SNS COLLEGE OF TECHNOLOGY,
COIMBATORE –35

Traveling Salesman Problem

Traveling Salesman Problem

- The **Traveling Salesman Problem** (TSP) is to find the **shortest** tour through a given set of *n* cities that visits each city exactly once before returning to the city where it started.

Traveling Salesman Problem

• The problem can be modeled by a weighted graph, with the graph's vertices representing the cities and the edge weights specifying the distances.

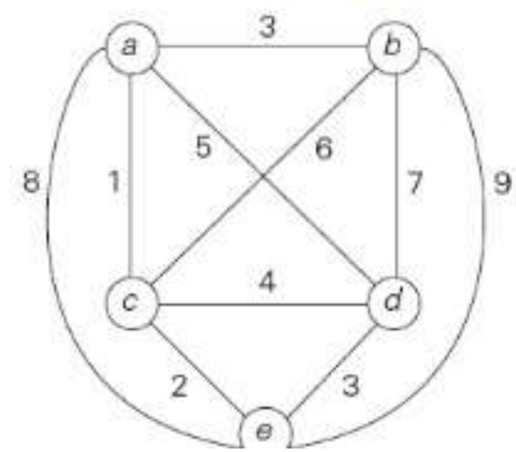
• Then the TSP problem can be stated as the problem of finding the shortest ***Hamiltonian circuit*** of the graph.

• A Hamiltonian circuit is defined as a cycle that passes through all the vertices of the graph exactly once.

Traveling Salesman Problem

- For each city i, $1 \le i \le n$, find the sum s_i of the distances from city i to the two nearest cities; compute the sum s of these n numbers; divide the result by 2 ($lb = \lceil s / 2 \rceil$).
- **Example**:

$$lb = \lceil [(1 + 3) + (3 + 6) + (1 + 2) + (3 + 4) + (2 + 3)] / 2 \rceil = 14$$



53

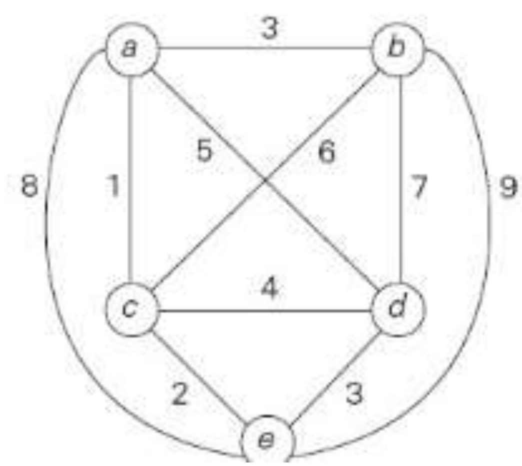
Traveling Salesman Problem

- If we know that a subset of a tour that must include a particular edge of a given graph, we update the lower bound by modifying the sums s_i's incident to two vertices of the edge.

