SNS COLLEGE OF TECHNOLOGY

An Autonomous Institution

Coimbatore-35

Accredited by NBA – AICTE and Accredited by NAAC – UGC with ‘A+’ Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECB212 – DIGITAL SIGNAL PROCESSING

II YEAR/ IV SEMESTER

UNIT 5 – DSP APPLICATIONS

TOPIC – MULTIRATE DSP – UPSAMPLING (INTERPOLATION)

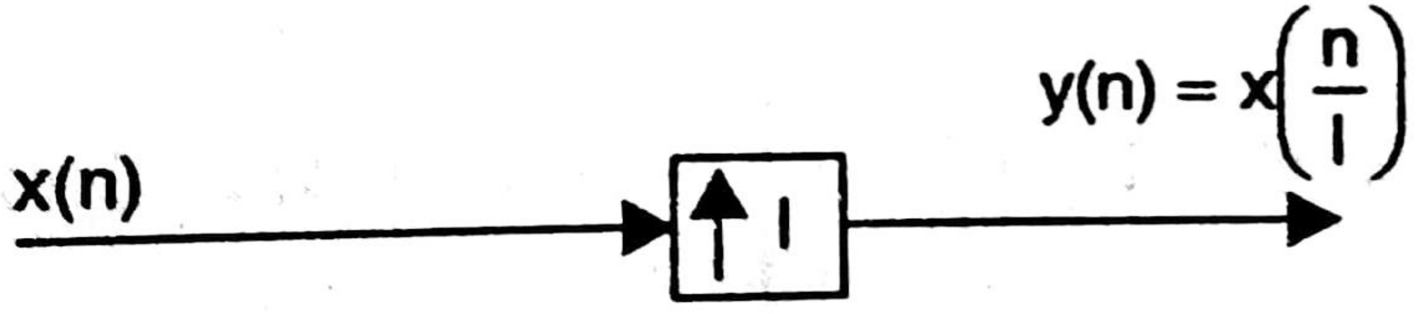
DECIMATION & INTERPOLATION

- **Downsampling or decimation** is the process of reducing the sampling rate by an integer factor D
- **Upsampling or interpolation** is the process of increasing the sampling rate by an integer factor I
- **Advantages of Multirate Processing:**

1. The reduction in number of computations
2. The reduction in memory requirement (or storage) for filter coefficients and intermediate results
3. The reduction in the order of the system
4. The finite word length effects are reduced

UPSAMPLING (OR) INTERPOLATION

- **Upsampling (or Interpolation)** is the process of increasing the samples of the discrete time signal
- Let, x(n) = Discrete time signal
- I = Sampling rate multiplication factor (and I is an integer)
- Now, x(n/I) = Upsampled version of x(n)
- The device which performs the process of upsampling is called a upsampler (or interpolator)
- The upsampler can be represented as



EXAMPLE

Consider the discrete time signal,

$$x(n) = \{1, 2, 3, 4\}$$

Determine the upsampled version of the signals for the sampling rate multiplication factor,

a) $I = 2$ b) $I = 3$ c) $I = 4$

Solution

Given that,

$$x(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$$

$\therefore$ When $n = 0$, $x(n) = x(0) = 1$

When $n = 1$, $x(n) = x(1) = 2$

When $n = 2$, $x(n) = x(2) = 3$

When $n = 3$, $x(n) = x(3) = 4$

EXAMPLE

a) Sampling rate multiplication factor, I = 2.

Now, $x\left(\frac{n}{I}\right) = x\left(\frac{n}{2}\right)$ = Discrete time signal interpolated by multiplication factor 2.

Let, $x\left(\frac{n}{2}\right) = x_{I2}(n)$

$\therefore$ When $n = 0$, $x_{I2}(n) = x_{I2}(0) = x\left(\frac{0}{2}\right) = x(0) = 1$

