Singular point :-

A point $z=a$ is said to be a singular point if $f(z)$ is not analytic at that point.

$f(z) = 1/z$, which is not analytic at $z=0$.

types of singularities

1) Isolated singularities :-

A point $z=a$ is said to be isolated singularities. if these ~~are few~~

$\rightarrow$ $f(z)$ is not analytic at $z=0$.

$\rightarrow$ $f(z)$ is analytic at all points for some neighbourhood of that point.

ex :- $f(z) = \dfrac{z}{(z-1)(z-2)}$

2) Pole :-

A point $z=a$ is said to be a pole of order n, if we can find the positive integer such that $\lim\limits_{z \to a} (z-a)^n f(z) \neq 0$

Eg :- $\dfrac{z-1}{(z-2)(z-3)^4}$

3) Essential singularities :-

A point $z=a$ is said to be an essential singularities if laurentz series of $f(z)$ about $z=a$ which possess infinite number of terms in the principle part (terms that contain (-ve) powers).

Eg :- $e^{1/4}$ at $z=0$

4) Removable singularity :

The singular point $z=a$ is called a removable singularity if $\lim_{z \to a} f(z)$ exist.

Eg :- $\dfrac{\sin z}{z}$ $\qquad \dfrac{z - \frac{z^3}{3!} + \frac{z^5}{5!}}{z}$

At the point $z=0$

$e^x = 1 + x + x^2/2 + \dots$

$e^{1/2} = 1 + 1/2 + \frac{(1/2)^2}{2} + \dots$

classify the singularities of the following

1. $\dfrac{\sin z - z}{z^3}$ – Removable

2. $\dfrac{\tan z}{z}$ – Removable

3. $\sin\left(\dfrac{1}{z+1}\right)$ - essential.

Residue :-

If $f(z) = a$ is pole of order 1, then $[\text{Res } f(z)]_{z=a} = \lim_{z \to a} [z-a] f(z)$.

If $z = a$ is a pole of order m, then $[\text{Res } (f(z))]_{z=a} = \frac{1}{(m-1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} (z-a)^m f(z)$

1, find the residue of $f(z) = \frac{z+1}{z(z-2)}$ at the point $z=2$.

Soln:- Here $z(z-2) = 0$

$z=0$ and $z-2=0$

$z=2$

$z=0$ is a pole of order 1

$z=2$ is a pole of order 1

$$[\text{Res } f(z)] = \lim_{z \to 2} (z-2) \frac{z+1}{z(z-2)}$$

$$= \lim_{z \to 2} \frac{2+1}{z} = \frac{2+1}{2} = 3/2$$