2D Transformation
3D Transformation

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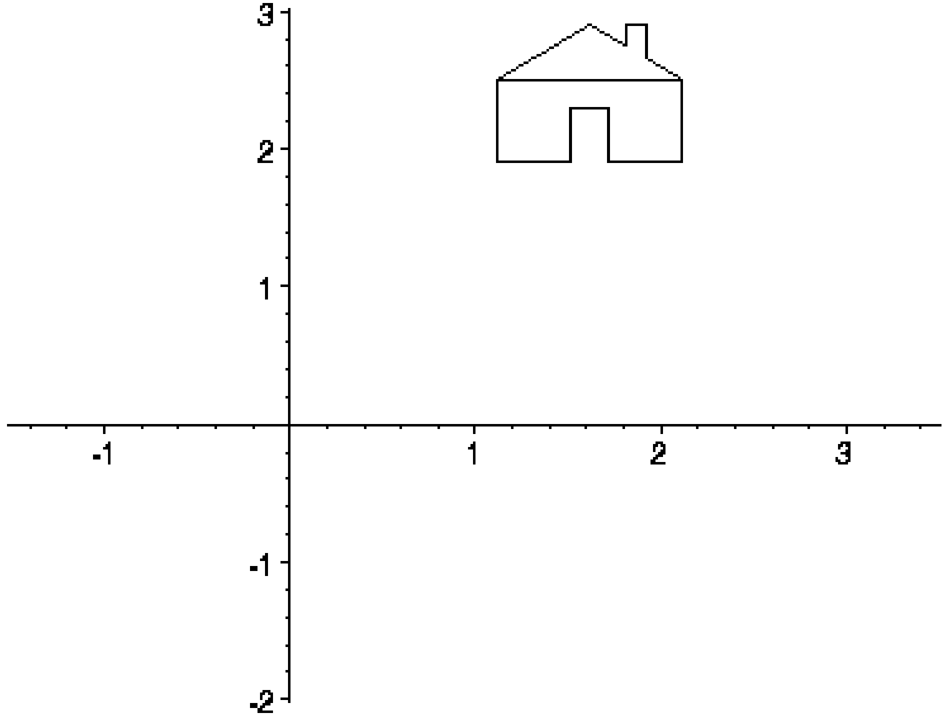
Transformation

- Transformations are a fundamental part of the computer graphics.
- Transformations are the movement of the object in Cartesian plane .



Transformation

- Transformations are of 6 Kinds:
  - **Geometric Transformation**
    - Translation
    - Rotation about the Origin
    - Scaling with respect to the Origin
    - Mirror Reflection about an Axix
  - **Inverse Geometric Transformation**
    - Translation
    - Rotation about the Origin
    - Scaling with respect to the Origin
    - Mirror Reflection about an Axix
  - **Co-ordinate Transformation**
    - Translation
    - Rotation about the Origin
    - Scaling with respect to the Origin
    - Mirror Reflection about an Axix
  - **Inverse Co-ordinate Transformation**
    - Translation
    - Rotation about the Origin
    - Scaling with respect to the Origin
    - Mirror Reflection about an Axix
  - **Composite Transformation**
  - **Instance Transformation**

Types of Transformation

- There are two types of transformation in computer graphics.
  (1) 2D transformation
  (2) 3D transformation

- Types of 2D and 3D transformation
  1. Translation
  2. Rotation
  3. Scaling
  4. Shearing
  5. Mirror reflection

Why We Use Transformation

- Transformation are used to position objects , to shape object, to change viewing positions , and even how something is viewed.

- In simple words transformation is used for
  (1) Modeling
  (2) viewing

3D Transformation

- When the transformation takes place on a 3D plane. It is called 3D transformation.
- Generalize from 2D by including **z** coordinate

Straight forward for translation and scale, rotation more difficult

$$\begin{bmatrix} a & b & c & t_x \\ d & e & f & t_y \\ g & h & i & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Homogeneous coordinates: 4 components