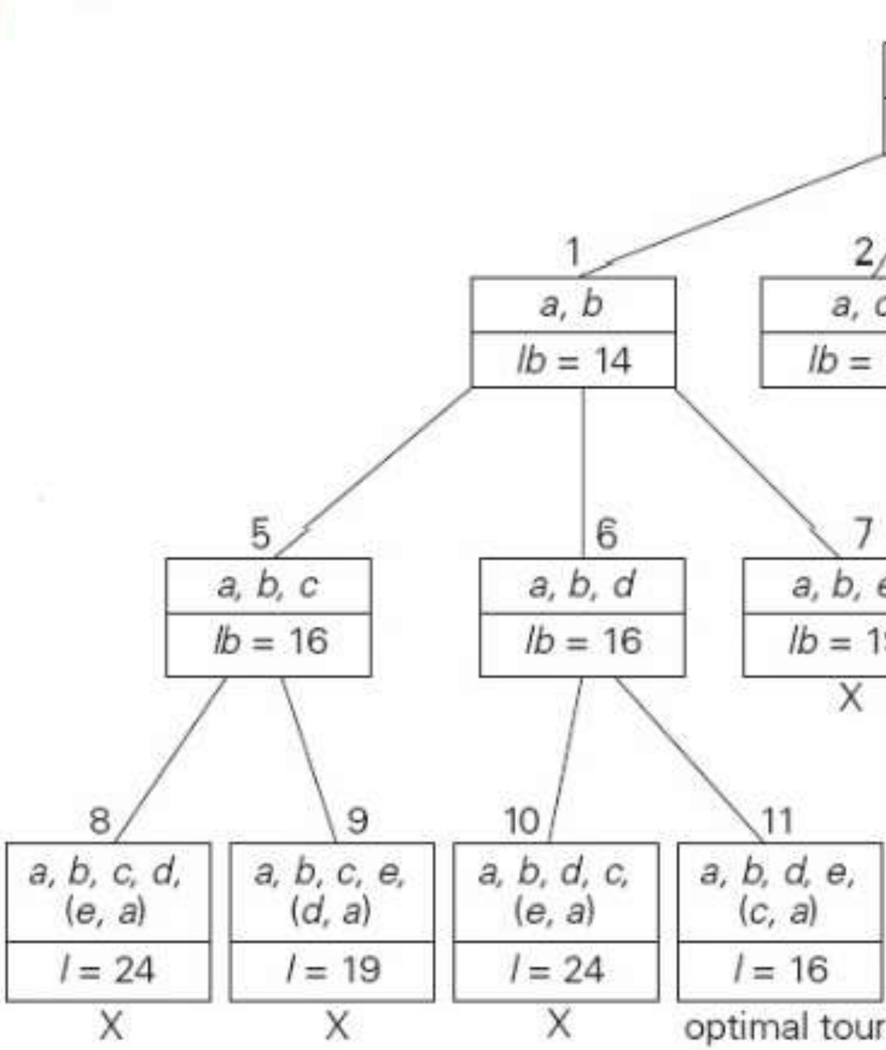
Traveling Salesman Problem

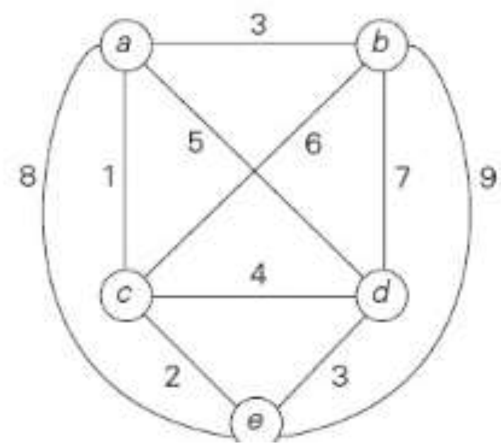
- For example, if a Hamiltonian circuit of the graph in Figure 12.9a that must include edge (*a*, *d*), we update the lower bound by modifying the sums s_1 and s_4 because these sums are incident to two vertices *a* and *d*:

$$\lceil [(1 + 5) + (3 + 6) + (1 + 2) + (3 + 5) + (2 + 3)] / 2 \rceil = 16$$



Traveling Salesman Problem





Traveling Salesman Problem

Node 0: $lb = \lceil [(1 + 3) + (3 + 6) + (1 + 2) + (3 + 4) + (2 + 3)] / 2 \rceil = \lceil 28/2 \rceil = 14.$

Node 1: $lb = \lceil [(1 + 3) + (3 + 6) + (1 + 2) + (3 + 4) + (2 + 3)] / 2 \rceil = \lceil 28/2 \rceil = 14.$

Node 2: $lb = \lceil [(1 + 3) + (3 + 6) + (1 + 2) + (3 + 4) + (2 + 3)] / 2 \rceil = \lceil 28/2 \rceil = 14.$

Traveling Salesman Problem

Node 3: $lb = \lceil [(1 + 5) + (3 + 6) + (1 + 2) + (3 + 5) + (2 + 3)] / 2 \rceil = \lceil 31/2 \rceil = 16.$ ×

Node 4: $lb = \lceil [(1 + 3) + (3 + 6) + (1 + 2) + (3 + 4) + (2 + 3)] / 2 \rceil = \lceil 38/2 \rceil = 19.$ ×

Node 5: $lb = \lceil [(1 + 3) + (3 + 6) + (1 + 6) + (3 + 4) + (2 + 3)] / 2 \rceil = \lceil 32/2 \rceil = 16.$

Node 6: $lb = \lceil [(1 + 3) + (3 + 7) + (1 + 2) + (3 + 7) + (2 + 3)] / 2 \rceil = \lceil 32/2 \rceil = 16.$

Node 7: $lb = \lceil [(1 + 3) + (3 + 9) + (1 + 2) + (3 + 4) + (2 + 9)] / 2 \rceil = \lceil 37/2 \rceil = 19.$ ×

Traveling Salesman Problem

//Leaf nodes

Node 8: $l = 3 + 6 + 4 + 3 + 8 = 24$.

Node 9: $l = 3 + 6 + 2 + 3 + 5 = 19$.

Node 10: $l = 3 + 7 + 4 + 2 + 8 = 24$.

Node 11: $l = 3 + 7 + 3 + 2 + 1 = 16$. ✓

Solution: tour = (a, b, d, e, c, a), tour length = 16