CS 430/536
Computer Graphics I

Polygon Clipping and Filling

Week 3, Lecture 5

David Breen, William Regli and Maxim Peysakhov

Geometric and Intelligent Computing Laboratory

Department of Computer Science

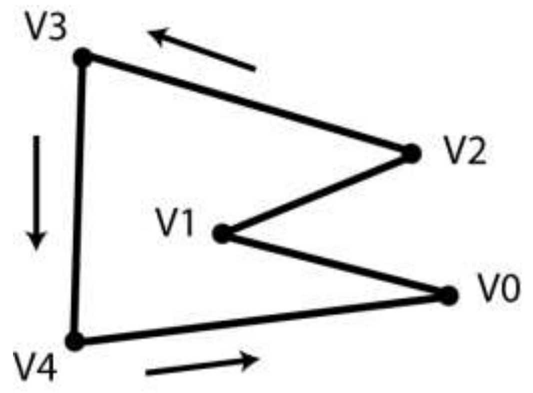
Drexel University

`http://gicl.cs.drexel.edu`

Outline

- Polygon clipping
  - Sutherland-Hodgman,
  - Weiler-Atherton
- Polygon filling
  - Scan filling polygons
  - Flood filling polygons
- Introduction and discussion of homework #2

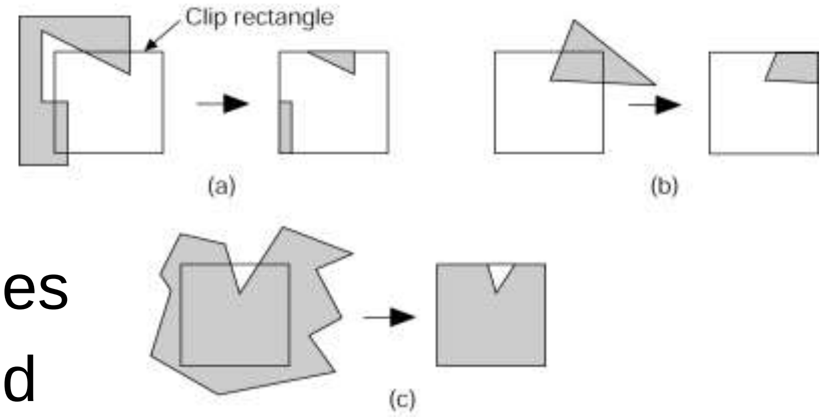
Polygon



- Ordered set of vertices (points)
  - Usually counter-clockwise
- Two consecutive vertices define an edge
- Left side of edge is inside
- Right side is outside
- Last vertex implicitly connected to first
- In 3D vertices are co-planar

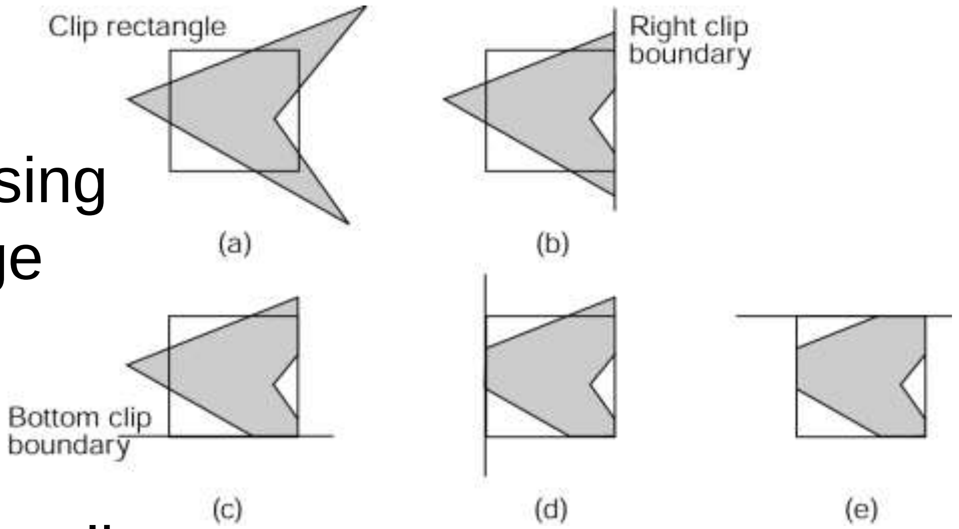
Polygon Clipping

- Lots of different cases
- Issues
  - Edges of polygon need to be tested against clipping rectangle
  - May need to add new edges
  - Edges discarded or divided
  - Multiple polygons can result from a single polygon



The Sutherland-Hodgman Polygon-Clipping Algorithm

- Divide and Conquer
- Idea:
  - Clip single polygon using single infinite clip edge
  - Repeat 4 times
- Note the generality:
  - 2D convex n-gons can clip arbitrary n-gons
  - 3D convex polyhedra can clip arbitrary polyhedra



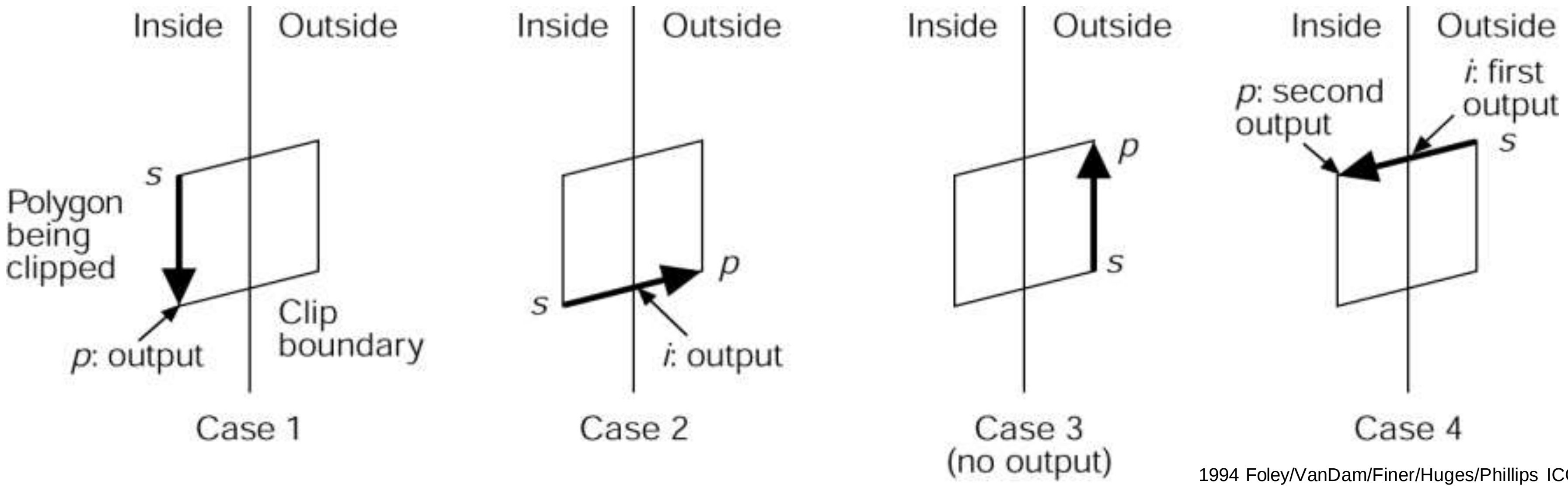
Sutherland-Hodgman Algorithm

- Input:
  - $v_1, v_2, \dots v_n$ the vertices defining the polygon
  - Single infinite clip edge w/ inside/outside info
- Output:
  - $v'_1, v'_2, \dots v'_m$, vertices of the clipped polygon
- Do this 4 (or n_e) times