Transformation matrices: 4×4 elements

3D Translation

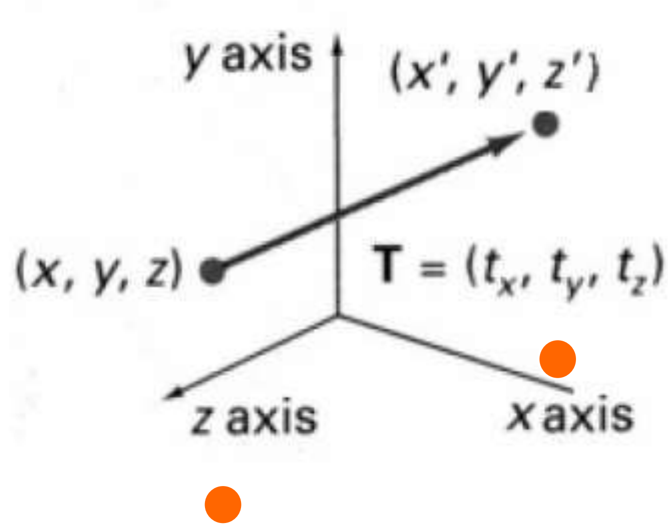
- Moving of object is called translation.
- In 3 dimensional homogeneous coordinate representation, a point is transformed from position

$$P = (x, y , z) \text{ to } P' = (x', y', z')$$

- This can be written as:-

Using **P' = T . P**

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



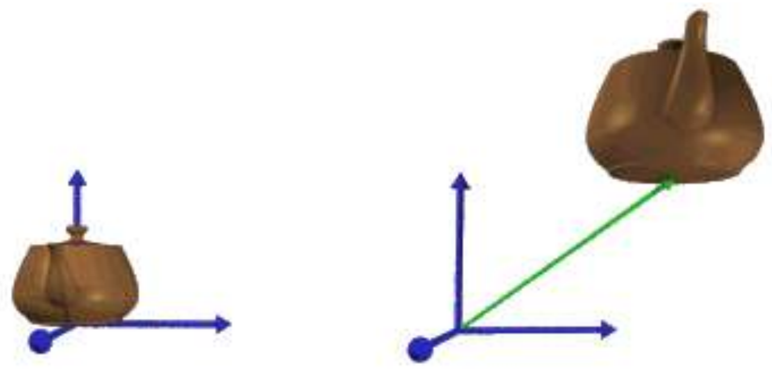
3D Translation

- The matrix representation is equivalent to the three equation.

  $\boldsymbol{x' = x + t_x}$,

  $\boldsymbol{y' = y + t_y}$,

  $\boldsymbol{z' = z + t_z}$ where parameter t_x , t_y , t_z are specifying translation distance for the coordinate direction x , y , z are assigned any real value.

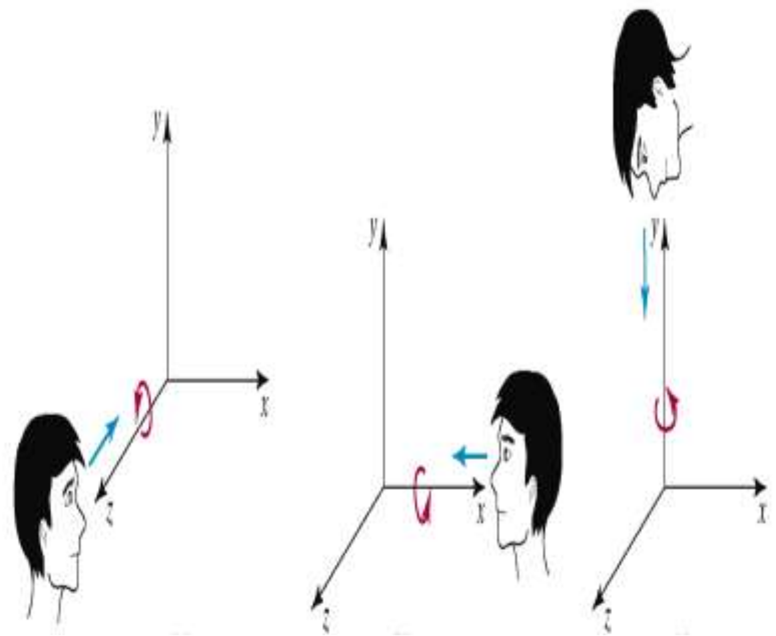


3D Rotation

- Where an object is to be rotated about an axis that is parallel to one of the coordinate axis, we can obtain the desired rotation with the following transformation sequence.

