UNIT – 1

Two marks

1. Define i) Discrete random variable

   ii) Continuous random variable

   i) Let X be a random variable, if the number of possible values of X is finite or count ably finite, then X is called a discrete random variable.

   ii) A random variable X is called the continuous random variable, if x takes all its possible values in an interval.

2. Define probability mass function (PMF):

   Let X be the discrete random variable taking the values ,X_1 , X_2 ……….

   Then the number P (X_i) = P(X =X_i) is called the probability mass function of X and it satisfies the following conditions.

   i) P (X_i) ≥ 0 for all;

   ii) $\sum_{i=1}^{\infty}$ P (X_i) = 1

3. Define probability Density function (PDF):

   Let x be a continuous random variable. The Function f(x) is called the probability density function (PDF) of the random variable x if it satisfies.

   i) f(x) ≥ 0

   ii) $\int_{-\infty}^{\infty} f(x)dx = 1$

4. Define cumulative distribution function (CDF):

   Let x be a random variable. The cumulative distribution function, denoted by F(X)and is given by F(X)=P(X≤x)

5. If x is a discrete R.V having the p.m.f

X:	-1	0	1

P(X):	k	2k	3k

Find P(x≥ 0)

Answer: $6k = 1 \Rightarrow k = \frac{1}{6}$

$P[x \geq 0] = 2k + 3k \Rightarrow P[x \geq 0] = \frac{1}{6}$

6. The random variable x has the p.m.f. P (x)=$\frac{x}{15}$, x=1,2,3,4,5 and = 0 else where.

Find P $[\frac{1}{2} < x < \frac{5}{2} / x > 1]$.**Answer:**

$P[\frac{1}{2} < x < \frac{5}{2} / x > 1] = \frac{P[x=2]}{P(x>1)} = \frac{P[x=2]}{1-P(x \leq 1)} = \frac{2/15}{1-1/15} = \frac{1}{7}$

7. If the probability distribution of X is given as :

X	1	2	3	4
P(X)	0.4	0.3	0.2	0.1

Find P $[\frac{1}{2} < x < \frac{7}{2} / x > 1]$.

Answer :

$P[\frac{1}{2} < x < \frac{7}{2} / x > 1] = \frac{P[1<x<7/2]}{P(x>1)} = \frac{P(x=2)+P(x=3)}{1-P(x=1)} = \frac{0.5}{0.6} = \frac{5}{6}$

8. A.R.V. X has the probability function

X	-2	-1	0	1
P(X)	0.4	k	0.2	0.3

Find k and the mean value of X

Answer:

k=0.1 Mean $= \sum xP(x) = \frac{1}{10}[-8-1+0+3] = -0.6$

9. If the p.d.fof a R.V. X is $f(x) = \frac{x}{2}$ in $0 \leq x \leq 2$, find

P$[x > 1.5/x > 1]$.

Answer :

P$[x > 1.5/x > 1] = \frac{P[x>1.5]}{P(x>1)} = \frac{\int_{1.5}^{2} \frac{x}{2}dx}{\int_{1}^{2} \frac{x}{2}dx} = \frac{4-2.25}{4-1} = 0.5833$

10.If the p.d.f of a R.V.X is given by *f*(x) = {1/4,-2<x<2.0, else where. Find P[|X|>1].

Answer:

P[|X|>1] =1- P[|X|<1] = 1-$\int_{-1}^{1} \frac{1}{4} dx = \frac{1}{2}$

11. If *f*(x) = kx^2, 0<x<3 is to be density function, Find the value of k.

Answer:

$\int_0^3 kx^2 dx$= 1 ⇒ 9k = 1 ∴ k = $\frac{1}{9}$

12. If the c.d.f. of a R.V X is given by F(x) = 0 for x<0;= $\frac{x^2}{16}$for 0$\leq x < 4$ $and =$ $1\ for\ x\ \geq 4, find\ \ P(X > 1/X < 3)$.

Answer:

$$P(X > 1/X < 3) = \frac{P[1<X<3]}{P[0<X<3]} = \frac{F(3)-F(1)}{F(3)-F(0)} = \frac{8/16}{9/16} = \frac{8}{9}.$$

13.The cumulative distribution of X is F(x) = $\frac{x^3+1}{9}$ $, -1, < X < 2\ and$ $=$ 0, otherwise. Find P[0 <X<1].

Answer:

P[0<X<1] = F(1) – F(0) = $\frac{2}{9} - \frac{1}{9} = \frac{1}{9}$

14. A Continuous R.V X that can assume any value between x=2 and x=5 had the p.d.f f(x) = k(1+x). Find P(x<4).

Answer:

$$\int_2^3 k(1+x)dx = 1 \Rightarrow \frac{27k}{2} = 1 \therefore k = \frac{2}{27}$$

P[X<4] = $\int_2^4 \frac{2}{27}(1+x)dx = \frac{16}{27}$

15. The c.d.f of X is given by F (x) = $\begin{cases} 0, x > 0 \\ x^2, \ 0 \le x \le 1 \\ 1, x > 1 \end{cases}$ Find the p.d.f of x, and obtain P(X>0.75).

Answer:

F(x) = $\frac{d}{dx}$F(x) = $\begin{cases} 2x, 0 \le x \le 1 \\ 0, otherwise \end{cases}$

P[x<0.75] = 1- P[X≤ $0.75] = 1 - F(0.75) = 1 - (0.75)^2 = 0.4375$

16. Check whether f(x)=$\frac{1}{4}xe^{-x/2}$for 0<x<∞ can be the p.d.f of X.

Answer:

$= \int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{\infty} \frac{x}{4} e^{-x/2}\, dx = \int_0^{\infty} te^{-1}\, dt$ where t = $\frac{x}{2}$

=$(-\, te^{-1} - e^{-1})_0^{\infty}$ = - [0-1]=1

$$\therefore f(\,x\,) is\ the\ p.d.f\ of\ X.$$

17. A continuous R.V X has a p.d.f f(x) = $3x^2, 0 \le x \le 1$. Find b such that P(X>b)= 0.05.

Answer:

$$3\int_b^1 x^2dx = 0.05 \Rightarrow 1 - b^3 = 0.05 \Rightarrow b^3 = 0.95 \therefore b = (0.95)^{\frac{1}{3}}$$

18. Let X be a random variable taking values -1, 0 and 1 such that P(X=-1) = 2P(X=0) = P(X=1). Find the mean of 2X-5.

Answer:

$$\sum P(X=x)=1 \Rightarrow 5P(X=0)=1 \therefore P(X=0)=\frac{1}{5}$$

Probability distribution of X:

X	-1	0	1
P(X)	2/5	1/5	2/5

$$Mean = E(x) = \sum xp(x) = -1\left(\frac{2}{5}\right)+0\left(\frac{1}{5}\right)+1\left(\frac{2}{5}\right)=0$$

E[2X-5] = 2E(X) – 5 = 2[0] – 5 = -5.

19. Find the cumulative distribution function F(x) corresponding to the p.d.f.

F(x) = $\frac{1}{\pi(1+x^2)}$, -∞ $< x < \infty$.