When $n = 1$, $x_{I2}(n) = x_{I2}(1) = x\left(\frac{1}{2}\right) = x(0.5) = 0$

When $n = 2$, $x_{I2}(n) = x_{I2}(2) = x\left(\frac{2}{2}\right) = x(1) = 2$

When $n = 3$, $x_{I2}(n) = x_{I2}(3) = x\left(\frac{3}{2}\right) = x(1.5) = 0$

When $n = 4$, $x_{I2}(n) = x_{I2}(4) = x\left(\frac{4}{2}\right) = x(2) = 3$

When $n = 5$, $x_{I2}(n) = x_{I2}(5) = x\left(\frac{5}{2}\right) = x(2.5) = 0$

When $n = 6$, $x_{I2}(n) = x_{I2}(6) = x\left(\frac{6}{2}\right) = x(3) = 4$

When $n = 7$, $x_{I2}(n) = x_{I2}(7) = x\left(\frac{7}{2}\right) = x(3.5) = 0$

$$\therefore x\left(\frac{n}{2}\right) = x_{I2}(n) = \{\underset{\uparrow}{1}, 0, 2, 0, 3, 0, 4, 0\}$$

EXAMPLE

b) Sampling rate multiplication factor, I = 3.

Now, $x\left(\frac{n}{I}\right) = x\left(\frac{n}{3}\right)$ = Discrete time signal interpolated by multiplication factor 3.

Let, $x\left(\frac{n}{3}\right) = x_{I3}(n)$

$\therefore$ When $n = 0$, $x_{I3}(n) = x_{I3}(0) = x\left(\frac{0}{3}\right) = x(0) = 1$

When $n = 1$, $x_{I3}(n) = x_{I3}(1) = x\left(\frac{1}{3}\right) = x(0.3) = 0$

When $n = 2$, $x_{I3}(n) = x_{I3}(2) = x\left(\frac{2}{3}\right) = x(0.7) = 0$

When $n = 3$, $x_{I3}(n) = x_{I3}(3) = x\left(\frac{3}{3}\right) = x(1) = 2$

When $n = 4$, $x_{I3}(n) = x_{I3}(4) = x\left(\frac{4}{3}\right) = x(1.3) = 0$

When $n = 5$, $x_{I3}(n) = x_{I3}(5) = x\left(\frac{5}{3}\right) = x(1.7) = 0$

When $n = 6$, $x_{I3}(n) = x_{I3}(6) = x\left(\frac{6}{3}\right) = x(2) = 3$

When $n = 7$, $x_{I3}(n) = x_{I3}(7) = x\left(\frac{7}{3}\right) = x(2.3) = 0$

When $n = 8$, $x_{I3}(n) = x_{I3}(8) = x\left(\frac{8}{3}\right) = x(2.7) = 0$

When $n = 9$, $x_{I3}(n) = x_{I3}(9) = x\left(\frac{9}{3}\right) = x(3) = 4$

When $n = 10$, $x_{I3}(n) = x_{I3}(10) = x\left(\frac{10}{3}\right) = x(3.3) = 0$

When $n = 11$, $x_{I3}(n) = x_{I3}(11) = x\left(\frac{11}{3}\right) = x(3.7) = 0$

$$\therefore x\left(\frac{n}{3}\right) = x_{I3}(n) = \{\underset{\uparrow}{1}, 0, 0, 2, 0, 0, 3, 0, 0, 4, 0, 0\}$$

EXAMPLE

c) Sampling rate multiplication factor, $I = 4$.

Now, $x\left(\frac{n}{I}\right) = x\left(\frac{n}{4}\right)$ = Discrete time signal interpolated by multiplication factor 4.

Let, $x\left(\frac{n}{4}\right) = x_{I4}(n)$

$\therefore$ When $n = 0$, $x_{I4}(n) = x_{I4}(0) = x\left(\frac{0}{4}\right) = x(0) = 1$

When $n = 1$, $x_{I4}(n) = x_{I4}(1) = x\left(\frac{1}{4}\right) = x(0.25) = 0$

When $n = 2$, $x_{I4}(n) = x_{I4}(2) = x\left(\frac{2}{4}\right) = x(0.5) = 0$

When $n = 3$, $x_{I4}(n) = x_{I4}(3) = x\left(\frac{3}{4}\right) = x(0.75) = 0$

When $n = 4$, $x_{I4}(n) = x_{I4}(4) = x\left(\frac{4}{4}\right) = x(1) = 2$

When $n = 5$, $x_{I4}(n) = x_{I4}(5) = x\left(\frac{5}{4}\right) = x(1.25) = 0$

When $n = 6$, $x_{I4}(n) = x_{I4}(6) = x\left(\frac{6}{4}\right) = x(1.5) = 0$

When $n = 7$, $x_{I4}(n) = x_{I4}(7) = x\left(\frac{7}{4}\right) = x(1.75) = 0$

When $n = 8$, $x_{I4}(n) = x_{I4}(8) = x\left(\frac{8}{4}\right) = x(2) = 3$

When $n = 9$, $x_{I4}(n) = x_{I4}(9) = x\left(\frac{9}{4}\right) = x(2.25) = 0$

When $n = 10$, $x_{I4}(n) = x_{I4}(10) = x\left(\frac{10}{4}\right) = x(2.5) = 0$