Coordinate axis rotation

Z- axis Rotation (Roll)
Y-axis Rotation (Yaw)
X-axis Rotation (Pitch)



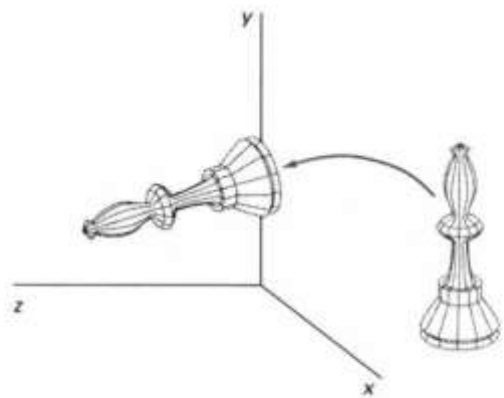
X-Axis Rotation

The equation for X-axis rotation

x' = x

y' = y cosθ – z sinθ

z' = y sinθ + z cosθ



$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Y-Axis Rotation

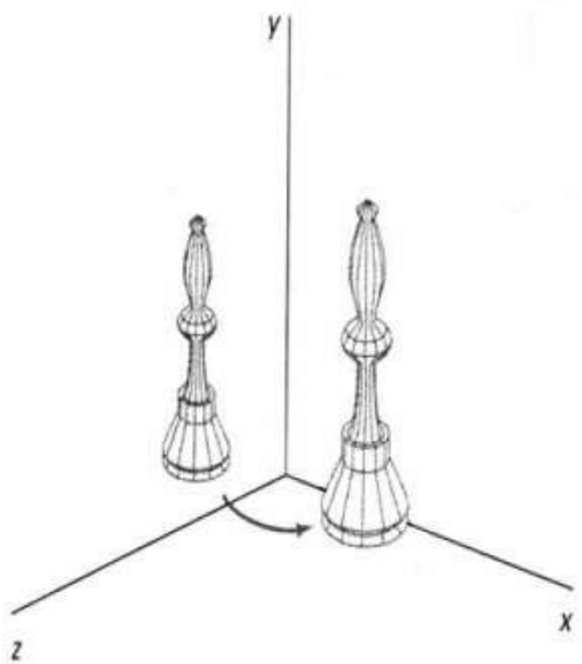
The equation for Y-axis rotaion

x’ = x cosθ + z sinθ

y’ = y

z’ = z cosθ - x sinθ

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



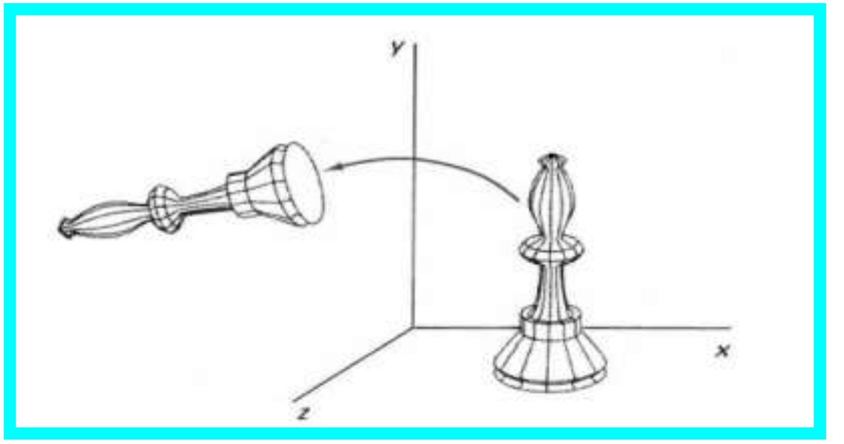
Z-Axis Rotation

The equation for Z-axis rotation

$x' = x\cos\theta - y\sin\theta$

$y' = x\sin\theta + y\cos\theta$

$z' = z$

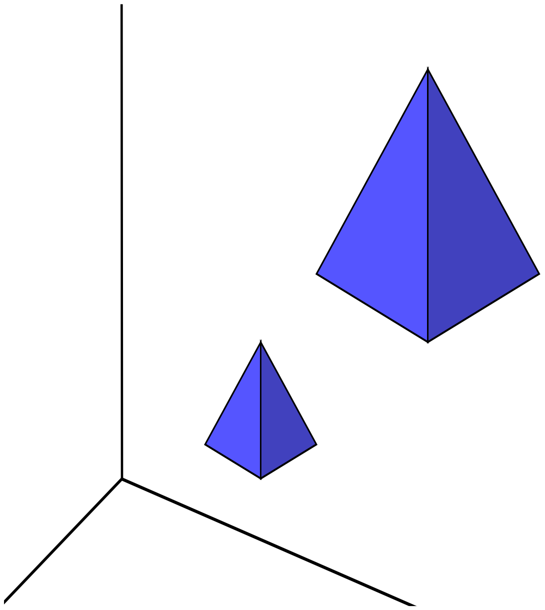


$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

3D Scaling

- Changes the size of the object and repositions the object relative to the coordinate origin.

