



## UNIT-4 COMPLEX INTEGRATION

### PART B

1. Using Cauchy's Integral formula, evaluate  $\int_c \frac{(z+4)dz}{(z^2+2z+5)}$ , where  $c$  is the circle  $|z+1+i|=2$ .
2. Using Cauchy's Integral formula, evaluate  $\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-2)(z-3)} dz$ , where  $c$  is the circle  $|z|=4$ .
3. Evaluate  $\int_c \frac{zdz}{(z-2)}$ , where  $c$  is the circle  $|z-2|=3/2$ , by using Cauchy's integral formula.
4. Using Cauchy's Integral formula, evaluate  $\int_c \frac{zdz}{(z-1)(z-2)^2}$ , where  $c$  is the circle  $|z-2|=1/2$ .
5. Evaluate  $\int_c \frac{dz}{(z-3)^2}$ , where  $c$  is the circle  $|z|=1$ .
6. Evaluate  $\frac{1}{2\pi i} \int_c \frac{z^2+5}{z-3} dz$  where  $c$  is  $|z|=4$ , using Cauchy Integral formula.
7. Use residue theorem to evaluate  $\int \frac{3z^2+z-1}{(z^2-1)(z-3)} dz$  around the circle  $|z|=2$ .
8. Evaluate  $\int_c \frac{z-2}{z(z-1)} dz$  where  $c$  is the circle  $|z|=3$ .
9. Find the residue of  $\frac{z+2}{(z+1)^2(z-2)}$  at its poles.



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10. Obtain the residue of the function  $f(z) = (z-3) / (z+1)(z+2)$  at its pole.
11. Evaluate  $\int_{|z|=3} \frac{\sin \pi z^2 + \cos \pi z^2}{(z+1)(z+2)} dz$ , using Cauchy's residue theorem.
12. Determine the residues at poles of the function  $f(z) = (z+4) / (z-1)(z-2)$ .
13. Evaluate  $\int_c \frac{2}{(z-1)(z+3)} dz$ , where  $c$  is  $|z-1|=2$ .
14. Evaluate  $\int_c \frac{z dz}{(z-1)^2(z+1)}$  where  $c$  is  $|z|=2$ .
15. Evaluate  $\int_c \frac{z^2+1}{(z^2-1)} dz$ , where  $c$  is the circle  $|z-i|=1$ .
16. Evaluate  $\int_c \frac{e^z dz}{(z^2 + \pi^2)^2}$ , where  $c$  is the circle  $|z|=4$  by using Cauchy's residue theorem.
17. Expand  $\frac{z-1}{z+2}$  in Taylor Series about the Point  $z=1$ .
18. Find the Laurent's Series expansion of  $f(z) = \frac{z}{(z^2+1)(z^2+4)}$  in the region  $1 < |z| < 2$ .
19. Find the Laurent's Series expansion of  $f(z) = \frac{1}{z^2+3z+2}$  in the region  $1 < |z| < 2$ .
20. Obtain the Laurent's series expansion of  $f(z) = 4z / (z^2-1)(z-4)$  in the region  $2 < |z-1| < 3$  and  $|z-1| > 4$ .



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21. Expand  $f(z) = \frac{z^2 - 1}{(z+2)(z+3)}$  in a Laurent's series for  $2 < |z| < 3$ .
22. Find the Laurent's series expansion of  $f(z) = 1/(z-z^2)$  in the region  $1 < |z+1| < 2$  and  $|z+1| > 2$ .
23. Find the Laurent's series expansion of  $f(z) = e^{2z}/(z-1)^3$  about  $z=1$ .
24. Find Laurent's series expansion of  $\frac{z-1}{(z+2)(z+3)}$  valid in the region  $2 < |z| < 3$ .
25. Find Laurent's series expansion of  $f(z) = \frac{7z-2}{z(z-2)(z+1)}$  in  $2 < |z| < 3$ .
26. Expand  $\frac{1}{z(z-1)}$  as Laurent's series about  $z=0$  in the annulus  $0 < |z| < 1$ .
27. Expand into Laurent's series expansion of  $\frac{z^2 - 1}{(z+2)(z+3)}$  in  $|z| < 2$ .
28. Evaluate  $\int_0^{2\pi} \frac{\cos 2\theta}{(5-4\cos\theta)} d\theta$ , using contour integration.
29. Using the method of contour integration prove that  $\int_0^{2\pi} \frac{d\theta}{5-4\cos\theta} = \frac{2\pi}{3}$ .
30. Using the method of contour integration prove that  $\int_0^{2\pi} \frac{\cos 3\theta d\theta}{5-4\cos\theta} = \frac{\pi}{12}$ .
31. Evaluate  $\int_0^{2\pi} \frac{d\theta}{13+5\cos\theta}$  by using contour integration.



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32. Prove that  $\int_0^{2\pi} \frac{d\theta}{a + b \cos \theta} = \frac{2\pi}{\sqrt{a^2 - b^2}}$ ,  $a > b > 0$  using Contour integration.