When $n = 11$, $x_{I4}(n) = x_{I4}(11) = x\left(\frac{11}{4}\right) = x(2.75) = 0$

When $n = 12$, $x_{I4}(n) = x_{I4}(12) = x\left(\frac{12}{4}\right) = x(3) = 4$

When $n = 13$, $x_{I4}(n) = x_{I4}(13) = x\left(\frac{13}{4}\right) = x(3.25) = 0$

When $n = 14$, $x_{I4}(n) = x_{I4}(14) = x\left(\frac{14}{4}\right) = x(3.5) = 0$

When $n = 15$, $x_{I4}(n) = x_{I4}(15) = x\left(\frac{15}{4}\right) = x(3.75) = 0$

$$\therefore x\left(\frac{n}{4}\right) = x_{I4}(n) = \{\underset{\uparrow}{1}, 0, 0, 0, 2, 0, 0, 0, 3, 0, 0, 0, 4, 0, 0, 0\}$$

EXAMPLE

Consider the discrete time signal shown in fig 1. Sketch the upsampled version of the signals for the sampling rate multiplication factor, **a)** $I = 2$ **b)** $I = 3$.

Solution

From fig 1, we can write the samples of given sequence as shown below.

$$x(n) = \{\underset{\uparrow}{1}, -1, 2, -2\}$$

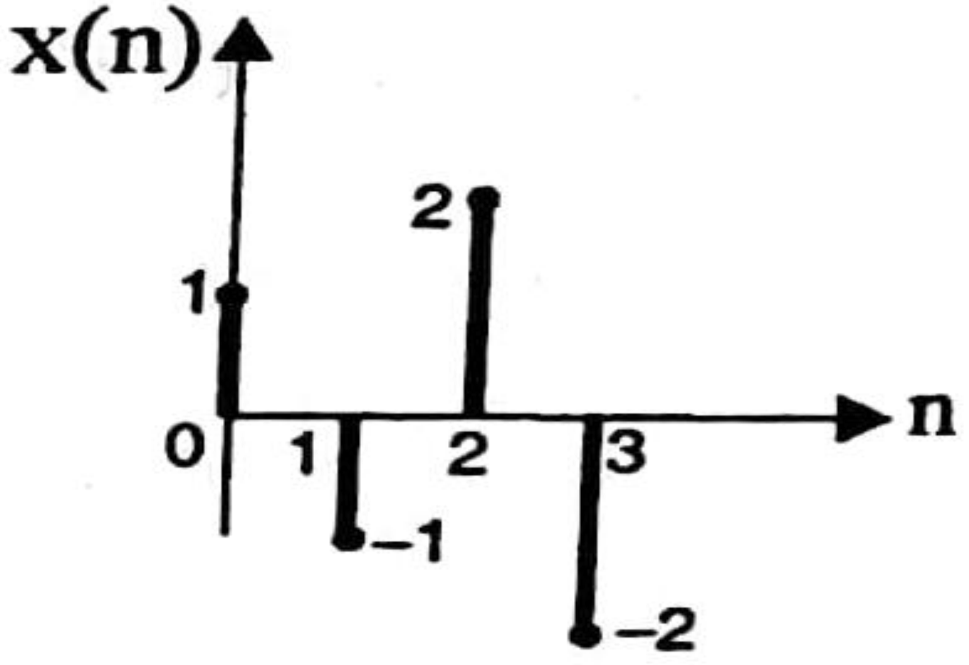


Fig 1.

$\therefore$ When $n = 0$, $x(n) = x(0) = 1$

When $n = 1$, $x(n) = x(1) = -1$

When $n = 2$, $x(n) = x(2) = 2$

When $n = 3$, $x(n) = x(3) = -2$

EXAMPLE

a) Sampling rate multiplication factor, I = 2.

Now, $x\left(\frac{n}{I}\right) = x\left(\frac{n}{2}\right)$ = Discrete time signal interpolated by multiplication factor 2.