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



3D Scaling

- The equations for scaling

$$x' = x \,.\, sx$$

$S_{sx,sy,sz}$ □ $y' = y \,.\, sy$

$$z' = z \,.\, sz$$

3D Reflection

- Reflection in computer graphics is used to emulate reflective objects like mirrors and shiny surfaces
- Reflection may be:
  - an x-axis, y-axis , z-axis. and also
  - in the planes xy-plane, yz-plane, and zx-plane.

- Reflection relative to a given Axis are
  - equivalent to 180 Degree rotations.

3D Reflection

- Reflection about x-axis:-

x'=x   y'=-y   z'=-z

$$\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

Reflection about y-axis:-

y'=y   x'=-x   z'=-z

3D Reflection

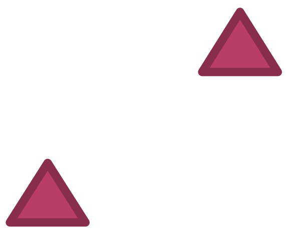
- The matrix for reflection about y-axis:-

$$\begin{matrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

- Reflection about z-axis:-

x’=-x   y’=-y   z’=z

$$\begin{matrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$



3D Shearing

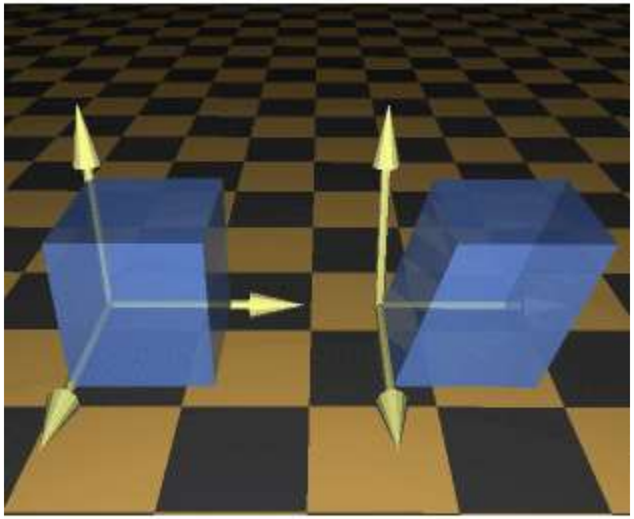
- **Modify object shapes**
- **Useful for perspective projections**
- When an object is viewed from different directions and at different distances, the appearance of the object will be different.
- Such view is called perspective view.
- Perspective projections mimic what the human eyes see.

e.g. draw a cube (3D) on a screen (2D) Alter the values for **x** and **y** by an amount proportional to the distance from z_{ref}

3D Shearing

- Matrix for 3d shearing
- Where a and b can be assigned any real value.

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & a & 0 \\ 0 & 1 & b & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



3D Shearing

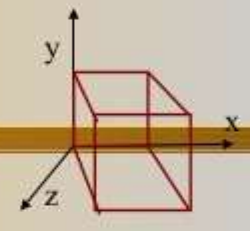
- In (y, z) w.r.t. x value $SH_{yz} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ sh_y & 1 & 0 & 0 \\ sh_z & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

- In (z, x) w.r.t. y value $SH_{xz} = \begin{bmatrix} 1 & sh_x & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & sh_z & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

- In (x, y) w.r.t. z value $SH_{xy} = \begin{bmatrix} 1 & 0 & sh_x & 0 \\ 0 & 1 & sh_y & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

3D Shearing

Shear along Z-axis



$$SH_{xy}(sh_x, sh_y) \;*\; P \;=\; P'$$

$$\Downarrow \qquad\qquad \Downarrow \qquad\qquad \Downarrow$$

$$\begin{bmatrix} 1 & 0 & sh_x & 0 \\ 0 & 1 & sh_y & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x + z * sh_x \\ y + z * sh_y \\ z \\ 1 \end{bmatrix}$$

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We have Learnt Transformation