Answer

F(x) = $\int_{-\infty}^{x} f(x)dx = \frac{1}{\pi}\int_{-\infty}^{x}\frac{dx}{1+x^2} = \frac{1}{\pi}[tan^{-1}\text{x}]$

$= \frac{1}{\pi}[\frac{\pi}{2} + tan^{-1}\text{ x}]$

20. The diameter of an electric cable, say X is assumed to a continues R.V with p.d.f of given by f(x) = kx(1-x), 0≤ $x \le 1$. Determine k and P$\left(x \le \frac{1}{3}\right)$

Answer:

$$\int_0^1 kx(1-x)dx = 1 \Rightarrow k\left[\frac{1}{2}-\frac{1}{3}\right]=1 \quad \therefore k=6$$

$$P\left[X \le \frac{1}{3}\right] = 6\int_0^{1/3}(x-x^2)dx = 6\left[\frac{x^2}{2}-\frac{x^3}{3}\right]_0^{1/3} = [(3x^2 - 2x^3)]_0^{1/3} = \frac{1}{3}-\frac{2}{27}=\frac{7}{27}$$

21. A random variable Xhas the p.d.f f(x) given by f(x)=$\begin{cases} Cxe^{-x}, if\ x > 0 \\ 0, if\ x \le 0 \end{cases}$. Find the value of C and C.D.F of X.

Answer:

$$C\int_0^{\infty} xe^{-x}dx = 1 \Rightarrow C[x(-e^{-x}]_0^{\infty} = 1$$

$$\therefore C[-0+1] = 1 \Rightarrow C = 1$$

$C.D.F : F(x) = \int_0^x f(x)dx = 1 - (1+x)e^{-x}$for x ≥ 0.

22. State the properties of cumulative distribution function.

Answer:

i) F(-∞)=0 and F(∞) $= 1$.

ii) F(∞) is non – decreasing function of X.

iii) If F(∞)$is\ the\ p.d.f$ of X, then f(x)=$F'(x)$

iv) P[a$\leq X \leq b$] = F(b) – F(a)

23. Define the raw and central moments of R.Vand state the relation between them.

Answer:

Raw moment μ'_r = E[X^r]

Central momentμ_r= E[$\{X - E(X)\}^r$].

$\mu_r = \mu'_r$- r$C_1\mu'_{r-1}\,\mu'_r$+ r$C_2\mu'_{r-2}(\mu'_r)^2$ - …….+$(-1)^r(\mu'_1)^r$

24.The first three moments of a R.V.X about 2 are 1, 16, -40. Find the mean, variance of X. Hence find μ_3.

Answer:

E(X) = $\mu'_1 + A \Rightarrow\ Mean = 1 + 2 = 3$

Variance = E(X^2)–[E((X)]$^2 = 16 - 1 = 15$

$\mu_3 = \mu'_3 - 3\mu'_2\mu'_1 + 2(\mu'_1)^3 = -86$

25. Find the r-th moment about origin of the R.V X with p.d.f$f(x) = \begin{cases} Ce^{-ax}, x \geq 0 \\ 0, \ else\ where \end{cases}$

Answer:

$$\int_0^\infty C\, e^{-ax}dx = 1 \Rightarrow C = a$$

$$\mu'_r = \int_0^\infty x^r\, f(x)dx = a\int_0^\infty x^{(r+1)-1}\, e^{-ax}dx = \frac{\sqrt{(r+1)}}{a^r} = \frac{r!}{a^r}$$

26. A C.R.V X has the p.d.f f(x)=$kx^2e^{-x}, x > 0$. Find the r-th moment about the origin.

Answer:

$$\int_0^\infty kx^2e^{-x}dx = 1 \Rightarrow k = \frac{1}{2}$$

$$\mu'_1 = E[X^r] = \frac{1}{2}\int_0^\infty x^{r+2}\, e^{-x}\, dx = \frac{1}{2}\sqrt{(r+3)} = \frac{(r+2)!}{2}$$

27.If X and Y are independent R,V's and Z = X+Y, prove that $M_x(t)M_y(t)$.

Answer:

$$M_z(t) = E[e^{tz}] = E\left[e^{t(X+Y)}\right] = E[e^{tx}]E[e^{ty}]$$
$$= M_x(t)M_y(t).$$

28.If the MGF of X is $M_x(t)$and if Y=aX+b show that $M_y(t) = e^{bt}M_x(at)$.

Answer:

$M_y(t) = E[e^{ty}] \ = E[e^{bt}e^{axt}] = e^{bt}E[e^{(at)X}] = e^{bt}M_x(at).$

29.If a R.V X has the MGF M(t)=$\frac{3}{3-t}$, obtain the mean and variance of X.

Answer:

M(t) = $\frac{3}{3[1-\frac{t}{3}]}$ =1+$\frac{t}{3}+\frac{t^2}{9}+\ \ldots..$

E(x) = Co-efficient of $\frac{t}{1!}$in (1) = $\frac{1}{3}$

E(X^2) = co-efficient of $\frac{t^2}{2!}$in (1) = $\frac{1}{9}$

∴ Mean = $\frac{1}{3}$ and V(X) = E(X^2) – [E(X)]^2 = $\frac{1}{9}$

30. If the r-th moment of a C.R.V X about the origin is r!, find the M.G. F of X.

Answer:

$$M_x(t) = \sum_{r=0}^{\infty} E[X^r]\ .\frac{t^r}{r!} = \sum_{r=0}^{\infty} t^r$$

$$= 1 + t + t^2 + \cdots \ = (1-t)^{-1} = \frac{1}{1-t}$$

31. If the MGF of a R.V. X is $\frac{2}{2-t}$, Find the standard deviation of x.

Answer:

$M_x(t)$ = $\frac{2}{2-t}$ = $(1-\frac{t}{2})^{-1} = 1+\frac{t}{2}+\frac{t^2}{4}+\cdots$

$E(X)$ = $\frac{1}{2}$; E(x^2)= $\frac{1}{2}$; V(X) = $\frac{1}{4}$ ⇒ $S.D\ of\ X = \frac{1}{2}$

32. Find the M.G.F of the R.V X having p.d.f $f(x) = \begin{cases} \frac{1}{3}, -1 < x < 2 \\ 0,\ \ else\ where \end{cases}$

Answer:

$M_x(t)$ = $\int_{-1}^{2}\frac{1}{3}e^{tx}dx$ = $\frac{1}{3t}[e^{2t}-e^{-t}]\ for\ t \neq 0$

When t=0 , $M_x(t) = \int_{-1}^{2}\frac{1}{3}\ dx = 1$

$$\therefore M_x(t) = \begin{cases} \frac{e^{2t} - e^{-t}}{3t}, t \neq 0 \\ 1, t = 0 \end{cases}$$

33. Find the MGF of a R.V X whose moments are given by $\mu'_r = (r = 1)!$

Answer:

$$M_x(t) = \sum_{r=0}^{\infty} E[X^r] \cdot \frac{t^r}{r!} = \sum_{r=0}^{\infty} (r+1)t^r$$

$$= 1 + 2t + 3t^2 + \cdots = (1-t)^{-2}$$

$$\therefore M_x(t) = \frac{1}{(1-t)^2}$$

34. Give an example to show that if p.d.f exists but M.G.F. does not exist.

Answer:

P(x)= $\begin{cases} \frac{6}{\pi^2 x^2}, x = 1,2,\ldots \\ 0, otherwise \end{cases}$

$$\sum P(x) = \frac{6}{\pi^2} \Rightarrow \sum_{x=1}^{\infty} \frac{1}{x^2} = \frac{6}{\pi^2}\left[\frac{\pi^2}{6}\right] = 1$$

∴ P(x) is a p.d.f.

