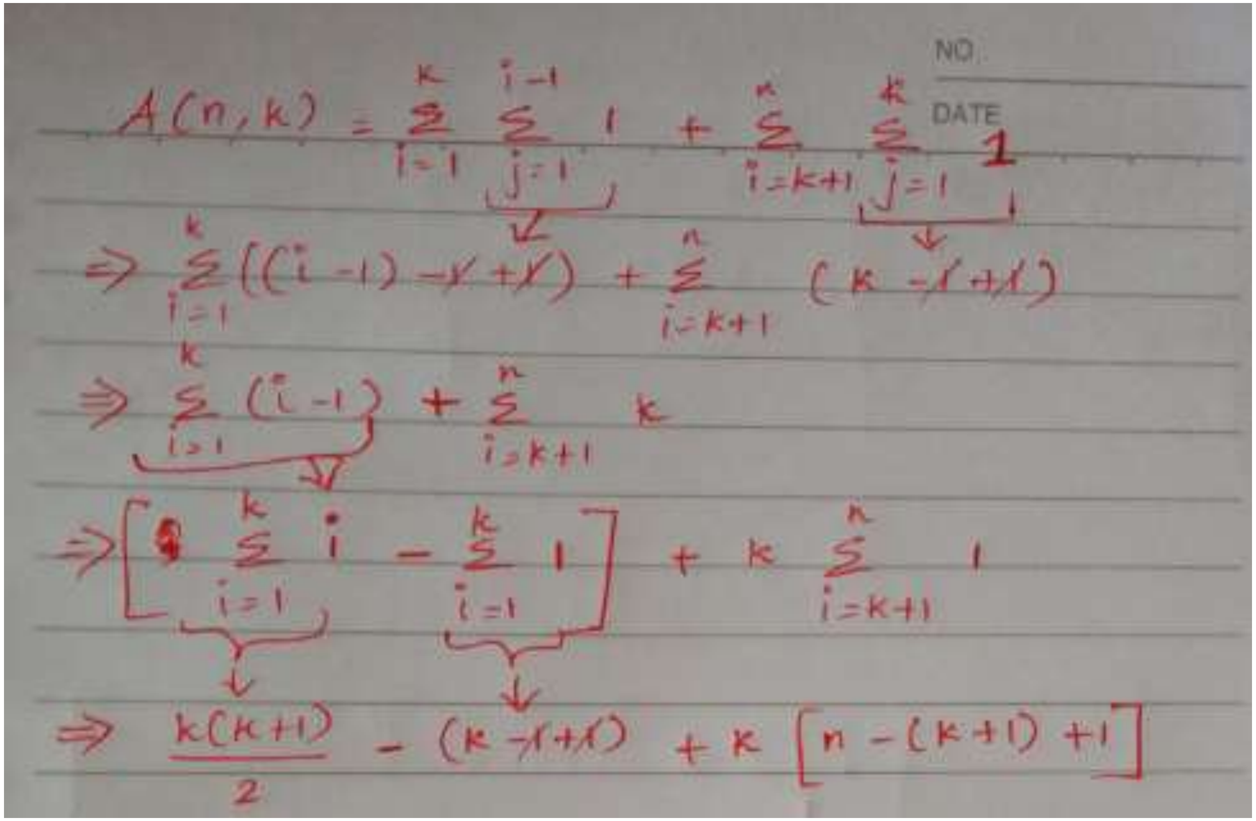
	0	1	2	3	4	5	...	$(k-1)$	k
0	1								
1	1	1							
2	1	2	1						
3	1	3	3	1					
4	1	4	6	4	1				
5									
⋮									
k	1								1
⋮									
$(n-1)$	1							$C(n-1, k-1)$	$C(n-1,k)$
n	1								$C(n,k)$

Computing a Binomial Coefficient - Algorithm

Algorithm *Binomial*(n, k)
 for $i \leftarrow 0$ **to** n **do** // fill out the table row wise
  for $i = 0$ **to** $\min(i, k)$ **do**
   if $j{=}{=}0$ or $j{=}{=}i$ **then** $C[i, j] \leftarrow 1$ // IC
   else $C[i, j] \leftarrow C[i\text{-}1, j\text{-}1] + C[i\text{-}1, j]$ // recursive relation
 return $C[n, k]$

Computing a Binomial Coefficient - Analysis

- Cost of the algorithm – table
- Sum – 2 parts (upper and lower triangle)
- $A(n, k)$ = sum for upper triangle + sum for the lower rectangle



Computing a Binomial Coefficient - Analysis