- Traverse vertices (edges)
- Add vertices one-at-a-time to output polygon
  - Use inside/outside info
  - Edge intersections

Sutherland-Hodgman Algorithm

- Can be done incrementally
- If first point inside add. If outside, don't add
- Move around polygon from v_1 to v_n and back to v_1
- Check v_i, v_{i+1} wrt the clip edge
- Need v_i, v_{i+1}'s inside/outside status
- Add vertex one at a time. There are 4 cases:



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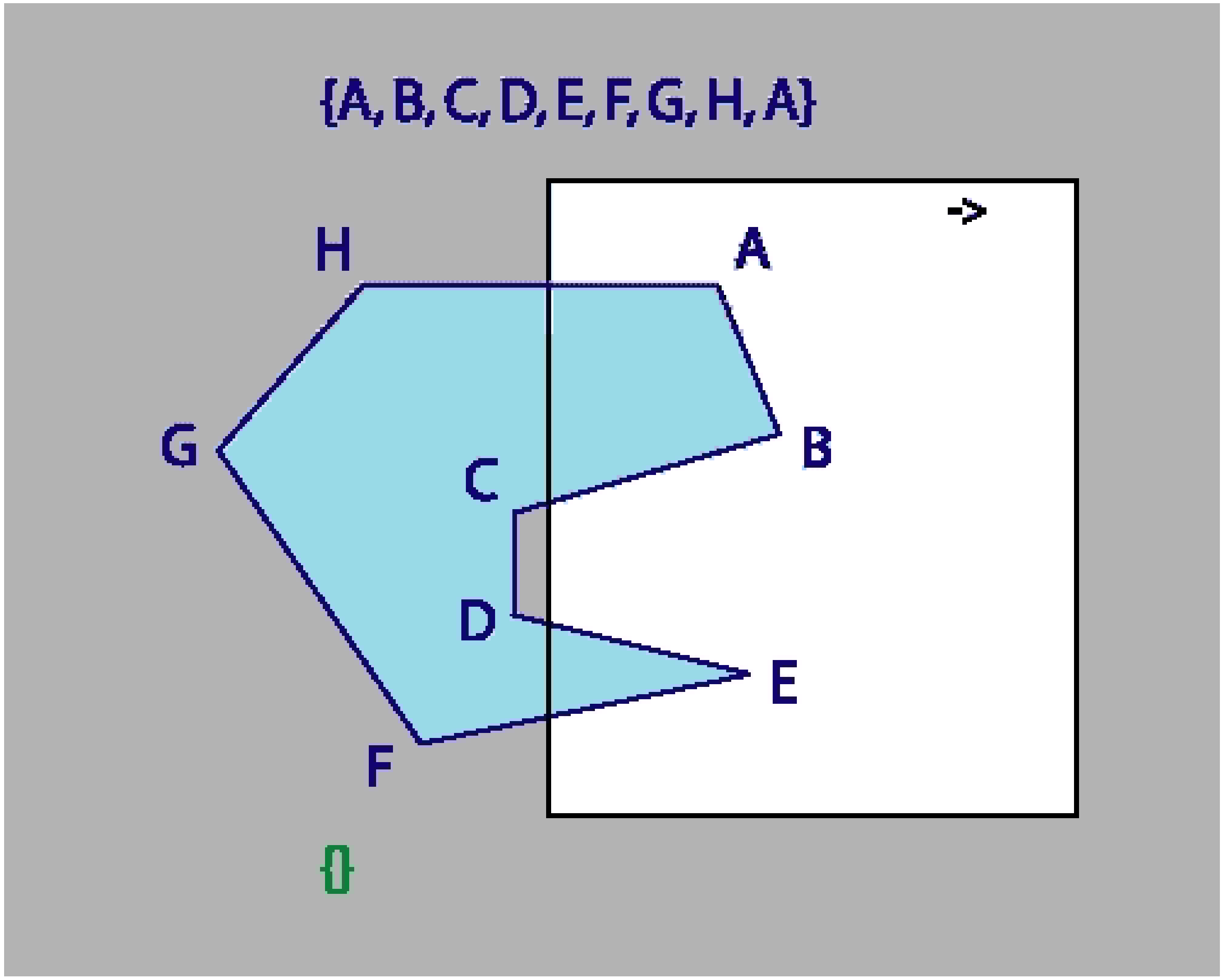
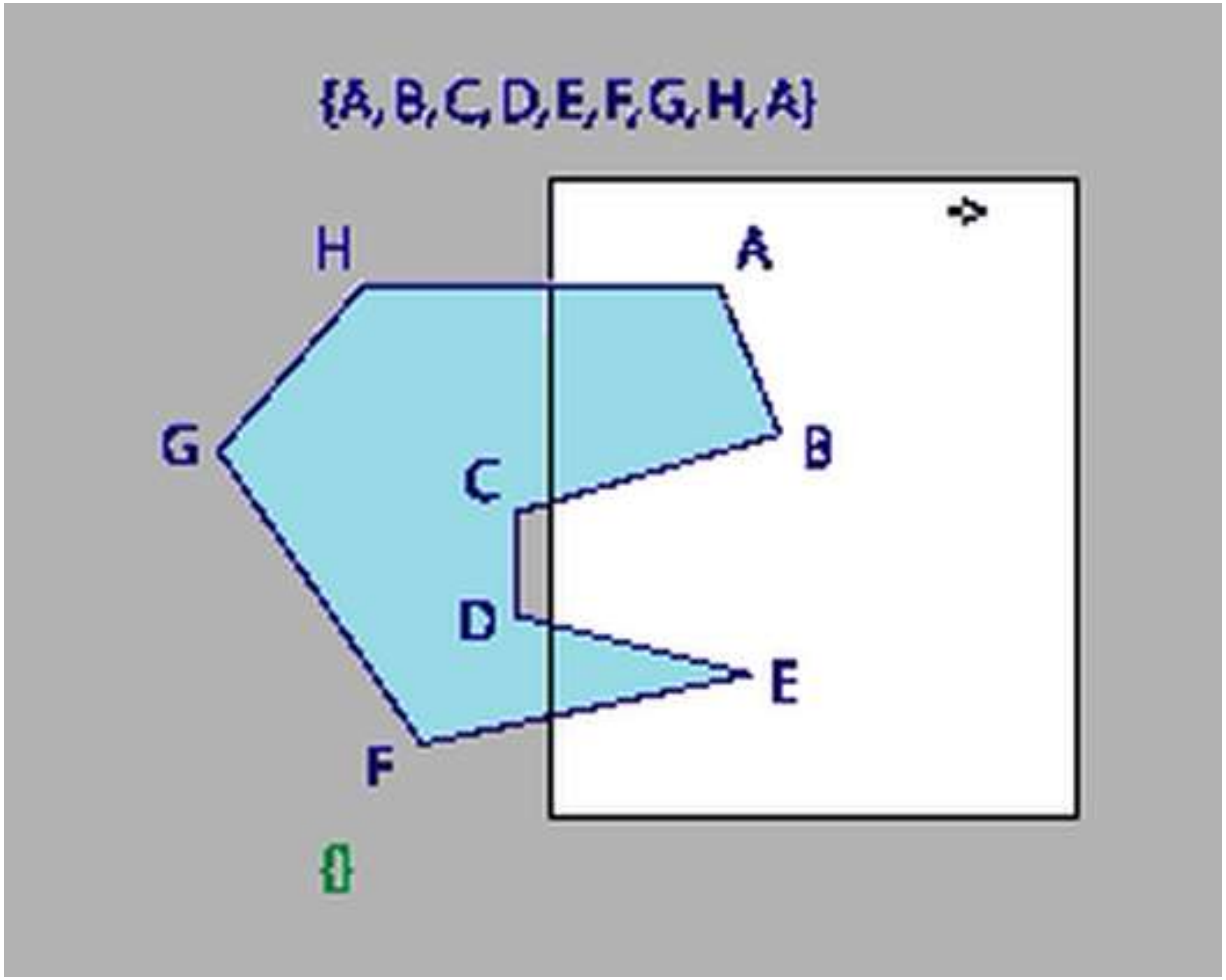
Sutherland-Hodgman Algorithm

