(1) $x^2$, $(0, 2l)$

$a_0 = \frac{8l^2}{3}$ ; $a_n = \frac{4l^2}{n^2\pi^2}$ ; $b_n = \frac{-4l}{n\pi}$

Problems on $(0, 2l)$

1. $f(x) = (l-x)^2$, $(0, 2l)$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos\frac{n\pi x}{l} + b_n \sin\frac{n\pi x}{l}\right] \rightarrow (1)$$

$$a_0 = \frac{1}{l}\int_0^{2l} f(x)\, dx$$

$$= \frac{1}{l}\int_0^{2l} (l-x)^2\, dx = \frac{1}{l}\left[\frac{(l-x)^3}{-3}\right]_0^{2l}$$

$$= \frac{-1}{3l}\left[(-l)^3 - l^3\right] = \frac{2l^3}{3l}$$

$$a_0 = \frac{2}{3} l^2$$

$$a_n = \frac{1}{l}\int_0^{2l} f(x) \cdot \cos\frac{n\pi x}{l}\, dx$$

$$= \frac{1}{l}\int_0^{2l} (l-x)^2 \cos\frac{n\pi x}{l}\, dx$$

| | |
|---|---|
| $u = (l-x)^2$ | $v = \cos\frac{n\pi x}{l}$ |
| $u' = -2(l-x)$ | $v_1 = \frac{l}{n\pi}\sin\frac{n\pi x}{l}$ |
| $u'' = 2$ | $v_2 = \frac{-l^2}{n^2\pi^2}\cos\frac{n\pi x}{l}$ |
| $u''' = 0$ | $v_3 = \frac{-l^3}{n^3\pi^3}\sin\frac{n\pi x}{l}$ |

$$= \frac{1}{l}\left[(l-x)^2 \frac{l}{n\pi} \sin\frac{n\pi x}{l} + 2(l-x)\frac{-l^2}{n^2\pi^2}\cos\frac{n\pi x}{l} + 2\left(\frac{-l^3}{n^3\pi^3}\sin\frac{n\pi x}{l}\right) - 0\right]_0^{2l}$$

$$= \frac{1}{l}\left[+\frac{2l^3}{n^2\pi^2} + \frac{2l^3}{n^2\pi^2}\right] = \frac{1}{l}\left[\frac{4l^3}{n^2\pi^2}\right]$$

$$a_n = \frac{4l^2}{n^2\pi^2}$$

$$b_n = \frac{1}{l}\int_0^{2l} f(x)\sin\frac{n\pi x}{l}\,dx$$

$$= \frac{1}{l}\int_0^{2l}(l-x)^2 \sin\frac{n\pi x}{l}\,dx$$

$\int uv\,dx = uv_1 - u'v_2 + u''v_3 - \dots$

$u = (l-x)^2$ | $v = \sin\frac{n\pi x}{l}$

$u' = -2(l-x)$ | $v_1 = -\frac{l}{n\pi}\cos\frac{n\pi x}{l}$

$u'' = 2$ | $v_2 = \frac{-l^2}{n^2\pi^2}\sin\frac{n\pi x}{l}$

$u''' = 0$ | $v_3 = \frac{l^3}{n^3\pi^3}\cos\frac{n\pi x}{l}$

$$= \frac{1}{l}\left[(l-x)^2\left(\frac{-l}{n\pi}\cos\frac{n\pi x}{l}\right) - 2(l-x)\frac{l^2}{n^2\pi^2}\sin\frac{n\pi x}{l} + 2\frac{l^3}{n^3\pi^3}\cos\frac{n\pi x}{l}\right]_0^{2l}$$

$$= \frac{1}{l}\left[\left(-\frac{l^3}{n\pi} + \frac{2l^3}{n^3\pi^3}\right) - \left(\frac{-l^3}{n\pi} + \frac{2l^3}{n^3\pi^3}\right)\right]$$

$$b_n = 0$$

$$(1) \Rightarrow \quad f(x) = \frac{2l^2/3}{2} + \sum_{n=1}^{\infty}\left[\frac{4l^2}{n^2\pi^2}\cos nx + 0\right]$$

$$= \frac{l^2}{3} + \frac{4l^2}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\cos nx$$

---

2). Obtain the fourier series for $f(x) = \begin{cases} l-x, & 0 < x \le l \\ 0, & l \le x < 2l \end{cases}$

Soln. :

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos\frac{n\pi x}{l} + b_n \sin\frac{n\pi x}{l}\right] \rightarrow (1)$$

$$a_0 = \frac{1}{l}\int_0^{2l} f(x)\,dx$$

$$= \frac{1}{l}\left[\int_0^{l}(l-x)\,dx + 0\right] = \frac{1}{l}\left[\int_0^{l}(l-x)\,dx\right]$$

$$= \frac{1}{l}\left[\frac{(l-x)^2}{-2}\right]_0^{l}$$

$$= \frac{-1}{2l}\left[0 - l^2\right]$$

$$a_0 = l/2$$

$$a_n = \frac{1}{l}\int_0^{2l} f(x)\cos\frac{n\pi x}{l}\,dx$$

$$= \frac{1}{l}\int_0^{l}(l-x)\cos\frac{n\pi x}{l}\,dx$$

$\int u\,v\,dx = uv_1 - u'v_2 + u''v_3 - \dots$

$u = l-x$, $v = \cos\frac{n\pi x}{l}$

$u' = -1$, $v_1 = \frac{l}{n\pi}\sin n\pi x/l$

$u'' = 0$, $v_2 = \frac{-l^2}{n^2\pi^2}\cos\frac{n\pi x}{l}$

$$= \frac{1}{l}\left[(l-x)\frac{l}{n\pi}\sin\frac{n\pi x}{l} + 1\left(\frac{-l^2}{n^2\pi^2}\cos\frac{n\pi x}{l}\right)\right]_0^{l}$$

