1. **State Cauchy's Residue theorem.**
   If f(z) be analytic at all points inside and on a simple closed curve C,except

for a finite number of isolated singularities $z_1, z_2, \ldots, z_n$ inside C,then

$$\int_c f(z)dz = 2\pi i[\text{sum of the residues of } f(z) \text{ at } z_1, z_2, \ldots, z_n]$$

$$= 2\pi i \sum_{i=1}^{n} R_i \text{ where } R_i \text{ is the residue of } f(z) \text{ at } z = z_i$$

2. **Calculate the residue of** $f(z) = \frac{e^{2z}}{(z+1)^2}$ **at the pole**

Given $f(z) = \frac{e^{2z}}{(z+1)^2}$

Z=-1 is a pole of order 2

$$\text{Res}\{f(z)\}_{z=-1} = \lim_{z\to -1} \frac{d}{dz}\left[(z+1)^2 \frac{e^{2z}}{(z+1)^2}\right]$$

$$= \lim_{z\to -1} \frac{d}{dz}\left[e^{2z}\right]$$

$$= \lim_{z\to -1} 2e^{2z}$$

$$= 2e^{-2}$$

3. **Find the residue of** $f(z) = \frac{z^2}{(z-1)^2(z+2)}$ **at z=-2.**

Z=1 is a pole of order 2

Z=-2 is a pole of order 1

$$\text{Res}\{f(z)\}_{z=-2} = \lim_{z\to -2}\left[(z+2) \frac{z^2}{(z-1)^2(z+2)}\right]$$

$$= \lim_{z \to -2} \frac{z^2}{(z-1)^2}$$

$$= \frac{(-2)^2}{(-2-1)^2} = \frac{4}{9} = \frac{2}{3}$$

4. Find the residue of $f(z) = \frac{z+1}{z(z+2)}$

Z=0 is a pole of order 1

Z=2 is a pole of order 1

$$\operatorname{Res}\{f(z)\}_{z=0} = \lim_{z \to 0} z \frac{z+1}{z(z-2)}$$

$$\operatorname{Res}\{f(z)\}_{z=2} = \lim_{z \to 2} (z-2) \frac{z+1}{z(z-2)} = \frac{-3}{2}$$

Z=2 is a simple pole

$$\operatorname{Res}\{f(z)\}_{z=2} = \lim_{z \to 2} (z-2) \frac{4}{z^3(z-2)} = \frac{1}{2}$$

5. Obtain the residues of the function $f(z) = \frac{z-3}{(z+1)(z-2)}$ **at its poles.**

Z=-1 is a pole of order 1

Z=-2 is a pole of order 1

$$\operatorname{Res}\{f(z)\}_{z=-1} = \lim_{z \to -1} (z+1) \frac{(z-3)}{(z+1)(z+2)} = -4$$

$$\operatorname{Res}\{f(z)\}_{z=-2} = \lim_{z \to -2} (z+2) \frac{(z-3)}{(z+1)(z+2)} = -5$$

6. **Consider the function $f(z)=\frac{\sin z}{z^4}$ .Find the pole and its order.**

$$f(z)=\frac{\sin z}{z^4}=\frac{1}{z^4}\left[z-\frac{z^3}{3}+\frac{z^5}{5}-..........\right]$$

$$=\frac{1}{z^3}\left[1-\frac{z^2}{3}+\frac{z^4}{5}-..........\right]$$

Z=0 is a pole of order 3

7. **Define pole and simple poles.**

A point z=a is said to be a pole f(z) of order n if we can find a positive integer n such that $\lim_{z\to a}(z-a)^n f(z)\neq 0$

A pole of order one is called a simple pole.

$$f(z)=\frac{1}{(z-4)^2(z-3)^3(z-1)}$$

Example :

Hence z=1 is a simple pole of order 1

Z=4 is a simple pole of order 2

Z=3 is a simple pole of order 3.

8. **Find the regularities of the function** $f(z)=\frac{\cot \pi z}{(z-a)^3}$

Given $f(z)=\frac{\cot \pi z}{(z-a)^3}=\frac{\cos \pi z}{\sin \pi z(z-a)^3}$

Singular points are poles and are given by Dr=0.

$$\sin \pi z=0,\quad (z-a)^3=0 \Rightarrow z=a \text{ is a singular pole of order } 3$$
$$\sin \pi z=\sin n \sin n\pi \quad \text{where } n=0,\pm1,\pm2,........$$
$$\pi z=n\pi \Rightarrow n=\pi$$
$$z=n=0,\pm1,\pm2,........ \text{ are simple poles.}$$

9. Find the principal part and residue at the pole of $f(z)=\frac{2z+3}{(z+2)^2}=(2z+3)(z+2)^{-2}$

[since principal part is negative powers]

$(z+2)^{-2} \Rightarrow z=-2$ is a singular pole of order 2

$$\text{Res}\{f(z)\}_{z=-2} = \lim_{z\to -2}\frac{d}{dz}\left[(z+2)^2 . \frac{(2z+3)}{(z+2)^2}\right] = 2$$

10. Evaluate $\int_c \frac{dz}{z-2}$ **where C is the square with vertices (0,0),(1,0),(1,1) and (0,1).**

Given $\int_c \frac{dz}{z-2}$

Here $f(z) = 1$

$a = 2$ lies outside

$\therefore$ By Cauchy's integral formula $\int_c \frac{dz}{z-2} = 0.$