- foreach polygon ***P***   ***P' = P***
  - foreach clipping edge (there are 4) {
    - Clip polygon ***P'*** to clipping edge
      - foreach edge in polygon ***P'***
        - » Check clipping cases (there are 4)
          - » Case 1 : Output v_{i+1}
          - » Case 2 : Output intersection point
          - » Case 3 : No output
          - » Case 4 : Output intersection point & v_{i+1}}

Sutherland-Hodgman Algorithm

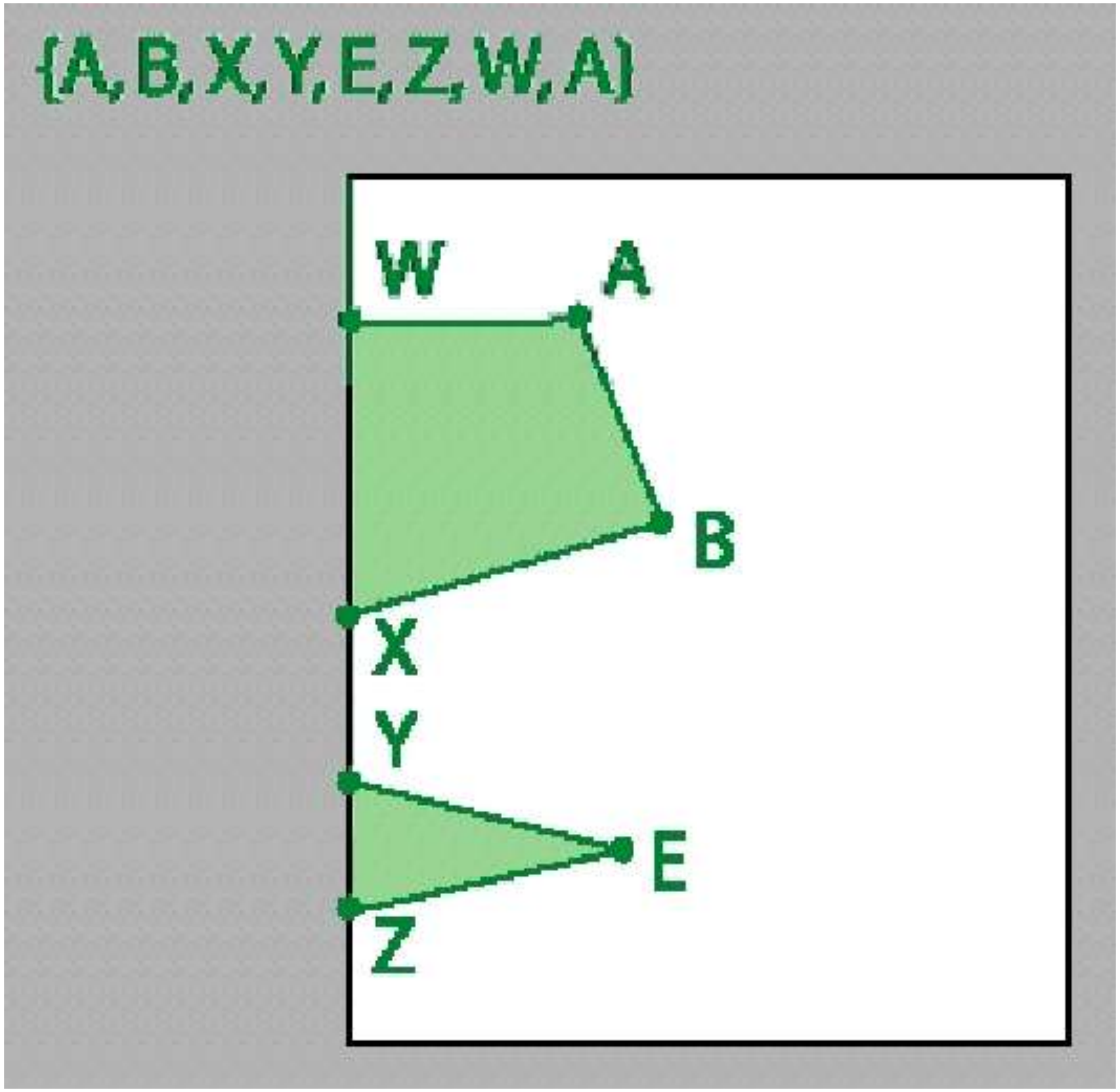


Sutherland-Hodgman Algorithm



Animated by Max Peysakhov @ Drexel University

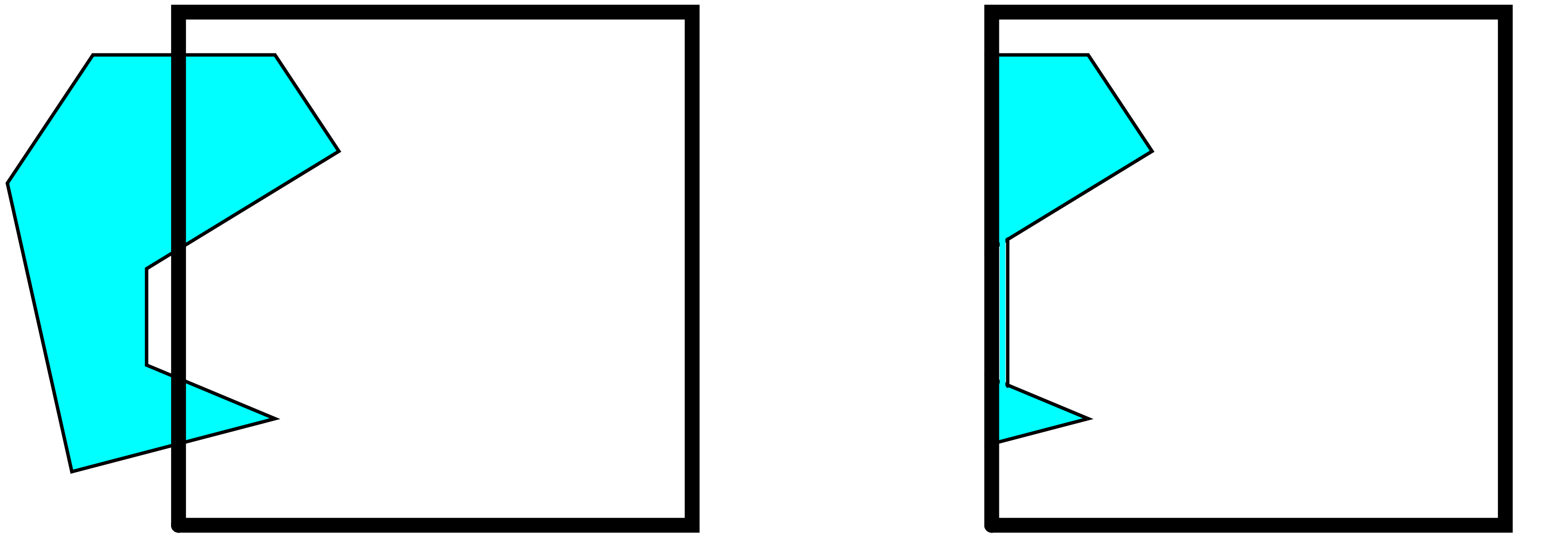
Final Result



Note: Edges XY and ZW!

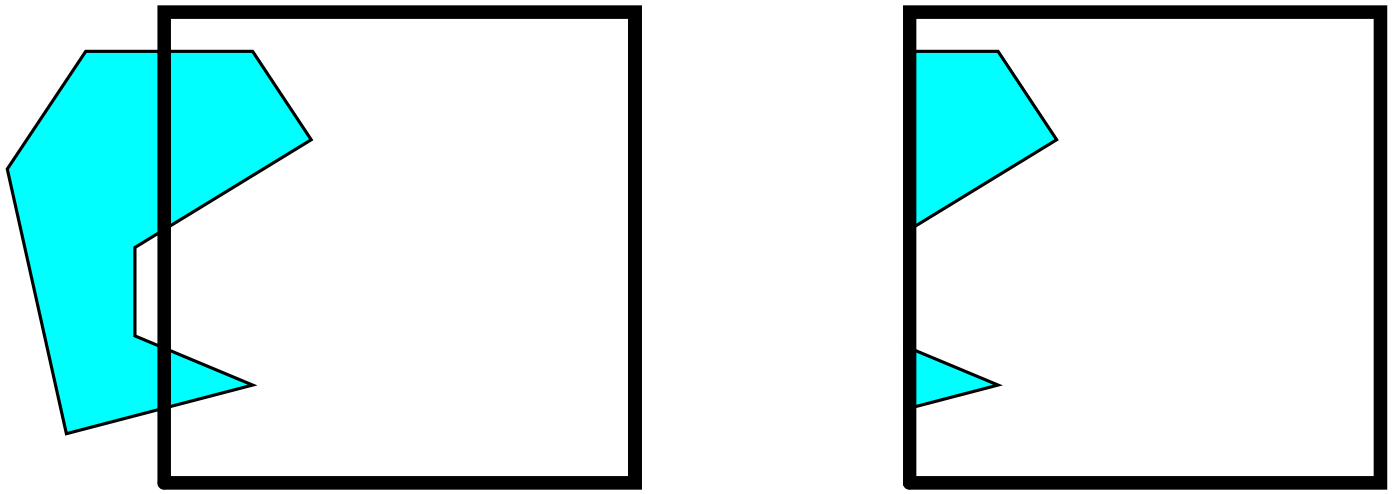
Issues with Sutherland-Hodgman Algorithm

- Clipping a concave polygon
- Can produce two CONNECTED areas



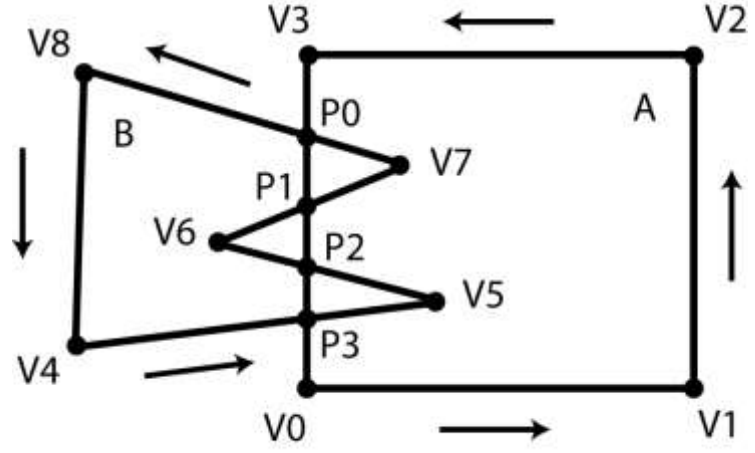
Weiler-Atherton Algorithm

- General clipping algorithm for concave polygons with holes
- Produces multiple polygons (with holes)
- Make linked list data structure
- Traverse to make new polygon(s)



Weiler-Atherton Algorithm

- Given polygons A and B as linked list of vertices (counter-clockwise order)
- Find all edge intersections & place in list
- Insert as “intersection” nodes
- Nodes point to A & B
- Determine in/out status of vertices



Intersection Special Cases

- If “intersecting” edges are parallel, ignore
- Intersection point is a vertex
  - Vertex of A lies on a vertex or edge of B
  - Edge of A runs through a vertex of B
  - Replace vertex with an intersection node

Weiler-Atherton Algorithm: Union

- Find a vertex of A outside of B
- Traverse linked list
- At each intersection point switch to other polygon
- Do until return to starting vertex
- All visited vertices and nodes define union’ed polygon

Weiler-Atherton Algorithm: Intersection

- Start at intersection point
  - If connected to an “inside” vertex, go there
  - Else step to an intersection point
  - If neither, stop
- Traverse linked list
- At each intersection point switch to other polygon and remove intersection point from list
- Do until return to starting intersection point
- If intersection list not empty, pick another one
- All visited vertices and nodes define and’ed polygon