But $M_x(t) = \frac{6}{\pi^2} \sum \frac{e^{tx}}{x^2}$, which is a divergent series

$$\therefore M_x(t)\ doesnt\ exist.$$

35. The moment generating function of a random variable X is given by $M_x(t) = \frac{1}{3}e^t + \frac{4}{15}e^{3t} + \frac{2}{15}e^{4t} + \frac{4}{15}e^{5t}$. Find the probability density function of X.

Answer:

X	1	2	3	4
P(X)	1/3	4/15	2/15	4/15

36. Let $M_x(t)\frac{1}{(1-t)}, t < 1$ be the M.G.F of a R.V X. Find the MGF of the RV Y=2X+1.

Answer:

If Y =aX+b, $M_y(t) = e^{bt}M_x(at)\ \therefore M_y(t) = \frac{e^t}{1-2t}$.

37. Suppose the MGF of a RV X is of the form $M_x(t) = (0.4e^t + 0.6)^8$. What is the MGF of the random variable Y=3X+2.

Answer:

$$M_y(t) = e^{2t}M_x(3t) = e^{2t}[(0.4)e^{3t} = 0.6)]^8$$

38. The moment generating function of a RV X is $\left[\frac{1}{5} + \frac{4e^t}{5}\right]^{15}$. Find the MGF of Y=2X + 3.

Answer:

If Y = 2X + 3, then $M_y(t) = e^{3t}M_x(2t)$.

$$\therefore M_y(t) = e^{3t}\left[\frac{1}{5} + \frac{4e^t}{5}\right]^{15}$$

39. If a random variable takes the values -1, 0 and 1 with equal probabilities, find the MGF of X.

Answer:

$M_x(t) = \sum \quad e^{tx}\ P(x) = \frac{1}{3}e^{-1} + \frac{1}{3} + \frac{1}{3}e^1 = \frac{1}{3}[1 + e^1 + e^{-1}]$

1. Determine the binomial distribution whose mean is 9 and whose standard deviation is $\frac{3}{2}$.

Answer:

np = 9 and npq = $\frac{9}{4}$. q = $\frac{npq}{np} = \frac{1}{4} \Rightarrow p = 1 - q = \frac{3}{4}$

np = 9 ⇒ n= $9 \times \frac{4}{3} = 12$

$\therefore P[X=r] = 12\, C_r \,.\left[\frac{3}{4}\right]^r \left[\frac{1}{4}\right]^{12-r}, r = 0,1,2, \ldots..12$

2. A die is thrown 3 times. If getting a '6' is considered a success, find the probability of atleast two successes.

Answer:

P=1/6; q= 5/6; n=3.

P[atleast two successes] = P(2) + P(3)

$$= 3C_2 \,.\left[\frac{1}{6}\right]^2 \frac{5}{6} + 3C_3 \,.\left[\frac{1}{6}\right]^3 = \frac{2}{27}$$

3. Find the MGF of binomial distribution.

Answer:

$M_x(t) = \sum_{r=0}^{n} nC_r \,.\; (pe^t)^r \;. q^{n-r}$

$= (q + pe^t)^n$

4. For a random variable X, $M_x(t) = \frac{1}{81}(e^t + 2)^4$, find P[X≤ 2].

Answer:

$$M_x(t) = \left(\frac{2}{3} + \frac{1}{3}e^t\right)^4.$$

For Binomial distribution, $M_x(t) = (q + pe^t)$

∴ n=4, q=2/3, p=1/3

∴ P[X≤2] = P(0) + P(1) + P(2)

$$= \left(\frac{2}{3}\right)^4 + 4C_1 \frac{1}{3}\left(\frac{2}{3}\right)^3 + 4C_1\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^2$$

$$= \frac{1}{81}[16 + 32 + 24] = \frac{72}{81}$$

= 0.8889

5. The mean and variance of a binomial variance are 4 and 4/3 respectively, find

P [X≥ 1] .

Answer:

np = 4, npq = $\frac{4}{3} \Rightarrow q = \frac{1}{3}$and $p = \frac{2}{3}$ $\therefore n = 4 \times \frac{3}{2} = 6.$

P[X≥ 1] = 1- P[X< 1] = 1 -P[X= 0]

= 1- $\left(\frac{1}{3}\right)^6$ = 0.9986

6. For a binomial distribution, mean is 6 and standard deviation is $\sqrt{2}$. Find the first two terms of the distribution.

Answer:

np = 6, npq = 2; $q = \frac{2}{3} \Rightarrow q = \frac{1}{3}$ $\therefore p = \frac{2}{3}$. Here n = 9.

The first two terms are $\left(\frac{1}{3}\right)^9, 9C_1\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^8$

7. A certain rare blood can be found in only 0.05% of people. If the population of a randomly selected group is 3000, what is the probability that atleast 2 people in the group have this rare blood type ?

Answer:

P=0.05% => p=0.0005; n = 3000; $\lambda = np$

⇒ $\lambda = 3000 \times \frac{5}{10000} = 1.5$

$P[X \geq 2] = 1 - P(X<2) = 1 - P(X=1)$

$= 1 - e^{-1.5}\left(1 + \frac{1.5}{1!}\right) = 0.4422$

8. It is known that 5% of the books bound at a certain bindery have defective bindings. Find the probability that 2 of 100 books bound by this bindery will have defective bindings.