Let, $x\left(\frac{n}{2}\right) = x_{I2}(n)$

$\therefore$ When $n = 0$, $x_{I2}(n) = x_{I2}(0) = x\left(\frac{0}{2}\right) = x(0) = 1$

When $n = 1$, $x_{I2}(n) = x_{I2}(1) = x\left(\frac{1}{2}\right) = x(0.5) = 0$

When $n = 2$, $x_{I2}(n) = x_{I2}(2) = x\left(\frac{2}{2}\right) = x(1) = -1$

When $n = 3$, $x_{I2}(n) = x_{I2}(3) = x\left(\frac{3}{2}\right) = x(1.5) = 0$

When $n = 4$, $x_{I2}(n) = x_{I2}(4) = x\left(\frac{4}{2}\right) = x(2) = 2$

When $n = 5$, $x_{I2}(n) = x_{I2}(5) = x\left(\frac{5}{2}\right) = x(2.5) = 0$

When $n = 6$, $x_{I2}(n) = x_{I2}(6) = x\left(\frac{6}{2}\right) = x(3) = -2$

When $n = 7$, $x_{I2}(n) = x_{I2}(7) = x\left(\frac{7}{2}\right) = x(3.5) = 0$

$$\therefore x\left(\frac{n}{2}\right) = x_{I2}(n) = \{\underset{\uparrow}{1},\ 0,\ -1,\ 0,\ 2,\ 0,\ -2,\ 0\} \quad .....(1)$$

EXAMPLE

b) Sampling rate multiplication factor, I = 3.

Now, $x\left(\frac{n}{I}\right) = x\left(\frac{n}{3}\right)$ = Discrete time signal interpolated by multiplication factor 3.

Let, $x\left(\frac{n}{2}\right) = x_{I3}(n)$

$\therefore$ When $n = 0$, $x_{I3}(n) = x_{I3}(0) = x\left(\frac{0}{3}\right) = x(0) = 1$