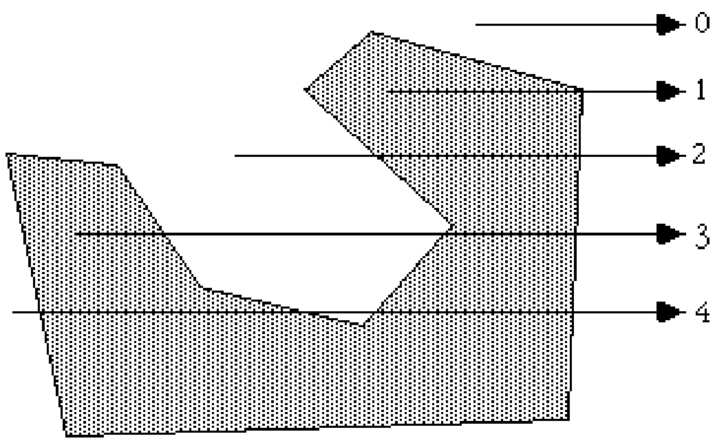
Boolean Special Cases

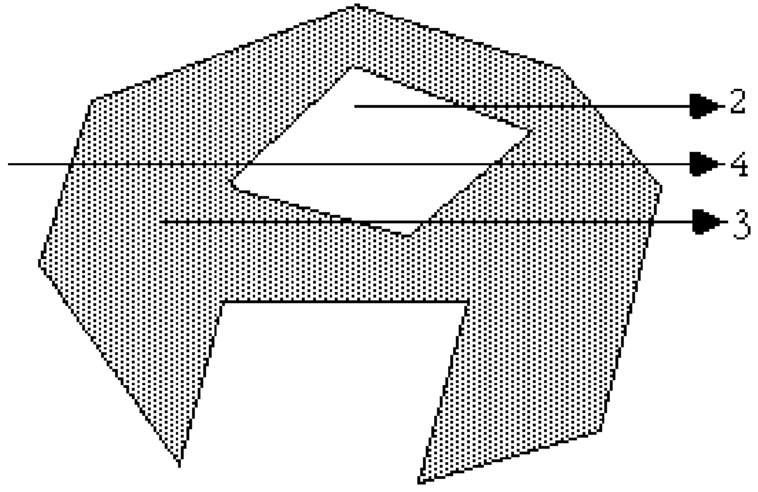
If polygons don’t intersect

- Union
  - If one inside the other, return polygon that surrounds the other
  - Else, return both polygons
- Intersection
  - If one inside the other, return polygon inside the other
  - Else, return no polygons

Point P Inside a Polygon?

- Connect P with another point P` that you know is outside polygon
- Intersect segment PP` with polygon edges
- Watch out for vertices!
- If # intersections is even (or 0) ⌫ Outside
- If odd ⌫ Inside





Edge clipping

- Re-use line clipping from HW1
  - *Similar triangles* method
  - Cyrus-Beck line clipping
- Yet another technique

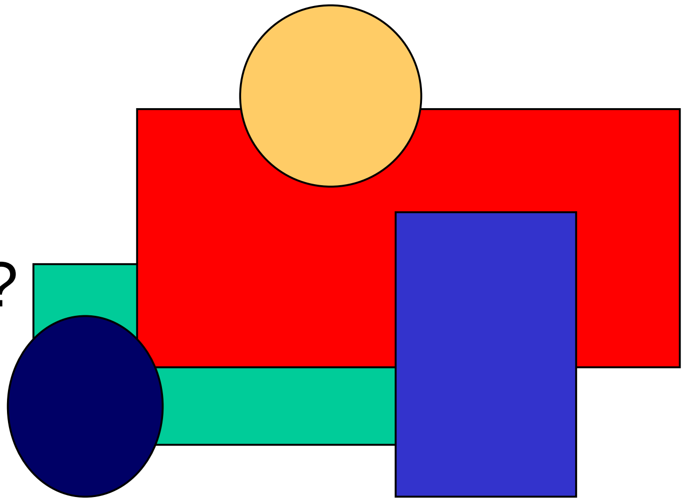
Intersecting Two Edges (1)

- Edge 0 : (P_0,P_1)
- Edge 2 : (P_2,P_3)
- $E_0 = P_0 + t_0 * (P_1-P_0)$ $D_0 \equiv (P_1-P_0)$
- $E_2 = P_2 + t_2 * (P_3-P_2)$ $D_2 \equiv (P_3-P_2)$
- $P_0 + t_0 * D_0 = P_2 + t_2 * D_2$
- $x_0 + dx_0 * t_0 = x_2 + dx_2 * t_2$
- $y_0 + dy_0 * t_0 = y_2 + dy_2 * t_2$

Intersecting Two Edges (2)

- Solve for t’s
- $t_0 = ((x_0 - x_2) * dy_2 + (y_2 - y_0) * dx_2) /$
  $(dy_0 * dx_2 - dx_0 * dy_2)$
- $t_2 = ((x_2 - x_0) * dy_0 + (y_0 - y_2) * dx_0) /$
  $(dy_2 * dx_0 - dx_2 * dy_0)$
- See http://www.vb-helper.com/howto_intersect_lines.html for derivation
- Edges intersect if $0 \leq t_0, t_2 \leq 1$
- Edges are parallel if denominator = 0

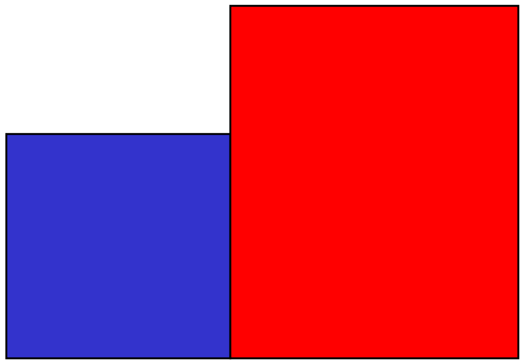
Filling Primitives: Rectangles, Polygons & Circles



- Two part process
  - Which pixels to fill?
  - What values to fill them with?
- Idea: ***Coherence***
  - *Spatial*: pixels are the same from pixel-to-pixel and scan-line to scan line;
  - *Span*: all pixels on a span get the same value
  - *Scan-line*: consecutive scan lines are the same
  - *Edge*: pixels are the same along edges