Answer:

$\lambda = np \Rightarrow \lambda = 100 \times 5/100 = 5$

$\therefore P[X=2] = \frac{5^2 e^{-5}}{2!} = 0.084$

9. If X is a poissonvariate such that P(X=2) = 9P(X=4) + 90P(X=6), find the variance .

Answer:

$\frac{e^{-\lambda}\lambda^2}{2!} = \frac{9e^{-\lambda}\lambda^4}{4!} + \frac{90e^{-\lambda}\lambda^6}{6!} \Longrightarrow \lambda^4 + 3\lambda^2 - 4 = 0$

$\Longrightarrow (\lambda^2 + 4)(\lambda^2 - 1) = 0$

$\therefore \lambda^2 = 1 \Rightarrow \text{variance} = \lambda = 1.$

10. The moment generating function of a random variable X is given by $M_x(t) = e^{3(e^t - 1)}$. Find P(X=1)

Answer:

$M_x(t) = e^{\lambda(e^t-1)} = e^{3(e^t-1)} \Longrightarrow \lambda = 3$

$P(X = 1) = \lambda e^{-\lambda} \Longrightarrow P(X=1) = 3e^{-3}.$

11. State the conditions under which the position distribution is a limiting case of the Binomial distribution.

Answer:

i) n →∞

ii) $p \to 0$

iii) $np = \lambda$, a constant

12. Show that the sum of 2 independent poisson variates is a poisson variates.

Answer:

Let $X \sim P(\lambda_1)$ and $Y \sim P(\lambda_2)$

Then $M_x(t) = e^{\lambda_1(e^t-1)}$; $M_y(t) = e^{\lambda_2(e^t-1)}$

$M_{x+y}(t) = M_x(t)M_y(t) = e^{(e^t-1)(\lambda_1+\lambda_2)}$

$\Longrightarrow$ X + Y is also a poissonvariate

13. In a book of 520 pages, 390 typo-graphical errors occur. Assuming poisson law for the number of errors per page, find the probability that a random sample of 5 pages will contain no error.

Answer:

$$\lambda = \frac{\mathbf{390}}{\mathbf{520}} = 0.75$$

P (X=x) = $\frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-0.75}(0.75)^x}{x!}$, x = 0.1.2,…

Required probability = $[\ P(X = 0)]^5 = (e^{-0.75})^5 = e^{-3.75}$

14. If X is a poissonvariate such that P(X=2)= 2/3 P(X=1) evaluate P(X=3).

Answer:

$$\frac{e^{-\lambda}\lambda^2}{2!} = \frac{2}{3}\frac{e^{-\lambda}\lambda}{1!} \Longrightarrow \lambda = \frac{4}{3}$$

$$\therefore P[X=3] = \frac{e^{-\lambda}\left(\frac{4}{3}\right)^3}{3!}$$

15.If for a poisson variate X, $E(X^2) = 6$, What is E(X)?

Answer:

$\lambda^2 + \lambda = 6 \Longrightarrow \lambda^2 + \lambda - 6$

$= 0 \Longrightarrow (\lambda + 3)(\lambda - 2) = 0 \Longrightarrow \lambda = 2, -3$

But $\lambda > 0$, $\lambda = 2$. E(X) = $\lambda = 2$

16. If X is a poisson variate with mean λ , show that $E(X^2) = \lambda E(X+1)$.

Answer:

$E(X^2) = \lambda^2 + \lambda$

$E(X+1) = E(X)+1 = \lambda + 1$

$\therefore E(X^2) = \lambda(\lambda + 1) = \lambda E(X+1)$

17. The time (in hours) required to repair a machine is exponentially distributed with parameter $\lambda = \frac{1}{2}$. What is the probability that a repair takes atleast 10 hours given that its duration exceeds 9 hours ?

Answer:

Let X be the R.V which represents the time to repair the machine.

$P[X \geq 10/x \geq 9] = P(X \geq 1)$ (by memory less property)

$= \int_1^\infty \frac{1}{2} e^{-\frac{x}{2}} dx = 0.6065$

18. The time (in hours) required to repair a machine is exponentially distributed with parameter $\lambda = \frac{1}{3}$. What is the probability that the repair time exceeds 3 hours ?

Answer:

X- represent the time to repair the machine

P.d.f of X, $f(x) = \frac{1}{3} e^{-\frac{x}{3}}$, x>0

$P(x>3) = \int_3^\infty \frac{1}{3} e^{-\frac{x}{3}} dx = e^{-1} = 0.3679$

19. Find the MGF of an exponential distribution with parameter λ.

Answer:

$M_x(t) = \lambda \int_0^\infty e^{tx} e^{-\lambda x} dx = \lambda \int_0^\infty e^{-(\lambda - x)x} dx$

$=\frac{\lambda}{\lambda - t} = \left(1 - \frac{t}{\lambda}\right)^{-1}$

20. Mention any four properties of normal distribution ?

Answer:

(1) The curve is bell shaped

(2) Mean,Median,Mode coincide.

(3) All odd central moments vanish

(4) X-axis is an asymptote to the normal curve

21. If X is normal variate with mean 30 and S.D 5, find P[26 < X < 40]

Answer:

P [26 < X < 40] = P [-0.8 ≤ Z ≤ 2] where Z = $\frac{X-30}{5}$

= P [0 ≤ Z ≤ 0.8] + P[0 ≤ Z ≤ 2]

= 0.2881 + 0.4772 = 0.7653

22. If X is a normal variate with mean 30 and s.d 5 , find P [$|X - 30|$≤5].

Answer:

P [$|X - 30|$≤ 5] = P [25 ≤ X ≤ 35] = P [-1 ≤ Z ≤ 1]

= 2P (2 ≤ Z≤ 1) = 2(0.3413) = 0.6826

23. X is normally distributed R.V with mean 12 and SD 4. Find P [X ≤ 20].

Answer:

P [X ≤ 20] = P [Z ≤ 2] where Z = $\frac{X-12}{4}$

= P [-∞ ≤ Z 0] + P [0 ≤ Z ≤ 2]

= 0.5 + 0.4772 = 0.9772

24. For a certain normal distribution, the first moment about 10 is 40 and the fourth moment about 50is 48. Find the mean and s.d of the distribution.

Answer:

Mean A + μ_1' ⇒ Mean = 10 + 40 = 50

$\mu_1'(about the point X = 50)$ = 48 ⇒ μ_4= 48

Since mean is 50, $3\sigma^4$ = 48

σ= 2.

25. If X is normally distributed with mean 8 and s.d4 , find P (10 ≤ X ≤ 15).

Answer:

P (10 ≤ X ≤ 15) = P [0.5 ≤ X ≤ 1.75]

= P [0.5 ≤ X ≤ 1.75] – P [0 ≤ X ≤ 0.5]

= 0.2684

26. X is a normal variate with mean 1 and variance 4 , Y is another normal variate independent of X with mean 2 and variance 3 , what is the distribution of

X + 2Y ?

Answer:

E [X + 2Y] = E (X) + 2E (Y) = 1 +4 = 5

V[X+2Y] = V (X) + 4V(Y) = 4+4(3) = 16

X + 2Y ~N(5,16) by additive property.

UNIT 2

Two marks

1. The bivariate random variable X and Y has the pdf f(x,y)={

$$f(x,y)=\begin{cases} kx^2(8-y), x<y<2x \\ 0\le x<2 \end{cases}$$ find k.