When $n = 1$, $x_{I3}(n) = x_{I3}(1) = x\left(\frac{1}{3}\right) = x(0.3) = 0$

When $n = 2$, $x_{I3}(n) = x_{I3}(2) = x\left(\frac{2}{3}\right) = x(0.7) = 0$

When $n = 3$, $x_{I3}(n) = x_{I3}(3) = x\left(\frac{3}{3}\right) = x(1) = -1$

When $n = 4$, $x_{I3}(n) = x_{I3}(4) = x\left(\frac{4}{3}\right) = x(1.3) = 0$

When $n = 5$, $x_{I3}(n) = x_{I3}(5) = x\left(\frac{5}{3}\right) = x(1.7) = 0$

When $n = 6$, $x_{I3}(n) = x_{I3}(6) = x\left(\frac{6}{3}\right) = x(2) = 2$

When $n = 7$, $x_{I3}(n) = x_{I3}(7) = x\left(\frac{7}{3}\right) = x(2.3) = 0$

When $n = 8$, $x_{I3}(n) = x_{I3}(8) = x\left(\frac{8}{3}\right) = x(2.7) = 0$

When $n = 9$, $x_{I3}(n) = x_{I3}(9) = x\left(\frac{9}{3}\right) = x(3) = -2$

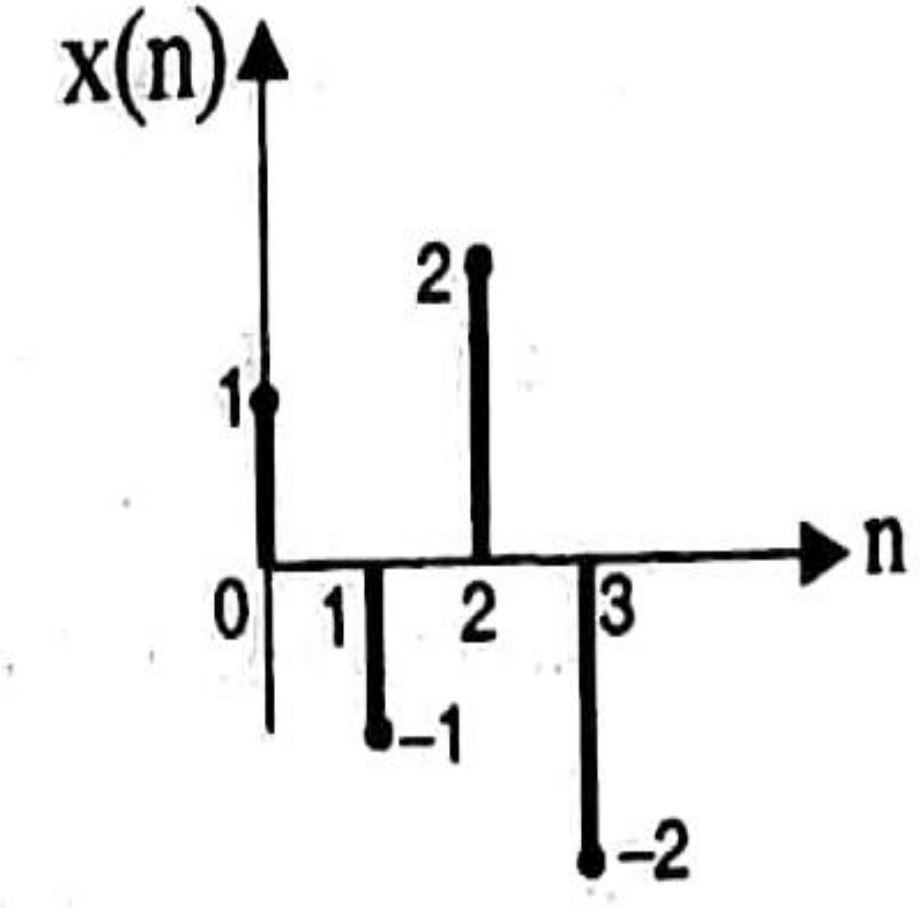
When $n = 10$, $x_{I3}(n) = x_{I3}(10) = x\left(\frac{10}{3}\right) = x(3.3) = 0$

When $n = 11$, $x_{I3}(n) = x_{I3}(11) = x\left(\frac{11}{3}\right) = x(3.7) = 0$

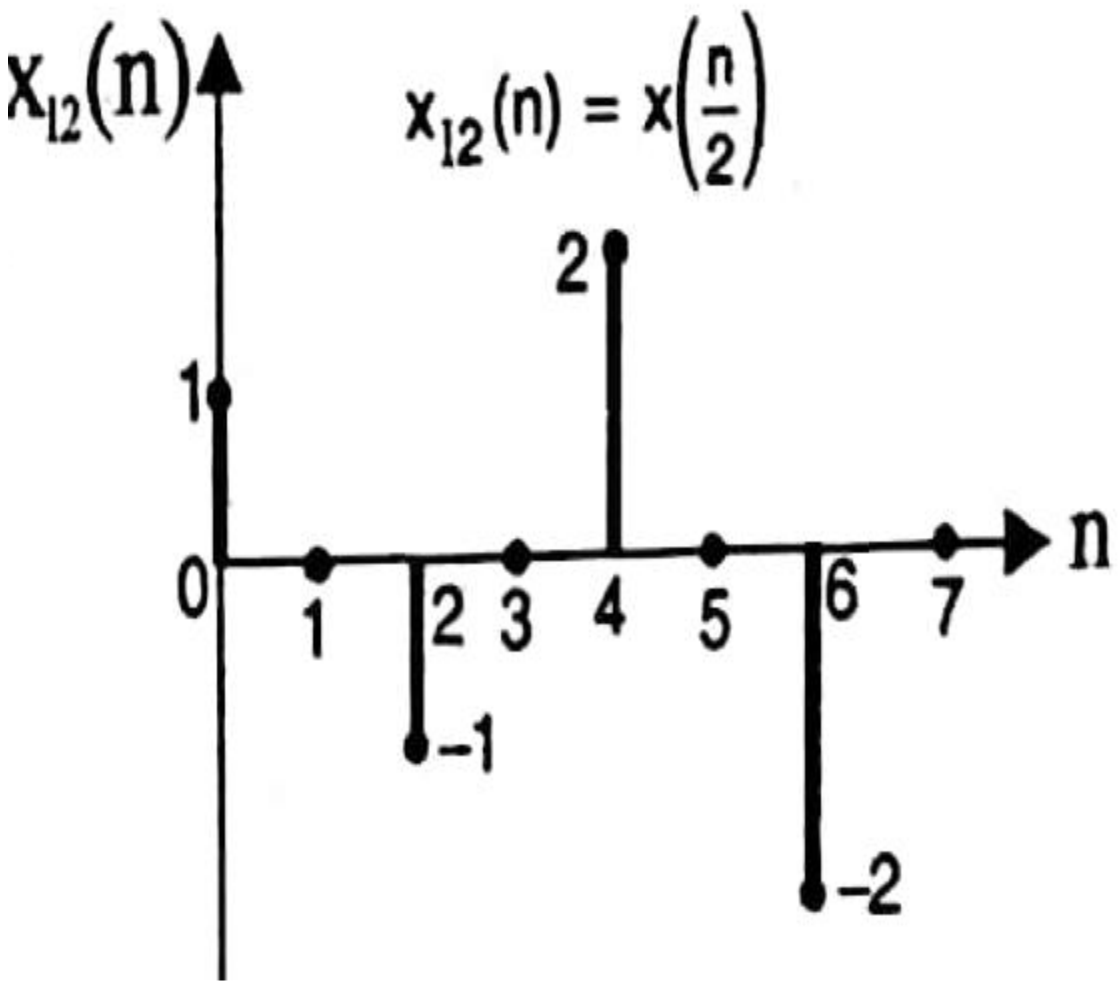
$$\therefore\ x\left(\frac{n}{3}\right) = x_{I3}(n) = \{\underset{\uparrow}{1},\ 0,\ 0,\ -1,\ 0,\ 0,\ 2,\ 0,\ 0,\ -2,\ 0,\ 0\} \qquad .....(2)$$