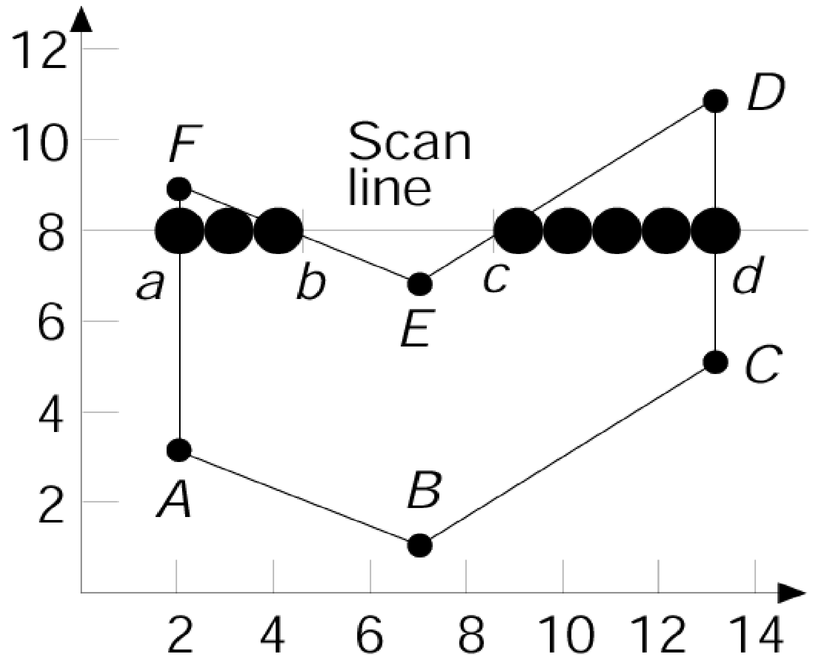
Scan Filling Primitives: Rectangles

- Easy algorithm
  - Fill from x_{min} to x_{max}
    Fill from y_{min} to y_{max}
- Issues
  - What if two adjacent rectangles share an edge?
  - Color the boundary pixels twice?
  - Rules:
    - Color only interior pixels
    - Color left and bottom edges

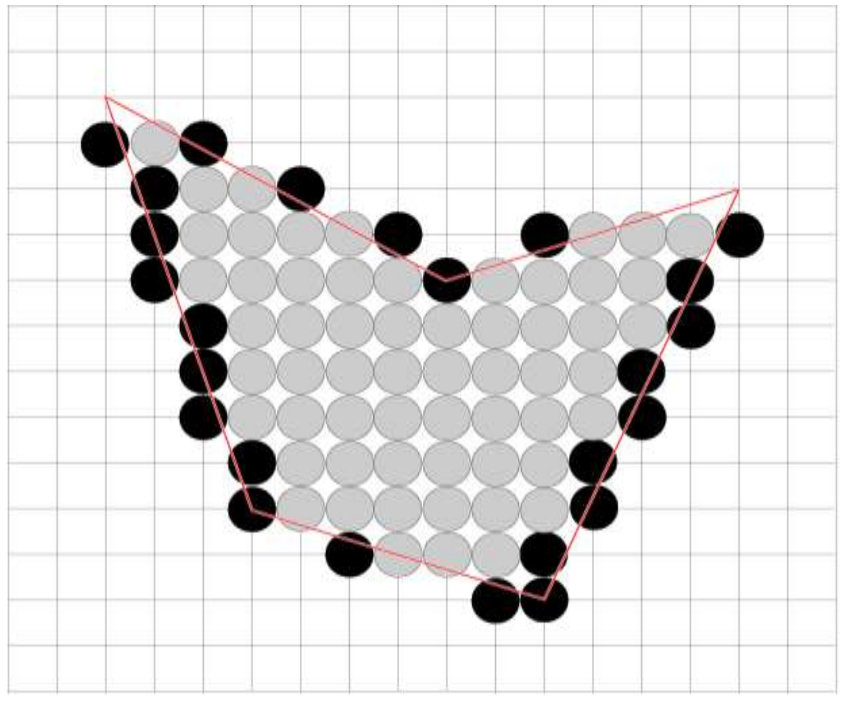


Scan Filling Primitives: Polygons

- Observe:
  - FA, DC intersections are integer
  - FE, ED intersections are not integer
- For each scan line, how to figure out which pixels are inside the polygon?



Scan Filling Polygons



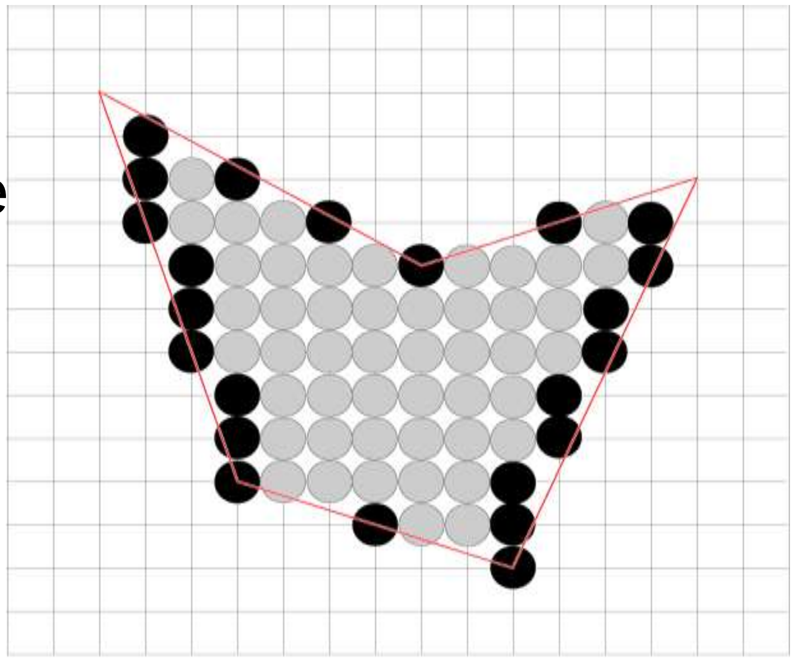
(a)

● Span extrema ● Other pixels in the span

- Idea #1: use midpoint algo on each edge, fill in between extrema points
- Note: many extrema pixels lie outside the polygon
- Why: midpoint algo has no sense of in/out

Scan Filling Polygons

- Idea #2: draw pixels only strictly inside
  - Find intersections of scan line with edges
  - Sort intersections by increasing *x* coordinate
  - Fill pixels on inside based on a parity bit
    - B_p initially even (off)
    - Invert at each intersect
    - Draw with odd, do not draw when even


(b)

● Span extrema ● Other pixels in the span

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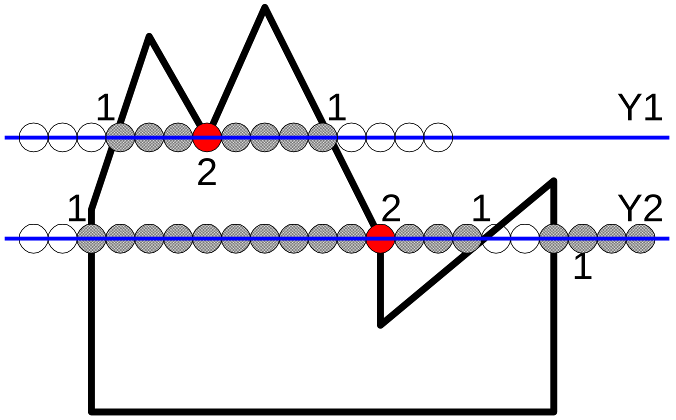
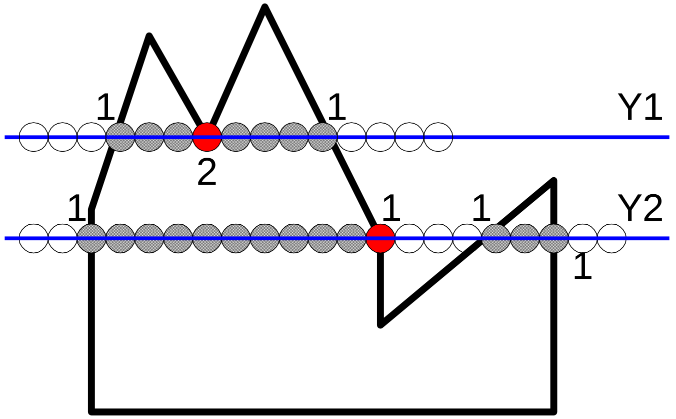
Scan Filling Polygons

