

SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution) Coimbatore-641035.



## UNIT-III COMPLEX DIFFERENTIATION

## HARMONIC FUNCTION

Laplace equation:  $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$  is called Laplace equation. Harmong coquation ; Any function with a variables having the and order portral derivatives which satisfies raplace eqn. B called a barmonge eqn. conjugate HastmonPC punction: If u and y core basimonic functions such that utiv is analytic, then each is called the conjugate barmonge function of the other. Hore u is conjugate barmonge of V and V is conjugate harmonic of u. ] prove that 4= @ casy & barmongc. Soln. Geven u= ex cos u  $\frac{\partial u}{\partial x} = e^{x} \cos y \qquad \left| \frac{\partial u}{\partial y} = -e^{x} \sin y \right| \\ \frac{\partial^{2} u}{\partial x^{2}} = e^{x} \cos y \qquad \left| \frac{\partial^{2} u}{\partial y^{2}} = -e^{x} \cos y \right|$  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = e^{\chi} \cos y - e^{\chi} \cos y$ . = 0 Hence u satisfees laplace equation. . The function 4 & barmonic. 2]. prove that  $u = \frac{1}{2} \log (x^2 + y^2)$  is harmonfe.



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Soln. Unver  $u = \frac{1}{2} \log (z^2 + y^2)$ .  $\frac{\partial u}{\partial x} = \frac{1}{2} \frac{1}{x^2 + y^2} (2x)$  $= \frac{\mathcal{X}}{\mathcal{X}^{\mathcal{R}} + \mathcal{Y}^{\mathcal{R}}}$   $\frac{\partial^{\mathcal{R}} u}{\partial x^{\mathcal{R}}} = \frac{(x^{\mathcal{R}} + \mathcal{Y}^{\mathcal{R}})(1) - x(\mathcal{Q}_{\mathcal{X}})}{(x^{\mathcal{R}} + \mathcal{Y}^{\mathcal{R}})^{\mathcal{R}}}$  $= \frac{2e^{2}+y^{2}-22e^{2}}{(2e^{2}+y^{2})^{2}}$  $= -\frac{x^{2}+y^{2}}{(x^{2}+y^{2})^{2}}$ <u> 84</u> = 1 3 (24) 1 101 101  $= \frac{y}{(x^2 + y^2)^2}$ =  $\frac{(x^2 + y^2)(1) - y(2y)}{(x^2 + y^2)^2}$  $= \frac{x^2 - y^2}{(x^2 + y^2)}$  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -\frac{x^2 + y^2}{(x^2 + y^2)^2} + \frac{x^2 - y^2}{(x^2 + y^2)^2}$  $= -\frac{x^2 + y^2 + x^2 - y^2}{(x^2 + y^2)^2}$ = 0Hence u satesfres laplace equ is boomoone u ((2) - 1