

SNS COLLEGE OF TECHNOLOGY



(An Autonomous Institution)
Coimbatore-641035.

UNIT-II COMPLEX DIFFERENTIATION

Harmonic Conjugate

construction of conjugate Harmonic function: * It the lead prott us given, then $V = \int \left[-\frac{\partial u}{\partial y} dx + \frac{\partial u}{\partial x} dy \right]$ * If the froagforault pout V is given, then u= [ov dx - ox dy] J. Show that u= y+e cosy is harmonec and bence find Ets conjugate boomonie. Soln. agres $\frac{\partial y}{\partial x} = e^{x} \cos y$ $\frac{\partial^{2} u}{\partial x^{2}} = e^{x} \cos y$ $\frac{\partial^{2} u}{\partial y^{2}} = -e^{x} \cos y$ $\frac{3^{2}u}{3x^{2}} + \frac{3^{2}u}{3y^{2}} = e^{x}\cos y - e^{x}\cos y$ - Hence u sattesfees laplace egn. : a & barmongc. $V = \int \int -\frac{\partial y}{\partial y} \, dx + \frac{\partial y}{\partial x} \, dy$ = [-(1-ex sqn y) dre+ ex coc y dy] = \ -dx + \ \ e^ x geny dx + \ \ e^ x coc y dy = -x + en siny + en siny + c V = 2ex sany - x + C



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of Show that u= cos or coshy & boomongs.

Fond & conjugate boomongs.

Given $u = \cos x \cosh y$ $\frac{\partial u}{\partial x} = -\sin x \cosh y$ $\frac{\partial^2 u}{\partial x^2} = -\cos x \cosh y$ $\frac{\partial^2 u}{\partial y^2} = \cos x \cosh y$

 $\frac{3^2 u}{8x^2} + \frac{3^2 u}{8y^2} = -\cos x \cosh y + \cos x \cosh y$

Hence u satectées laplace egn.

: u & basinosmac.

Now $V = \int \left[-\frac{\partial u}{\partial y} dx + \frac{\partial u}{\partial x} dy \right]$

= [- cos x spnhy dx - spnx cos hy dy]

= - SPAR SPANY + cas SPAR & SPANY do +C

V = -2 square squary + C

3]. Prove that $u = x^3 - 3xy^2 + 3x^2 - 3y^2 is$ bourmonic function. Find conjugate hormonic

Soln. Geren u= 23-324+324-34

$$\frac{\partial u}{\partial x} = 3x^{2} - 3y^{2} + 6x \left| \frac{\partial u}{\partial x^{2}} = -6x + 6 \right|$$

$$\frac{\partial u}{\partial y} = -6xy - 6y \left| \frac{\partial^{2} u}{\partial y^{2}} = -6x - 6 \right|$$



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$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 6x + 6 - 6x - 6$$

$$= 0$$

Hence u satisfies laplace egn. :. u 38 harmonic.

NOW,

$$V = \int \left[-\frac{\partial u}{\partial y} \, dx + \frac{\partial u}{\partial x} \, dy \right]$$

$$= \int \left[-\left(-6xy - 6y \right) \, dx + \left(3x^2 - 3y^2 + 6x \right) \, dy \right]$$

$$= \int \left(6xy + 6y \right) \, dx + \left(3x^2 - 3y^2 + 6x \right) \, dy$$

$$= \left(6x^2y + 6xy + 3x^2y - \frac{3y^3}{3} + 6xy \right)$$

$$V = 6x^2y + 12xy - y^3 + C$$

$$V = 6x^2y + 12xy - y^3 + C$$

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