EXAMPLE

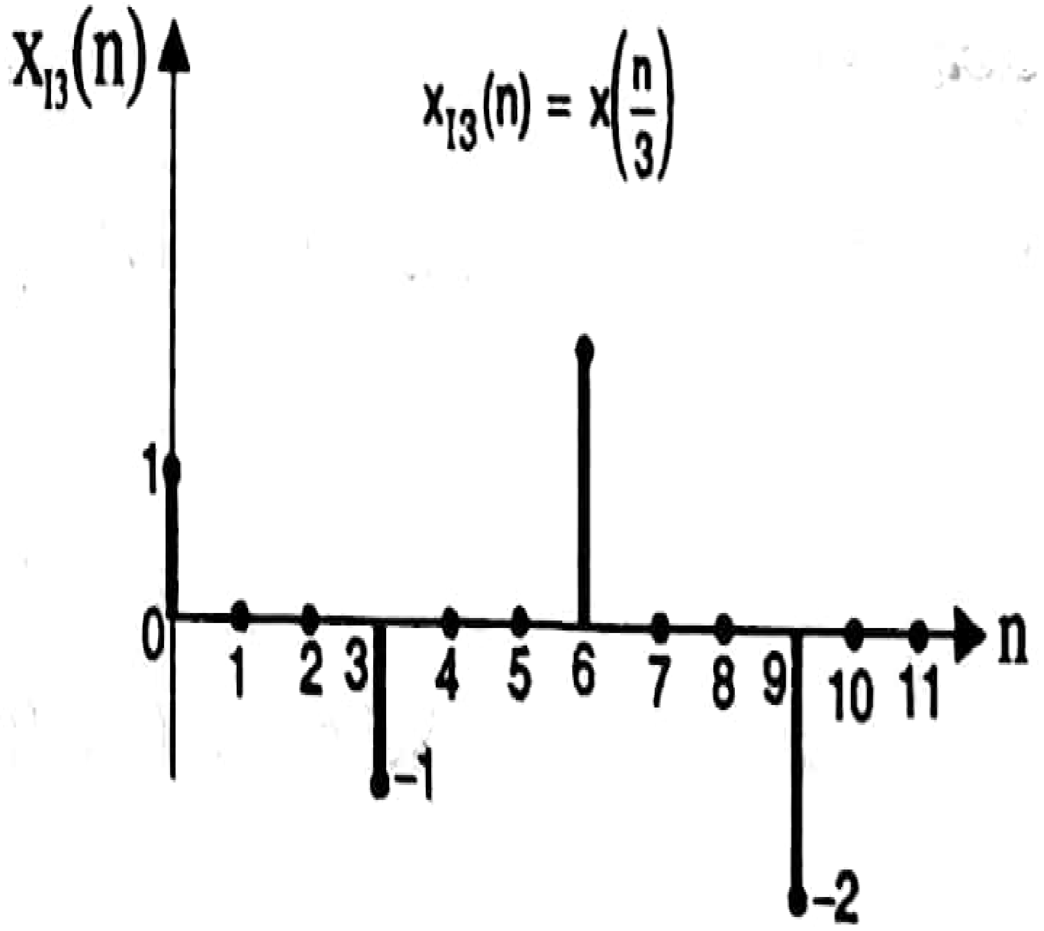
Samples of sequence



x(n) interpolated by 2



x(n) interpolated by 3



SPECTRUM OF UPSAMPLER

- Let x(n) be an input signal to the upsampler and y(n) be the output signal
- Let x(n/I) be the upsampled version of x(n) by an integer factor I