$$= \frac{1}{l}\left[\frac{-l^2}{n^2\pi^2}(-1)^n + \frac{l^2}{n^2\pi^2}\right]$$

$$= \frac{1}{l}\,\frac{l^2}{n^2\pi^2}\left[1-(-1)^n\right]$$

$$a_n = \frac{l}{n^2\pi^2}\left[1-(-1)^n\right]$$

$$b_n = \frac{1}{l}\int_0^{2l} f(x)\sin\frac{n\pi x}{l}\,dx$$

$$= \frac{1}{l}\int_0^{l}(l-x)\sin\frac{n\pi x}{l}\,dx$$

$\int u\,v\,dx = uv_1 - u'v_2 + u''v_3 - \dots$

$u = l-x$, $v = \sin\frac{n\pi x}{l}$

$u' = -1$, $v_1 = \frac{-l}{n\pi}\cos\frac{n\pi x}{l}$

$u'' = 0$, $v_2 = \frac{-l^2}{n^2\pi^2}\sin\frac{n\pi x}{l}$

$$= \frac{1}{l}\left[(l-x)\left(\frac{-l}{n\pi}\cos\frac{n\pi x}{l}\right) + 1\left(\frac{-l^2}{n^2\pi^2}\sin\frac{n\pi x}{l}\right)\right]_0^l$$

$$= \frac{1}{l}\left[+\frac{l^2}{n\pi}\right]$$

$$b_n = \frac{l}{n\pi}$$

$$\therefore f(x) = \frac{l/2}{2} + \sum_{n=1}^{\infty}\left[\frac{l}{n^2\pi^2}\{[1-(-1)^n]\cos\frac{n\pi x}{l} + \frac{l}{n\pi}\sin\frac{n\pi x}{l}\right]$$

$$= \frac{l}{4} + \frac{l}{\pi^2}\sum_{n=1}^{\infty}\frac{[1-(-1)^n]}{n^2}\cos\frac{n\pi x}{l} + \frac{l}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\frac{n\pi x}{l}$$

3]. Find the fourier series for $f(x) = 2x - x^2$, $0<x<2$

$2l = 2$

$l = 1$

Soln. :

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n\cos\frac{n\pi x}{l} + b_n\sin\frac{n\pi x}{l}\right]$$

$$a_0 = \frac{1}{l}\int_0^{2l} f(x)\,dx = \int_0^2 (2x - x^2)\,dx$$

$$= \left(2\frac{x^2}{2} - \frac{x^3}{3}\right)_0^2$$

$$= \left(4 - \frac{8}{3}\right) - 0$$

$$a_0 = \frac{4}{3}$$

$$a_n = \frac{1}{l}\int_0^{2l} f(x)\cos\frac{n\pi x}{l}\,dx = \int_0^2 (2x - x^2)\cos n\pi x\,dx$$

$$\int uv\,dx = uv_1 - u'v_2 + u''v_3 - \cdots$$

$u = 2x - x^2$, $v = \cos n\pi x$

$u' = 2 - 2x$, $v_1 = \frac{\sin(n\pi x)}{n\pi}$

$u'' = -2$, $v_2 = -\frac{\cos(n\pi x)}{n^2\pi^2}$

$u''' = 0$, $v_3 = -\frac{\sin(n\pi x)}{n^3\pi^3}$

$$= \left[(2x-x^2)\frac{\sin(n\pi x)}{n\pi} + (2-2x)\frac{\cos(n\pi x)}{n^2\pi^2} - 2\left(-\frac{\sin(n\pi x)}{n^3\pi^3}\right)\right]_0^2$$

$$= \frac{-2}{n^2\pi^2} - \frac{2}{n^2\pi^2}$$

$$a_n = \frac{-4}{n^2\pi^2}$$

$$b_n = \frac{1}{l}\int_0^{2l} f(x)\sin\frac{n\pi x}{l}\,dx$$

$$= \int_0^2 (2x-x^2)\sin(n\pi x)\,dx$$

$u = 2x - x^2$, $u' = 2-2x$, $u'' = -2$, $u''' = 0$

$v = \sin n\pi x$, $v_1 = -\frac{\cos n\pi x}{n\pi}$, $v_2 = -\frac{\sin n\pi x}{n^2\pi^2}$, $v_3 = \frac{\cos n\pi x}{n^3\pi^3}$

$$= \left[(2x-x^2)\left(-\frac{\cos n\pi x}{n\pi}\right) - (2-2x)\left(-\frac{\sin n\pi x}{n^2\pi^2}\right) - 2\left(\frac{\cos n\pi x}{n^3\pi^3}\right) + 0\right]_0^2$$

$$= \frac{-2}{n^3\pi^3} + \frac{2}{n^3\pi^3}$$

$$b_n = 0$$

$$f(x) = \frac{2}{3} - \frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\cos\frac{n\pi x}{l}$$