- Issues with Idea #2:
  - If at a fractional x value, how to pick which pixels are in interior?
  - Intersections at integer vertex coordinates?
  - Shared vertices?
  - Vertices that define a horizontal edge?

How to handle vertices?

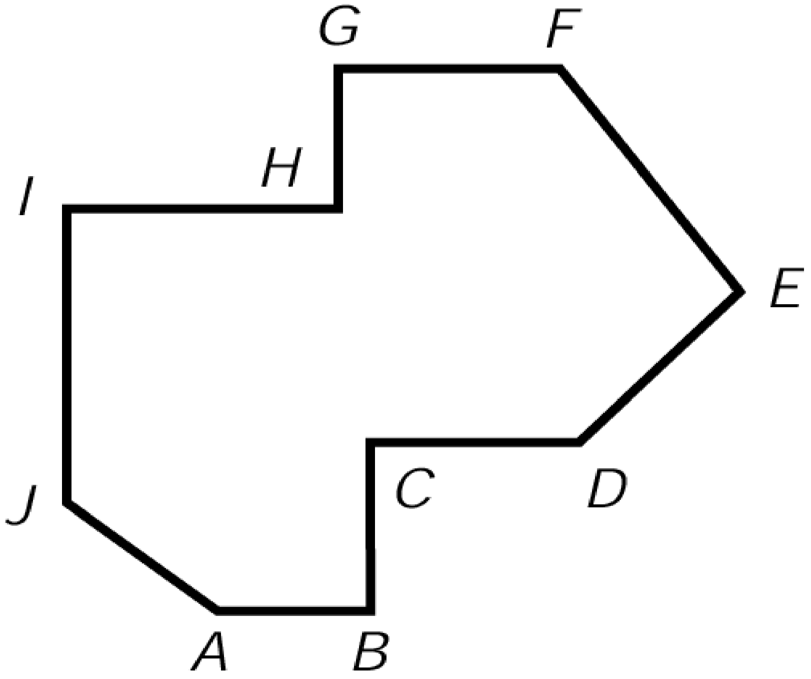
- Problem:
  - vertices are counted twice
- Solution:
  - If both neighboring vertices are on the same side of the scan line, don't count it
  - If both neighboring vertices are on different sides of a scan line, count it once
  - Compare current y value with y value of neighboring vertices





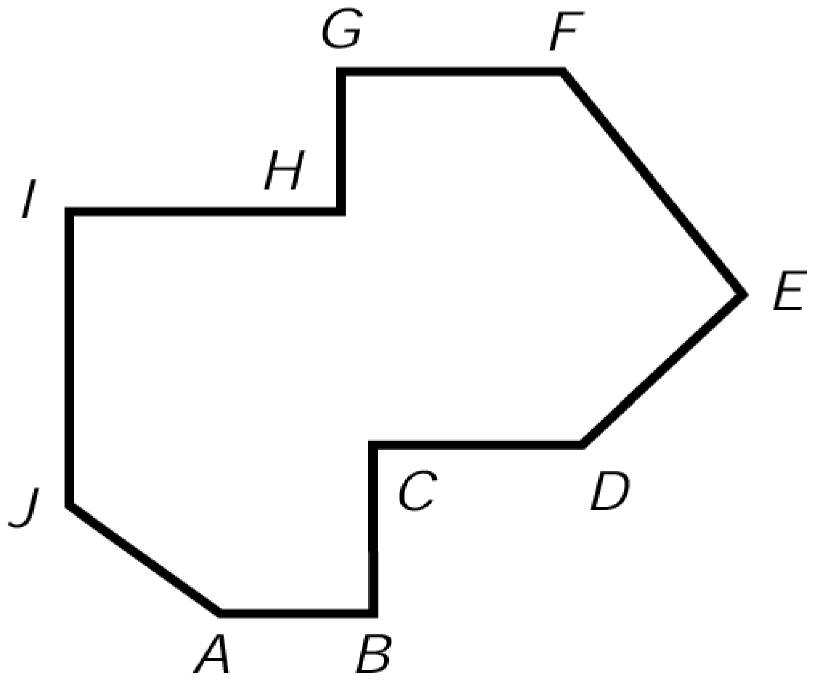
How to handle horizontal edges?

- Idea: don’t count their vertices
- Apply open and closed status to vertices to other edges
  - y_{min} vertex closed
  - y_{max} vertex is open
- On AB, A is at y_{min} for JA; AB does not contribute, B_p is odd and draw AB
- Edge BC has y_{min} at B, but AB does not contribute, B_p becomes even and drawing stops



How to handle horizontal edges?

- Start drawing at IJ (B_p becomes odd).
- C is y_{max} (open) for BC. B_p doesn't change.
- Ignore CD. D is y_{min} (closed) for DE. B_p becomes even. Stop drawing.
- I is y_{max} (open) for IJ. No drawing.
- Ignore IH. H is y_{min} (closed) for GH. B_p becomes odd. Draw to FE.
- Ignore GF. No drawing

