TAYLOR SERIES

If f(z) is analytic inside a circle C , with centre at z=a , then f(z) can be expressed as $f(z) = f(a) + \frac{(z-a)}{1!} f'(a) + \frac{(z-a)^2}{2!} f''(a) + ...$

which is convergent at every point inside C.This is called Taylor series of f(z) about z=a

NOTE:

The Taylor series of f(z) about z=a is called Maclaurent series.

Solved Problems:

1.Expand f(z)=log (1+z) as Taylor series about the point z=0

Solution:

Given f(z)= log(1+z) and z=0

Derivatives	Derivatives at z=0
$f(z) = \log(1+z)$	$f(0) = \log 1 = 0$
$f'(z) = \frac{1}{1+z}$	$f'(0) = \frac{1}{1+0} = 1$
$f''(z) = -\frac{1}{(1+z)^2}$	$f''(0) = -\frac{1}{(1+0)^2} = -1$
$f''(z) = \frac{2}{(1+z)^3}$	$f''(0) = \frac{2}{(1+0)^3} = 2$

By Taylor series,

$$f(z) = f(a) + \frac{(z-a)}{1!} f'(a) + \frac{(z-a)^2}{2!} f''(a) + ...$$

$$f(z) = f(0) + \frac{(z-0)}{1!} f'(0) + \frac{(z-0)^2}{2!} f''(0) + ...$$

$$= 0 + z(1) + \frac{z^2}{2}(-1) + \frac{z^3}{6}(2) + ...$$

$$= z - \frac{z^2}{2} + \frac{z^3}{3} + ...$$

2.Expand $f(z) = e^z$ as Taylor series about the point z=0

Solution:

Given $f(z) = e^z$ and z=0

Derivatives	Derivatives at z=0
$f(z) = e^z$	$f(0) = e^0 = 1$
$f'(z) = e^z$	$f'(0) = e^0 = 1$
$f''(z) = e^z$	$f''(0) = e^0 = 1$
$f'''(z) = e^z$	$f'''(0) = e^0 = 1$

By Taylor series,

$$f(z) = f(a) + \frac{(z-a)}{1!} f'(a) + \frac{(z-a)^2}{2!} f''(a) + ...$$

$$f(z) = f(0) + \frac{(z-0)}{1!} f'(0) + \frac{(z-0)^2}{2!} f''(0) + ...$$

$$= 1 + z(1) + \frac{z^2}{2}(1) + \frac{z^3}{6}(1) + ...$$

$$= 1 + z + \frac{z^2}{2} + \frac{z^3}{6} + ...$$

3.Expand $f(z) = \cos z$ as Taylor series about the point z=0

Solution:

Given $f(z) = \cos z$ and z=0

Derivatives	Derivatives at z=0
$f(z) = \cos z$	$f(0) = \cos 0 = 1$
$f'(z) = -\sin z$	$f'(0) = -\sin 0 = 0$
$f''(z) = -\cos z$	$f''(0) = -\cos 0 = -1$
$f'''(z) = \sin z$	$f'''(0) = \sin 0 = 0$

By Taylor series,

$$f(z) = f(a) + \frac{(z-a)}{1!} f'(a) + \frac{(z-a)^2}{2!} f''(a) + ...$$

$$f(z) = f(0) + \frac{(z-0)}{1!} f'(0) + \frac{(z-0)^2}{2!} f''(0) + ...$$

$$= 1 + z(0) + \frac{z^2}{2}(-1) + \frac{z^3}{6}(0) + ...$$

$$= 1 - \frac{z^2}{2} + ...$$