Ans:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dydx=1$$

$$\int_0^2\int_x^{2x} kx^2(8-y)\,dydx=1 \qquad k\int_0^2 x^2\left(8y-\frac{y^2}{2}\right)_x^{2x}dx=1$$

$$k\int_0^2 x^2\left(16x-\frac{4x^2}{2}-8x+\frac{x^2}{2}\right)dx=1 \quad k\int_0^2\left(16x^3-2x^4-8x^3+\frac{x^4}{2}\right)dx=1$$

$$k\int_0^2\left(8x^3-\frac{3x^4}{2}\right)dx=1 \quad k\left[\frac{8x^4}{4}-\frac{3x^5}{10}\right]_0^2=1$$

$$k\left[32-\frac{48}{5}\right]=1 \qquad k\left[\frac{112}{5}\right]=1$$

$$k\left[\frac{112}{5}\right]=1$$

$$k=\frac{5}{112}$$

2. The joint pdf of random variable x and y is given by $f(x,y)=kxye^{-(x^2+y^2)}, x>0, y>0$ find the value of k.

Ans:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)dxdy=1$$

$$\int_0^{\infty}\int_0^{\infty} kxye^{-(x^2+y^2)}dydx=1$$

$$k\int_0^{\infty} ye^{-y^2}dy\int_0^{\infty} xe^{-x^2}dx=1 \qquad \left[\int_0^{\infty} xe^{-x^2}dx=\frac{1}{2}\right]$$

$$k\frac{1}{2}.\frac{1}{2}=1, \quad k=4$$

3. If X and Y have joint pdf $f(x,y)=\begin{cases} x+y, 0<x<1, 0<y<1 \\ 0, \quad \text{otherwise} \end{cases}$. check whether X and Y are independent.

Ans:

The marginal density of X is

$$f(x)=\int_{-\infty}^{\infty} f(x,y)dy \qquad f(x)=\int_0^1 (x+y)dy$$

$$f(x)=\left[xy+\frac{y^2}{2}\right]_0^1 \qquad f(x)=x+\frac{1}{2}$$

$$f(y)=\int_{-\infty}^{\infty} f(x,y)dy \qquad f(y)=\int_0^1 (x+y)dy$$

$$f(y)=\left[\frac{x^2}{2}+xy\right]_0^1 \qquad f(y)=\frac{1}{2}+y$$

$$f(x).f(y)=\left(x+\frac{1}{2}\right)\left(y+\frac{1}{2}\right) \qquad f(x).f(y)\neq f(x,y)$$

4. Let X and Y have j.d.f f(x,y)=2, 0<x<y<1. Find m.d.f

Ans:

Marginal density of X is given by

$$f(x)=\int_{-\infty}^{\infty} f(x,y)dy=\int_x^1 2dy$$

$$=2[y]_x^1$$

$$=2(1-x), 0<x<1.$$

Marginal density function of Y is given by

$$f(y)=\int_{-\infty}^{\infty} f(x,y)dy=\int_{0}^{y} 2dx=2[x]_0^y=2y, 0<y<1.$$

5. The j.d.f of the random variables X and Y is given by

$$f(x,y)=\begin{cases} 8xy, 0<x<1, 0<y<x \\ 0, \quad \text{otherwise} \end{cases} .\text{find} f_x(x).$$

Ans:

$$f_x(x)=\int_{-\infty}^{\infty} f(x,y)dy$$

$$=\int_0^x 8xydy=8x\left(\frac{y^2}{2}\right)_0^x$$

$$=8x\left(\frac{x^2}{2}\right)$$

$$f_x(x)=4x^3, 0<x<1$$

6. Given $f(x,y)=\begin{cases} cx(x-y), 0<x<2, -x<y<x \\ 0, \quad \text{otherwise} \end{cases}$, find c.

Ans:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)dydx=1$$

$$\int_0^2\int_{-x}^{x} cx(x-y)dydx=1$$

$$c\int_0^2\left[x^2y-x.\frac{y^2}{2}\right]_{-x}^{x} dx=1$$

$$c\int_0^2\left[x^3-\frac{x^2}{2}+x^3+\frac{x^3}{2}\right]dx=1$$

$$c\int_0^2 2x^3dx=1$$

$$2c\left[\frac{x^4}{4}\right]_0^2=1 \qquad 2c\left[\frac{16}{4}\right]=1 \qquad c=\frac{1}{8}$$

6. The joint p.d.f of a bivariate random variable (X,Y) is given by $f(x,y)=\begin{cases}kxy, 0<x<1, 0<y<1\\ 0, \quad otherwise\end{cases}$, find K.

Ans:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)dxdy=1$$

$$\int_0^1\int_0^1 kxydxdy=1 \qquad k\int_0^1\left[\frac{x^2}{2}y\right]_0^1 dy=1$$

$$k\int_0^1 \frac{y}{2}dy=1 \qquad k\left[\frac{y^2}{4}\right]_0^1=1$$

$$k=4$$

7. If the joint pdf of (x,y) is $f(x,y)=\frac{1}{4}, 0<x,y<1$, find $p(x+y\le 1)$.

Ans:

$$p(x+y\le 1)=p(x\le 1-y)$$

$$=\int_0^1\int_0^{1-y} f(x,y)dxdy$$

$$=\int_0^1\int_0^{1-y}\frac{1}{4}dxdy=\frac{1}{4}\int_0^1[x]_0^{1-y}dy$$

$$=\frac{1}{4}\int_0^1[(1-y)]dy \qquad =\frac{1}{4}\left[y-\frac{y^2}{2}\right]_0^1$$

$$=\frac{1}{4}\left[1-\frac{1}{2}\right] \quad =\frac{1}{4}.\frac{1}{2} \; =\frac{1}{8}$$

8. Two random variables X and Y have joint pdf $f(x,y)=\begin{cases}\frac{xy}{96}, 0<x<4, 1<y<5\\ 0, \quad otherwise\end{cases}$, find E(x).

Ans:

$$E(x)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xf(x,y)dxdy$$

$$=\int_{1}^{5}\int_{0}^{4} x\left(\frac{xy}{96}\right)dxdy \qquad =\frac{1}{96}\int_{1}^{5}\left[y.\frac{x^3}{3}\right]_0^4 dy$$

$$=\frac{1}{96}\int_{1}^{5}\frac{64}{3}ydy \quad =\frac{64}{288}\left[\frac{y^2}{2}\right]_1^5$$

$$=\frac{2}{9}\left[\frac{25}{2}-\frac{1}{2}\right]=\frac{1}{9}(24)$$

$$=\frac{8}{3}.$$

9. Let X be a Random variable with pdf $f(x)=\frac{1}{2}, -1\leqslant x\leqslant 1$, and let $Y=X^2$, find E(Y).

Ans:

$Y=x^2$

$$E(Y)=E(x^2)$$

$$\int_{-\infty}^{\infty} x^2 f(x)dx=\int_{-\infty}^{\infty} x^2\left(\frac{1}{2}\right)dx=\frac{1}{2}\left(\frac{x^3}{3}\right)_{-1}^{1}=\frac{1}{6}(2)=\frac{1}{3}.$$

10. If the joint pdf of (x,y) is given by $f(x,y)=x+y, 0\le x,y\le 1$. Find $E(XY)$.

Ans:

$$E(XY)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xyf(x,y)dxdy$$

$$= \int_0^1 \int_0^1 xy(x+y)\,dxdy$$

$$= \int_0^1 \int_0^1 \left(x^2 y + xy^2\right) dxdy$$

$$= \int_0^1 \left(\frac{x^3}{3} y + \frac{x^2}{2} y^2 \right)_0^1 dy$$

$$= \int_0^1 \left(\frac{y}{3} + \frac{y^2}{2} \right)_0^1 dy$$

$$= \left(\frac{y^2}{6} + \frac{y^3}{6} \right)_0^1 = \frac{2}{6} = \frac{1}{3}.$$

11. Find the acute angle between the two lines of regression

    Ans:

    The acute angle between the two lines of regression is $\tan\theta = \frac{1-r^2}{r}\left(\frac{\sigma_x \sigma_y}{\sigma_x{}^2 + \sigma_y{}^2}\right)$.