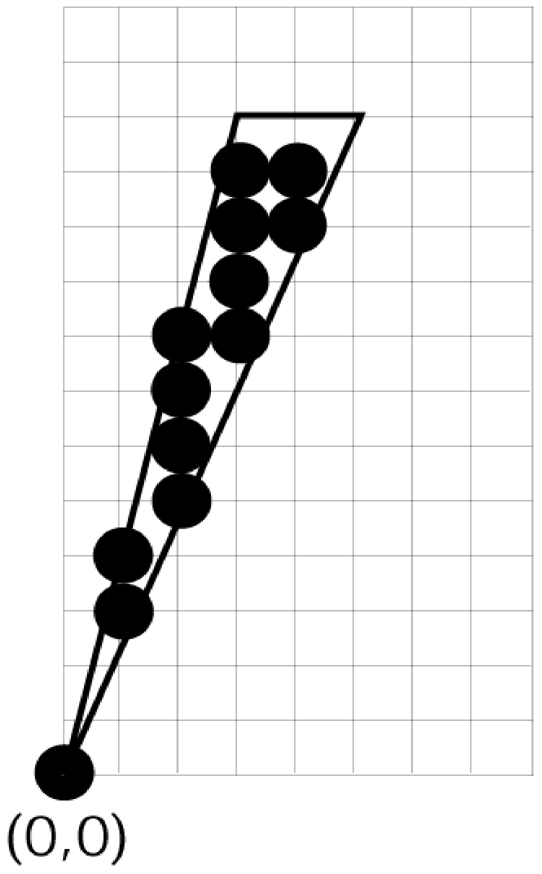


Polygon Filling Algorithm

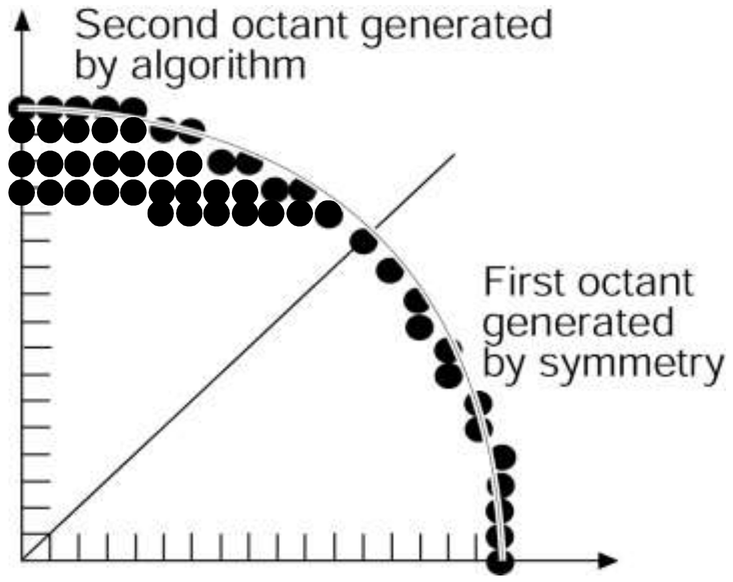
- For each polygon
  - For each edge, mark each scan-line that the edge crosses by examining its y_{min} and y_{max}
    - If edge is horizontal, ignore it
    - If y_{max} on scan-line, ignore it
    - If $y_{min} <= y < y_{max}$ add edge to scan-line y's edge list
  - For each scan-line between polygon's y_{min} and y_{max}
    - Calculate intersections with edges on list
    - Sort intersections in x
    - Perform parity-bit scan-line filling
    - Check for double intersection special case
  - Clear scan-lines' edge list

How to handle slivers?

- When the scan area does not have an “interior”
- Solution: use anti-aliasing
- But, to do so will require softening the rules about drawing only interior pixels

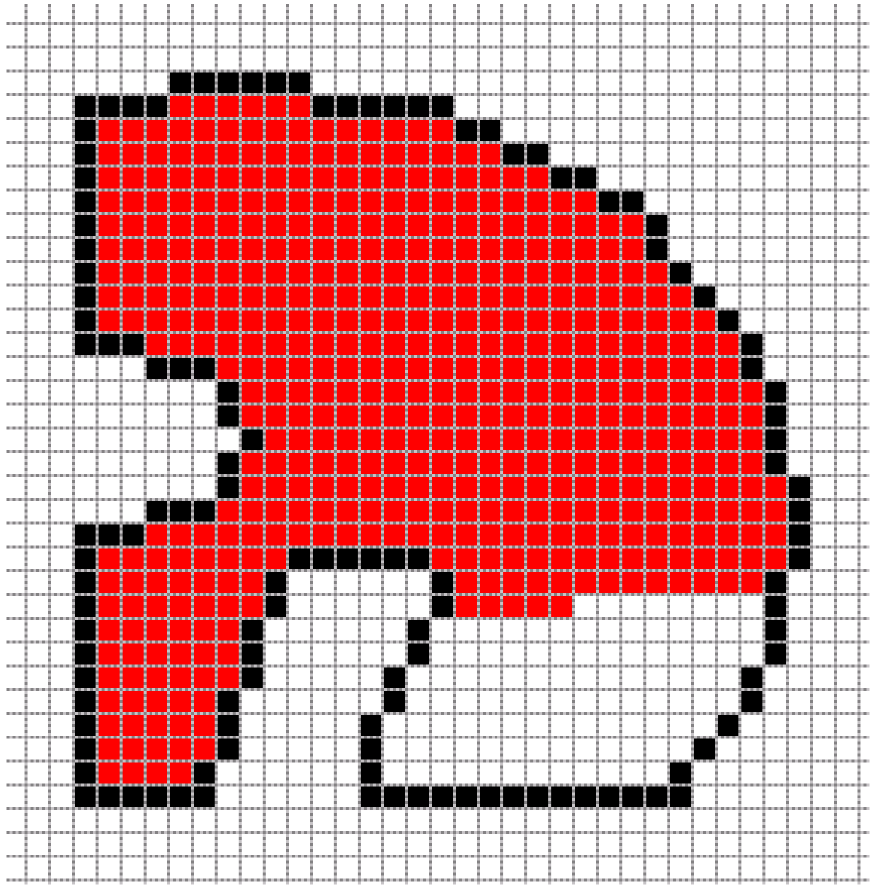


Scan Filling Curved Objects


Second octant generated by algorithm
First octant generated by symmetry

- Hard in general case
- Easier for circles and ellipses.
- Use midpoint Alg to generate boundary points.
- Fill in horizontal pixel spans
- Use symmetry

Boundary-Fill Algorithm



- Start with some internal point (x,y)
- Color it
- Check neighbors for filled or border color
- Color neighbors if OK
- Continue recursively

4 Connected Boundary-Fill Alg

```
Void BoundaryFill4( int x, int y, int fill,
  int bnd)
{
   If Color(x,y) != fill and Color(x,y) != bnd
   {
     SetColor(x,y) = fill;
     BoundaryFill4(x+1, y, fill, bnd);
     BoundaryFill4(x, y +1, fill, bnd);
     BoundaryFill4(x-1, y, fill, bnd);
     BoundaryFill4(x, y -1, fill, bnd);
   }
}
```

Boundary-Fill Algorithm

- Issues with recursive boundary-fill algorithm:
  - May make mistakes if parts of the space already filled with the Fill color
  - Requires very big stack size

- More efficient algorithms
  - First color contiguous span along one scan line
  - Only stack beginning positions of neighboring scan lines

Course Status

So far everything straight lines!

- How to model 2D curved objects?
  - Representation
    - Circles
    - Types of 2D Curves
    - Parametric Cubic Curves
    - Bézier Curves, (non)uniform, (non)rational
    - NURBS
  - Drawing of 2D Curves
    - Line drawing algorithms for complex curves
    - DeCasteljeau, Subdivision, De Boor

Homework #2

- Modify homework #1
- Add “moveto” and “lineto” commands
- They define closed polygons
- Clip polygons against window with Sutherland-Hodgman algorithm
- Display edges with HW1 line-drawing code