



SNS COLLEGE OF TECHNOLOGY



(AN AUTONOMOUS INSTITUTION)

COIMBATORE-35

DEPARTMENT OF MATHEMATICS

UNIT II
PART - A

1. Solve: $(D^2 - 2D + 2)y = 0$.
2. Solve: $(D^2 - 6D + 13)y = 0$.
3. Solve: $(D^2 + 4)y = 0$.
4. Solve: $(D^2 + 1)y = e^{-x}$.
5. Solve: $\frac{d^4 y}{dx^4} = 16y$.
6. Solve: $\frac{d^2 y}{dx^2} + 4y = e^{-2x}$.
7. Find the particular integral of $\frac{d^2 y}{dx^2} + 4y = \sin 2x$.
8. Find the particular integral of $(D^4 + D^2)y = \cos x$.
9. Find the particular integral of $(D^4 + D^2)y = \sin x$.
10. Solve: $(D^3 + 3D^2 + 3D + 1)y = 0$.
11. Solve: $(D^4 - 2D^3 + D^2)y = 0$.
12. Solve: $(D^4 + 2D^2 + 1)y = 0$.
13. Solve: $(4D^2 - 4D + 1)y = 4$.
14. Solve: $(D - 2)^2 y = e^{2x}$.

Answers

The Auxillary Equation is $m^2 - 2m + 2 = 0$

$$1) m = \frac{2 \pm \sqrt{4-8}}{2} = \frac{2 \pm 2i}{2} = 1 \pm i$$

The Complete Solution is $y = e^x (A \cos x + B \sin x)$

The Auxillary Equation is $m^2 - 6m + 13 = 0$

$$2) m = \frac{6 \pm \sqrt{36 - 52}}{2} = \frac{6 \pm 4i}{2} = 3 \pm 2i$$

The Complete Solution is $y = e^{3x} (A \cos 2x + B \sin 2x)$

The Auxillary Equation is $m^2 + 4 = 0$

$$3) m = \pm 2i$$

The Complete Solution is $y = A \cos 2x + B \sin 2x$

The Auxillary Equation is $m^2 + 1 = 0$

$$m = \pm i$$

$$4) \text{ The Complementary Function} = A \cos x + B \sin x$$

$$P.I = \frac{1}{D^2 + 1} e^{-x} = \frac{1}{2} e^{-x}$$

The Complete Solution is $y = A \cos x + B \sin x + \frac{1}{2} e^{-x}$

The Auxillary Equation is $m^4 - 2^4 = 0$

$$(m^2 - 2^2)(m^2 + 2^2) = 0$$

$$5) (m - 2)(m + 2)(m^2 + 4) = 0$$

$$m = -2, 2, \pm 2i$$

The Complementary Function = $Ae^{-2x} + Be^{2x} (C \cos 2x + D \sin 2x)$

The Auxillary Equation is $m^2 + 4 = 0$

$$(m^2 + 2^2) = 0$$

$$m = \pm 2i$$

$$6) \text{ The Complementary Function} = A \cos 2x + B \sin 2x$$

$$P.I = \frac{1}{D^2 + 4} e^{-2x} = \frac{1}{8} e^{-2x}$$

The Complete Solution is $y = A \cos 2x + B \sin 2x + \frac{1}{8} e^{-2x}$

$$7) P.I = \frac{1}{D^2 + 4} \sin 2x = \frac{x}{2D} \sin 2x = \frac{-x \cos 2x}{4}$$

$$8) P.I = \frac{1}{D^4 + D^2} \cos x = \frac{x}{4D^3 + 2D} \cos x = \frac{x}{-4D + 2D} \cos x = \frac{x}{-2D} \cos x = \frac{-x \sin x}{2}$$

$$9) P.I = \frac{1}{D^4 + D^2} \sin x = \frac{x}{4D^3 + 2D} \sin x = \frac{x}{-4D + 2D} \sin x = \frac{x}{-2D} \sin x = \frac{x \cos x}{2}$$

The Auxillary Equation is $m^3 + 3m^2 + 3m + 1 = 0$

$$-1 \left| \begin{array}{cccc} 1 & 3 & 3 & 1 \\ 0 & -1 & -2 & -1 \end{array} \right.$$

$$10) \quad \begin{array}{cccc|c} 1 & 2 & 1 & 0 & 0 \end{array}$$

$$m = -1, m^2 + 2m + 1 = 0$$

$$m = -1, m = -1, m = -1$$

The Complementary Function = $(Ax^2 + Bx + C)e^{-x}$

The Auxillary Equation is $m^4 - m^3 + m^2 = 0$

$$m^2(m^2 - m + 1) = 0$$

$$11) \quad m = 0, 0, m = \frac{1 \pm \sqrt{1-4}}{2} = \frac{1 \pm \sqrt{3}i}{2}$$

The Complete Solution is $y = (Ax + B) + e^{\frac{x}{2}} \left(C \cos \frac{\sqrt{3}}{2} x + D \sin \frac{\sqrt{3}}{2} x \right)$

The Auxillary Equation is $m^4 + 2m^2 + 1 = 0$

Take, $t = m^2$, Then $t^2 + 2t + 1 = 0$

$$12) \quad t = -1, -1$$

$$m^2 = -1, m^2 = -1$$

$$m = \pm i, \pm i$$

The Complete Solution is $y = (Ax + B) \cos x + (Cx + D) \sin x$

The Auxillary Equation is $4m^2 - 4m + 1 = 0$

$$m = \frac{4 \pm \sqrt{16-16}}{8} = \frac{1}{2} (\text{twice})$$

13) The Complementary function is $(Ax + B)e^{\frac{x}{2}}$

$$P.I = \frac{1}{4D^2 - 4D + 1} 4 = 4$$

The Complete Solution is $y = (Ax + B)e^{\frac{x}{2}} + 4$

The Auxillary Equation is $(m - 2)^2 = 0$

$m = 2$ (twice)