12. State the equation of the two regression lines. What is the formula for correlation coefficient

    Ans:

    X on Y is $\left(\mathbf{x} - \overline{\mathbf{x}}\right) = \mathbf{b}_{xy}\left(\mathbf{y} - \overline{\mathbf{y}}\right)$ and Y on X is $\left(\mathbf{y} - \overline{\mathbf{y}}\right) = \mathbf{b}_{yx}\left(\mathbf{x} - \overline{\mathbf{x}}\right)$.

    Correlation coefficient $\mathbf{r} = \sqrt{\mathbf{b}_{xy}.\mathbf{b}_{yx}}$.

13. If X and Y are independent random variables with variance 2 and 3. Find the variance of 3X+4Y.

    Ans:

    $Var(x) = 2$, $Var(y) = 3$

$$Var(3X + 4Y) = 3^2 Var(X) + 4^2 Var(Y)$$

$$= 9Var(X) + 16Var(Y)$$

$$= 9*2 + 16*3$$

$$= 66$$

14. The joint pdf of (X,Y) is given by $e^{-(x+y)}, 0 < x, y < \infty$. Are X and Y independent?

    Ans :

$$f(x) = \int_{-\infty}^{\infty} f(x,y)\,dy$$

$$= \int_0^\infty e^{-x} e^{-y} dy$$

$$= e^{-x}\left(-e^{-y}\right)_0^\infty = -e^{-x}\left(0-1\right) = e^{-x}.$$

$$f\left(y\right) = \int_{-\infty}^{\infty} f\left(x,y\right) dx$$

$$= \int_0^\infty e^{-x} e^{-y} dx$$

$$= e^{-y}\left(-e^{-x}\right)_0^\infty = -e^{-y}\left(0-1\right) = e^{-y}.$$

$f\left(x\right) f\left(y\right) = e^{-x} e^{-y} = e^{-(x+y)} = f\left(x,y\right)$ Therefore, X any Y are independent.

15. The two lines of regression are $8x - 10y + 66 = 0$, $40x - 18y - 214 = 0$. Find the mean value of X and Y.

Ans:

$8x - 10y = -66$ __________ (1)

$40x - 18y = 214$ ________(2)

Solving (1) and (2) , we get $x = 104, y = 17$

Mean of X = 13

Mean of Y = 17.

16. The two regression lines are $x = \frac{9}{20}y + \frac{107}{20}$, $y = \frac{4}{5}x + \frac{33}{5}$. Find correlation coefficient?

Ans:

$r = \sqrt{b_{xy}.b_{yx}}$

Here, $b_{xy} = \frac{9}{20}$, $b_{yx} = \frac{4}{5}$

$$r = \sqrt{\frac{9}{20} X \frac{4}{5}} = 0.6$$

UNIT 3
Two marks

1. **Define Random processes and give an example of a random process.**

A Random process is a collection of R.V $\{X(s,t)\}$ that are functions of a real variable namely time t where $s \in S$ and $t \in T$

Example:

$X(t) = A\cos(\omega t + \theta)$ where θ is uniformly distributed in $(0, 2\pi)$ where A and ω are constants.

2. State the four classifications of Random processes.

The Random processes is classified into four types

(i)Discrete random sequence

If both T and S are discrete then Random processes is called a discrete Random sequence.

(ii)Discrete random processes

If T is continuous and S is discrete then Random processes is called a Discrete Random processes.

(iii)Continuous random sequence

If T is discrete and S is continuous then Random processes is called a Continuous Random sequence.

(iv)Continuous random processes

If T &S are continuous then Random processes is called a continuous Random processes.

3. **Define stationary Random processes.**

If certain probability distributions or averages do not depend on t, then the random process $\{X(t)\}$ is called stationary.

4.**Define first order stationary Random processes.**

A random processes $\{X(t)\}$ is said to be a first order SSS process if $f(x_1, t_1 + \delta) = f(x_1, t_1)$ (i.e.) the first order density of a stationary process $\{X(t)\}$ is independent of time t

5.**Define second order stationary Random processes**

A RP $\{X(t)\}$ is said to be second order SSS if $f(x_1, x_2, t_1, t_2) = f(x_1, x_2, t_1 + h, t_2 + h)$ where $f(x_1, x_2, t_1, t_2)$ is the joint PDF of $\{X(t_1), X(t_2)\}$.

6. **Define strict sense stationary Random processes**

Sol: A RP $\{X(t)\}$ is called a SSS process if the joint distribution

$X(t_1)X(t_{21})X(t_3)........X(t_n)$ is the same as that of

$X(t_1+h)X(t_2+h)X(t_3+h)........X(t_n+h)$ for all $t_1,t_2,t_3........t_n$ and h >0 and for $n \geq 1$.

7. **Define wide sense stationary Random processes**

A RP $\{X(t)\}$ is called WSS if $E\{X(t)\}$ is constant and $E[X(t)X(t+\tau)] = R_{xx}(\tau)$ (i.e.) ACF is a function of τ only.

8. **Define jointly strict sense stationary Random processes**

Sol: Two real valued Random Processes $\{X(t)\}$ and $\{Y(t)\}$ are said to be jointly stationary in the strict sense if the joint distribution of the $\{X(t)\}$ and $\{Y(t)\}$ are invariant under translation of time.

9. **Define jointly wide sense stationary Random processes**

Sol: Two real valued Random Processes $\{X(t)\}$ and $\{Y(t)\}$ are said to be jointly stationary in the wide sense if each process is individually a WSS process and $R_{XY}(t_1,t_2)$ is a function of t_1,t_2 only.

10. **Define Evolutionary Random processes and give an example.**

Sol: A Random processes that is not stationary in any sense is called an Evolutionary process. Example: Poisson process.

11. **When is a random process said to be ergodic? Give an example**

Answer: A R.P {X(t)} is ergodic if its ensembled averages equal to appropriate time averages. Example: $X(t) = A\cos(\omega t + \theta)$ where θ is uniformly distributed in $(0,2\pi)$ is mean ergodic.

12. **Define Markov Process.**

Sol: If for $t_1 < t_2 < t_3 < t_4 .... < t_n < t$ then

$$P(X(t) \leq x / X(t_1) = x_1, X(t_2) = x_2, ...X(t_n) = x_n) = P(X(t) \leq x / X(t_n) = x_n)$$

Then the process $\{X(t)\}$ is called a Markov process.

13. **Define Markov chain.**

Sol: A Discrete parameter Markov process is called Markov chain.