$$\mathbf{y(n) = x(n/I)}$$

- By definition of Z-transform, y(n) can be expressed as

$$Y(z) = \sum_{n=-\infty}^{+\infty} y(n)\, z^{-n}$$

$$= \sum_{n=-\infty}^{+\infty} x\left(\frac{n}{I}\right) z^{-n}$$

$$= \sum_{m=-\infty}^{+\infty} x(m)\, z^{-mI}$$

$$= \sum_{n=-\infty}^{+\infty} x(n)\, z^{-nI}$$

$$= \sum_{n=-\infty}^{+\infty} x(n)\, (z^{I})^{-n}$$

$\boxed{\text{On substituting } y(n) = x\left(\frac{n}{I}\right) \text{ from equation}}$

Let, $m = \frac{n}{I} \Rightarrow n = mI$

when $n = -\infty, \quad m = -\infty$

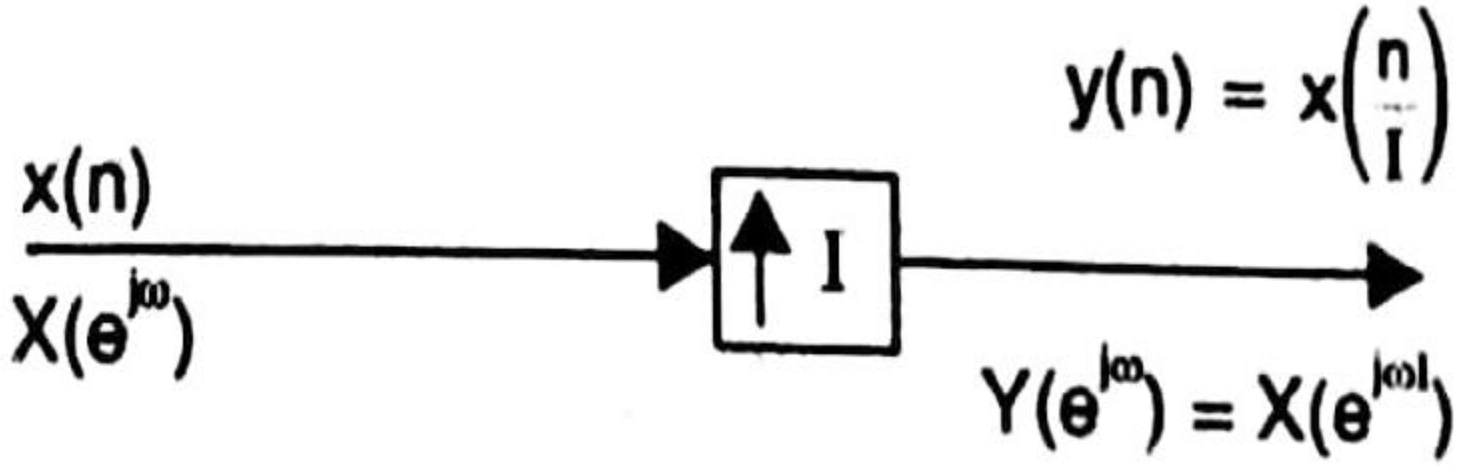
when $n = +\infty, \quad m = +\infty$

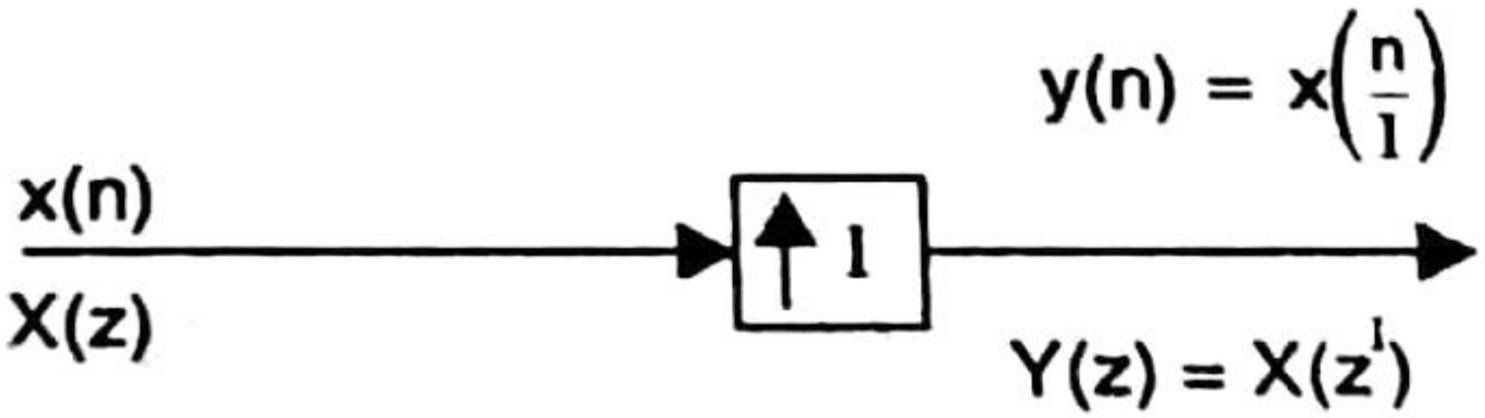
Let, $m \rightarrow n$

SPECTRUM OF UPSAMPLER

$$Y(e^{j\omega}) = X(e^{j\omega I})$$

$$Y(z) = X(z^{I})$$



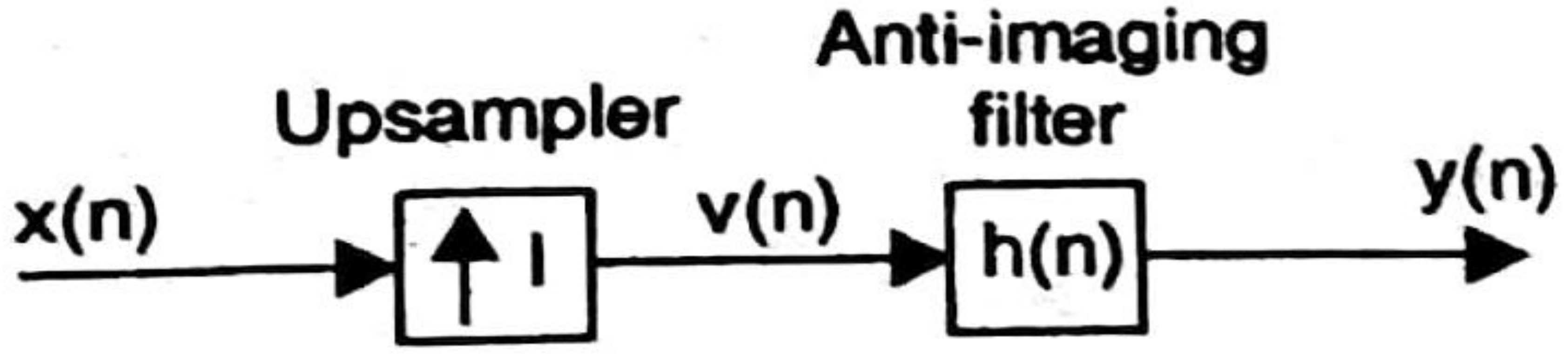


Frequency Domain Representation of upsampler

Z-Domain Representation of upsampler

ANTI-IMAGING FILTER

- The output spectrum of interpolator is compressed version of the input spectrum, Therefore, the spectrum of upsampled signal has multiple images in the period of 2**π**
- When upsampled by a factor of I, the output spectrum will have I images in a period of 2**π**, with each image bandlimited to **π/I**. Since the frequency spectrum in the range 0 to **π/I** are unique, we have to filter the other images
- Hence the output of upsampler is passed through a lowpass filter with a bandwidth of **π/I** . Since this lowpass filter is designed to avoid multiple images in the output spectrum, it is called anti-imaging filter



ASSESSMENT

1. Define multirate DSP.
2. The discrete time systems that employ sampling rate conversion while processing the discrete time signals are called --------------------
3. What is meant by sampling rate conversion.
4. List the two ways for sampling rate conversion in the digital domain
5. What is meant by downsampling and upsampling?
6. What are the advantages of multirate Processing?
7. Define anti-imaging filter.

THANK YOU