14) The Complementary funtion is $= (Ax + B)e^{2x}$

$$P.I = \frac{1}{(D-2)^2} e^{2x} = \frac{x}{2(D-2)} e^{2x} = \frac{x^2}{2D} e^{2x} = \frac{x^2}{4} e^{2x}$$

The Complete Solution is $y = (Ax + B)e^{2x} + \frac{x^2}{4} e^{2x}$

LEVEL-2, QUESTIONS

1. Find the particular integral of $(D^2+1)y = \cos(2x-1)$.
2. Find the particular integral of $y'' + 2y' + 5y = e^{-x} \cos 2x$.
3. Find the particular integral of $(D^2 + 4D + 5)y = e^{-2x} \cos x$.
4. Find the particular integral of $(D^2 + 1)y = \cosh 2x$.
5. Find the particular integral of $(D - 1)^2 y = \sinh 2x$.
6. Find the particular integral of $(D + 1)^2 y = e^{-x} \cos x$.
7. Find the particular integral of $(D^2 + 4)y = \cos^2 x$.
8. Find the particular integral of $(D^3 - 7D - 6)y = x$.
9. Find the particular integral of $(D^2 + 4D + 3)y = 2e^{-x}(x^2 + 2)$.
10. Find the particular integral of $(D^2 + 6D + 8)y = \cos^2 x$.
11. Find the particular integral of $(D - 3)^2 y = xe^{-2x}$.
12. Find the particular integral of $(D^2 + 4D + 4)y = xe^{-2x}$.

Answers

$$1) P.I = \frac{1}{(D^2+1)} \cos(2x-1) = \frac{1}{(-4+1)} \cos(2x-1) = -\frac{1}{3} \cos(2x-1)$$

$$2) P.I = \frac{1}{(D^2+2D+5)} e^{-x} \cos 2x = e^{-x} \frac{1}{((D-1)^2+2(D-1)+5)} \cos 2x$$
$$= e^{-x} \frac{1}{(D^2+4)} \cos 2x = e^{-x} \frac{x}{2D} \cos 2x = \frac{xe^{-x} \sin 2x}{4}$$

$$\begin{aligned}
 3) \quad P.I &= \frac{1}{(D^2+4D+5)} e^{-2x} \cos x = e^{-2x} \frac{1}{((D-2)^2+4(D-2)+5)} \cos x \\
 &= e^{-2x} \frac{1}{D^2+1} \cos x = e^{-2x} \frac{1}{-1+1} \cos x = \frac{x e^{-2x} \sin x}{2}
 \end{aligned}$$

$$\begin{aligned}
 P.I &= \frac{1}{(D^2+1)} \cosh 2x = \frac{1}{(D^2+1)} \left(\frac{e^{-2x} + e^{2x}}{2} \right) \\
 4) &= \frac{1}{2} \left\{ \frac{1}{D^2+1} e^{-2x} + \frac{1}{D^2+1} e^{2x} \right\} = \frac{1}{2} \left\{ \frac{1}{-4+1} e^{-2x} + \frac{1}{-4+1} e^{2x} \right\} \\
 &= \frac{1}{2} \left\{ \frac{1}{-3} e^{-2x} + \frac{1}{-3} e^{2x} \right\} = \frac{1}{-3} \left(\frac{e^{-2x} + e^{2x}}{2} \right) = \frac{\cosh 2x}{-3}
 \end{aligned}$$

$$\begin{aligned}
 P.I &= \frac{1}{(D-1)^2} \sinh 2x = \frac{1}{(D^2-2D+1)} \left(\frac{e^{-2x} - e^{2x}}{2} \right) \\
 5) &= \frac{1}{2} \left\{ \frac{1}{D^2-2D+1} e^{-2x} - \frac{1}{D^2-2D+1} e^{2x} \right\} = \frac{1}{2} \left\{ \frac{1}{-4-4+1} e^{-2x} - \frac{1}{-4-4+1} e^{2x} \right\} \\
 &= \frac{1}{2} \left\{ \frac{1}{-7} e^{-2x} - \frac{1}{-7} e^{2x} \right\} = -\frac{1}{7} \left(\frac{e^{-2x} - e^{2x}}{2} \right) = -\frac{1}{7} \sinh 2x
 \end{aligned}$$

$$\begin{aligned}
 6) \quad P.I &= \frac{1}{(D+1)^2} e^{-x} \cos x = e^{-x} \frac{1}{((D-1)+1)^2} \cos x \\
 &= e^{-x} \frac{1}{D^2} \cos x = e^{-x} \frac{1}{-1} \cos x = -e^{-x} \cos x
 \end{aligned}$$

$$\begin{aligned}
 P.I &= \frac{1}{(D^2+4)} \cos^2 x = \frac{1}{(D^2+4)} \left(\frac{1+\cos 2x}{2} \right) \\
 7) &= \frac{1}{2} \frac{1}{(D^2+4)} (1+\cos 2x) = \frac{1}{2} \left\{ \frac{1}{(D^2+4)} (1) + \frac{1}{(D^2+4)} \cos 2x \right\} \\
 &= \frac{1}{2} \left\{ \frac{1}{4} + \frac{1}{(-4+4)} \cos 2x \right\} = \frac{1}{2} \left\{ \frac{1}{4} + \frac{x}{2D} \cos 2x \right\} = \frac{1}{2} \left\{ \frac{1}{4} + \frac{x \sin 2x}{2} \right\} = \frac{1}{8} \{1 + x \cos 2x\}
 \end{aligned}$$