14. **Define one step transition probability.**

Sol: The one step probability $P[X_n = a_j / X_{n-1} = a_i]$ is called the one step probability from the state a_i to a_j at the n^{th} step and is denoted by $P_{ij}(n-1,n)$

15. **State the postulates of a Poisson process.**

Let $\{X(t)\}$ = number of times an event A say, occurred up to time 't' so that the sequence $\{X(t)\}$, $t \geq 0$ forms a Poisson process with parameter λ.

(i) P[1 occurrence in $(t, t+\Delta t)$]= $\lambda \Delta t$

(ii) P[0 occurrence in $(t, t+\Delta t)$]=1- $\lambda \Delta t$

(iii) P[2 or more occurrence in $(t, t+\Delta t)$]=0

(iv) X(t) is independent of the number of occurrences of the event in any interval prior and after the interval (0,t).

(v) The probability that the event occurs a specified number of times in (t_0,t_0+t) depends only on t, but not on t_0.

16. **State any two properties of Poisson process**

Sol: (i) The Poisson process is a Markov process

(ii) Sum of two independent Poisson processes is a Poisson process

(iii) The difference of two independent Poisson processes is not a Poisson process.

`17. **If the customers arrived at a counter in accordance with a Poisson process with a mean rate of 2 per minute, find the probability that the interval between two consecutive arrivals is more than one minute.**

Sol: The interval T between 2 consecutive arrivals follows an exponential distribution with

parameter $\lambda = 2, P(T > 1) = \int_1^\infty 2e^{-2t} dt = e^{-2} = 0.135.$

18. **A bank receives on an average $\lambda = 6$ bad checks per day, what are the probabilities that it will receive (i) 4 bad checks on any given day (ii) 10 bad checks over any 2 consecutive days.**

Sol: $P(X(t) = n) = \dfrac{e^{-\lambda t}.(\lambda t)^n}{n!} = \dfrac{e^{-6t}(6t)^n}{n!}, n = 0,1,2....$

(i) $P(X(1) = 4) = \dfrac{e^{-6}(6)^4}{4!} = 0.1338$

(ii) $P(X(2) = 10) = \dfrac{e^{-12}(12)^{10}}{10!} = 0.1048$

19. **Consider a Markov chain with two states and transition probability matr**

$P = \begin{bmatrix} 3/4 & 1/4 \\ 1/2 & 1/2 \end{bmatrix}$ **.Find the stationary probabilities of the chain.**

Sol: $(\pi_1, \pi_2)\begin{bmatrix} 3/4 & 1/4 \\ 1/2 & 1/2 \end{bmatrix} = (\pi_1, \pi_2) \qquad \pi_1 + \pi_2 = 1$

$$\frac{3}{4}\pi_1 + \frac{\pi_2}{4} = \pi_1 \Rightarrow \frac{\pi_1}{4} - \frac{\pi_2}{2} = 0. \qquad \therefore \pi_1 = 2\pi_2$$

$$\therefore \pi_1 = \frac{2}{3}, \pi_2 = \frac{1}{3}.$$

UNIT 4

Two marks

1. Define the ACF.

Answer:

Let $X(t_1)$ and $X(t_2)$ be two random variables. The autocorrelation of the random process $\{X(t)\}$ is

$$R_{XX}(t_1,t_2) = E[X(t_1)X(t_2)].$$

If $t_1 = t_2 = t$, $R_{XX}(t,t) = E[X^2(t)]$ is called as mean square value of the random process.

2. State any four properties of Autocorrelation function.

Answer:

1. $R_{XX}(-\tau) = R_{XX}(\tau)$
2. $|R(\tau)| \le R(0)$
3. $R(\tau)$ *is continuous for all* τ.
4. If $R(\tau)$ is ACF of a stationary RP $\{X(t)\}$ with no periodic components, then $\mu_X{}^2 = \lim_{\tau\to\infty} R(\tau)$.

3. Define the cross – correlation function.

Answer:

Let $\{X(t)\}$ and $\{Y(t)\}$ be two random processes. The cross-correlation is

$$R_{XY}(\tau) = E[X(t)Y(t-\tau)].$$

4. State any two properties of cross-correlation function.

Answer:

1. $R_{YX}(-\tau) = R_{XY}(\tau)$
2. $|R_{XY}(\tau)| \le \sqrt{R_{XX}(0)R_{YY}(0)} \le \frac{1}{2}[R_{XX}(0) + R_{YY}(0)]$

5. Given the ACF for a stationary process with no periodic component is $R_{XX}(\tau) = 25 + \frac{4}{1+6\tau^2}$ .find the mean and variance of the process{X(t)}

Answer:

By the property of ACF

$$\mu_x^{\ 2} = \lim_{\tau\to\infty} R_{XX}(\tau) = \lim_{\tau\to\infty} 25 + \frac{4}{1+6\tau^2} = 25$$

$$\mu_x = 5$$

$E\{X^2(t)\} = R_{xx}(0) = 25+4 = 29$

$Var\{X(t)\} = E\{X^2(t)\} - E^2\{X(t)\} = 29-25 = 4.$

6. ACF: $R_{XX}(\tau) = \frac{25\tau^2 + 36}{6.25\tau^2 + 4}$ .find mean and variance.

7. ACF: $R_{XX}(\tau) = 25 + \frac{4}{1+6\tau^2}$ .find mean and variance.

8. Define power spectral density.

Answer:

If $R_{XX}(\tau)$ is the ACF of a WSS process $\{X(t)\}$ then the power spectral density $S_{XX}(\omega)$ of the process $\{X(t)\}$, is defined by

$$S_{XX}(\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau) e^{-i\omega\tau} d\tau \quad \text{(or)} \quad S_{XX}(f) = \int_{-\infty}^{\infty} R_{XX}(\tau) e^{-i2\pi f\tau} d\tau$$

9. Express each of ACF and PSD of a stationary R.P in terms of the other.{(or) write down wiener khinchine relation }

Answer:

$R_{XX}(\tau)$ and $S_{XX}(\omega)$ are Fourier transform pairs.

i.e., $S_{XX}(\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau)e^{-i\omega\tau}\,d\tau$ and $R_{xx}(\tau) = \int_{-\infty}^{\infty} S_{XX}(\omega)e^{i\omega\tau}\,d\tau$

10. Define cross power spectral density of two random process {X(t)} and{Y(t)}.

Answer:

If {X(t)} and {Y(t)} are jointly stationary random processes with cross correlation function $R_{XY}(\tau)$,then cross power spectral density of {X(t)} and {Y(t)}is defined by

$$S_{XY}(\omega) = \int_{-\infty}^{\infty} R_{XY}(\tau)e^{-i\omega\tau}\,d\tau$$

11. State any two properties of power spectral density.

Answer:

i) $S(\omega) = S(-\omega)$

ii) $S(\omega) > 0$

iii)The spectral density of a process {X(t)},real or complex, is a real function of ω and non-negative.

12. If $R(\tau) = e^{-2\lambda|\tau|}$ is the ACF of a R.P{X(t)}, obtain the spectral density.

Answer:

$$S_{XX}(\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau)e^{-i\omega\tau}\,d\tau = \int_{-\infty}^{\infty} e^{-2\lambda|\tau|}e^{-i\omega\tau}\,d\tau = 2\int_{0}^{\infty} e^{-2\lambda\tau}\cos\omega\tau\,d\tau = \frac{4\lambda}{4\lambda^2+\omega^2}.$$

13. State any four properties of cross power density spectrum.

Answer:

i) $S_{XY}(\omega) = S_{YX}(-\omega) = S^*_{YX}(\omega)$

ii) Re[$S_{XY}(\omega)$] and Re[$S_{YX}(\omega)$] are even function of ω

iii) Im[$S_{XY}(\omega)$] and Im[$S_{YX}(\omega)$] are odd function of ω

iv) $S_{XY}(\omega)$=0and $S_{YX}(\omega)$=0 if X(t) and Y(t) are orthogonal.

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Unit 5

Two marks

14. Define a system and Define the linear system.

Answer:

A system is a functional relationship between the input X(t) and the output Y(t).i.e., $Y(t) = f[X(t)], -\infty < t < \infty$.

A System is a functional relationship between the input X(t) and the output Y(t).

If $f[a_1X_1(t)+ a_2X_2(t)] = a_1 f[X_1(t)]+ a_2 f[X_2(t)]$, then f is called a linear system.

15. Define time invariant system.

Answer:

If $Y(t+h) = f[X(t+h)]$ where $Y(t) = f[X(t)]$, then f is called the time invariant system.

16. Check whether the following system is linear .y(t)=t x(t)

Answer:

Consider two input functions $x_1(t)$ and $x_2(t)$. The corresponding outputs are

$y_1(t)=t\, x_1(t)$ and $y_2(t)=t\, x_2(t)$

Consider $y_3(t)$ as the linear combinations of the two inputs.

$y_3(t)= t[a_1 x_1(t)+a_2 x_2(t)]= a_1t\, x_1(t)+a_2\, t\, x_2(t)$ ……………..(1)

consider the linear combinations of the two outputs.

$a_1y(t)+a_2\, y_2(t)= a_1t\, x_1(t)+a_2\, t\, x_2(t)$ …………………(2)

From (1)and(2), (1)=(2)

The system y(t)=t x(t) is linear.

17.

Check whether the following system is linear .y(t)= x²(t)

Answer:

Consider two input functions $x_1(t)$ and $x_2(t)$. The corresponding outputs are

$y_1(t)=x_1^2(t)$ and $y_2(t)=x_2^2(t)$

Consider $y_3(t)$ as the linear combinations of the two inputs.

$y_3(t)= [a_1 x_1(t)+a_2 x_2(t)]^2= a_1^2 x_1^2(t)+a_2^2 x_2^2(t)+2\, a_1x_1(t)a_2x_2(t)$ ……..(1)

consider the linear combinations of the two outputs.

$a_1y(t)+a_2\, y_2(t)= a_1 x_1^2(t)+a_2 x_2^2(t)$ …………………(2)

From (1) and (2), $(1) \neq (2)$

The system $y(t)=x^2(t)$ is not linear.

18. Define the Linear Time Invariant System.

Answer:

A linear system is said to be also time-invariant if the form of its impulse response $h(t,u)$ does not depend on the time that the impulse is applied.

For linear time invariant system, $h(t,u) = h(t-u)$

If a system is such that its Input X(t) and its Output Y(t) are related by a Convolution integral,

i.e., if $Y(t) = \int_{-\infty}^{\infty} h(u)\, X(t-u)\, du$, then the system is a linear time-invariant system.

19. Find the ACF of the random process {X(t)}, if its power spectral density is given by

$$S(\omega) = \begin{cases} 1+\omega^2, & for\ |\omega| \le 1 \\ 0 & , for\ |\omega| > 1 \end{cases}$$

Solution:

$$R(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S(\omega)e^{i\omega\tau}\, d\omega = \frac{1}{2\pi}\int_{-1}^{1}\{1+\omega^2\}e^{i\omega\tau}\, d\omega = \frac{1}{2\pi}\int_{-1}^{1}\left\{e^{i\omega\tau} + \omega^2 e^{i\omega\tau}\right\}d\omega = \frac{1}{2\pi}\left\{\left[\frac{e^{i\omega\tau}}{i\tau}\right]_{-1}^{1} + \int_{-1}^{1}\omega^2 \cos\omega\tau\, d\omega\right\}$$

$$= \frac{1}{2\pi}\left\{\left[\frac{e^{i\omega\tau}}{i\tau}\right]_{-}^{1} + 2\int_{0}^{1}\omega^2 \cos\omega\tau\, d\omega\right\} = \frac{1}{2\pi}\left[\frac{e^{i\tau} - e^{-i\tau}}{i\tau}\right] + 2\left[\omega^2\frac{\sin\omega\tau}{\tau} + \frac{2\omega\cos\omega\tau}{\tau^2} - \frac{2\sin\omega\tau}{\tau^3}\right]_0^1$$

$$= \frac{1}{2\pi}\left[\frac{2\sin\tau}{\tau} + \frac{2\sin\tau}{\tau} + \frac{4\cos\tau}{\tau^2} - \frac{4\sin\tau}{\tau^3}\right] = \frac{1}{2\pi}\left[\frac{2\tau^2\sin\tau + 2\tau^2\sin\tau + \tau 4\cos\tau - 4\sin\tau}{\tau^3}\right]$$

$$= \frac{2\{\tau^2\sin\tau + \tau\cos\tau - \sin\tau\}}{\pi\tau^3}$$

20. Define Average power.

$$\text{Average power } = \frac{1}{2\pi}\int_{-\infty}^{\infty} S(\omega)\, d\omega = R(0)$$

21. A system has an impulse response $h(t) = e^{-\beta t}U(t)$, find the system transfer function.

Solution:

The unit step function $U(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$

$$h(t) = \begin{cases} 0, & t < 0 \\ e^{-\beta t}, & t \geq 0 \end{cases}$$

$$\therefore H(\omega) = \int_{-\infty}^{\infty} h(t)\, e^{-i\omega t}\, dt$$

$$= \int_{0}^{\infty} e^{-\beta t}\, e^{-i\omega t}\, dt$$

$$= \int_{0}^{\infty} e^{-(\beta + i\omega)t}\, dt$$

$$= \left[\frac{e^{-(\beta+i\omega)t}}{-(\beta+i\omega)}\right]_0^{\infty}$$

$$= -\frac{1}{\beta+i\omega}\left[e^{-(\beta+i\omega)t}\right]_0^{\infty}$$

$$= -\frac{1}{\beta+i\omega}\left[0-1\right]$$

$$= \frac{1}{\beta+i\omega}$$

22. State any properties of Linear time invariant system.

1. If X(t) is WSS process, then Y(t) is also WSS process.

If the input $x(t)$ and its output $y(t)$ are related by $y(t) = \int_{-\infty}^{\infty} h(u)\, x(t-u)\, du$, then the system is linear time invariant system

2.