



1. State Cauchy's Residue theorem.

If  $f(z)$  be analytic at all points inside and on a simple closed curve  $C$ , except

for a finite number of isolated singularities  $z_1, z_2, \dots, z_n$  inside  $C$ , then

$$\int_C f(z) dz = 2\pi i [\text{sum of the residues of } f(z) \text{ at } z_1, z_2, \dots, z_n]$$
$$= 2\pi i \sum_{i=1}^n R_i \text{ where } R_i \text{ is the residue of } f(z) \text{ at } z = z_i$$

2. Calculate the residue of  $f(z) = \frac{e^{2z}}{(z+1)^2}$  at the pole

Given  $f(z) = \frac{e^{2z}}{(z+1)^2}$

$z = -1$  is a pole of order 2

$$\begin{aligned} \text{Res } \{f(z)\}_{z=-1} &= \lim_{z \rightarrow -1} \frac{d}{dz} \left[ (z+1)^2 \frac{e^{2z}}{(z+1)^2} \right] \\ &= \lim_{z \rightarrow -1} \frac{d}{dz} [e^{2z}] \\ &= \lim_{z \rightarrow -1} 2e^{2z} \\ &= 2e^{-2} \end{aligned}$$

3. Find the residue of  $f(z) = \frac{z^2}{(z-1)^2(z+2)}$  at  $z = -2$ .

$z = 1$  is a pole of order 2

$z = -2$  is a pole of order 1

$$\text{Res } \{f(z)\}_{z=-2} = \lim_{z \rightarrow -2} \left[ (z+2) \frac{z^2}{(z-1)^2(z+2)} \right]$$



$$= \lim_{z \rightarrow -2} \frac{z^2}{(z-1)^2}$$

$$= \frac{(-2)^2}{(-2-1)^2} = \frac{4}{9} = \frac{2}{3}$$

4. Find the residue of  $f(z) = \frac{z+1}{z(z+2)}$   
 $Z=0$  is a pole of order 1

$Z=2$  is a pole of order 1

$$\text{Res}\{f(z)\}_{z=0} = \lim_{z \rightarrow 0} z \frac{z+1}{z(z-2)}$$

$$\text{Res}\{f(z)\}_{z=2} = \lim_{z \rightarrow 2} (z-2) \frac{z+1}{z(z-2)} = \frac{-3}{2}$$

$Z=2$  is a simple pole

$$\text{Res}\{f(z)\}_{z=2} = \lim_{z \rightarrow 2} (z-2) \frac{4}{z^3(z-2)} = \frac{1}{2}$$

5. Obtain the residues of the function  $f(z) = \frac{z-3}{(z+1)(z-2)}$  at its poles.  
 $Z=-1$  is a pole of order 1

$Z=-2$  is a pole of order 1

$$\text{Res}\{f(z)\}_{z=-1} = \lim_{z \rightarrow -1} (z+1) \frac{(z-3)}{(z+1)(z+2)} = -4$$

$$\text{Res}\{f(z)\}_{z=-2} = \lim_{z \rightarrow -2} (z+2) \frac{(z-3)}{(z+1)(z+2)} = -5$$



UNIT-4    **COMPLEX INTEGRATION**                      Cauchy Residues Theorem

6. Consider the function  $f(z) = \frac{\sin z}{z^4}$ . Find the pole and its order.

$$f(z) = \frac{\sin z}{z^4} = \frac{1}{z^4} \left[ z - \frac{z^3}{3} + \frac{z^5}{5} - \dots \right]$$

$$= \frac{1}{z^3} \left[ 1 - \frac{z^2}{3} + \frac{z^4}{5} - \dots \right]$$

**Z=0 is a pole of order 3**

7. Define pole and simple poles.

A point  $z=a$  is said to be a pole  $f(z)$  of order  $n$  if we can find a positive integer  $n$  such

that  $\lim_{z \rightarrow a} (z-a)^n f(z) \neq 0$

A pole of order one is called a simple pole.

$$f(z) = \frac{1}{(z-4)^2(z-3)^3(z-1)}$$

Example :

Hence  **$z=1$  is a simple pole of order 1**

**$Z=4$  is a simple pole of order 2**

**$Z=3$  is a simple pole of order 3.**

8. Find the regularities of the function  $f(z) = \frac{\cot \pi z}{(z-a)^3}$

Given  $f(z) = \frac{\cot \pi z}{(z-a)^3} = \frac{\cos \pi z}{\sin \pi z (z-a)^3}$

Singular points are poles and are given by  $\text{Dr}=0$ .

$\sin \pi z = 0, (z-a)^3 = 0 \Rightarrow z = a$  is a singular pole of order 3

$\sin \pi z = \sin n \pi$  where  $n = 0, \pm 1, \pm 2, \dots$

$\pi z = n \pi \Rightarrow n = \pi$

$z = n = 0, \pm 1, \pm 2, \dots$  are simple poles.



9. Find the principal part and residue at the pole of  $f(z) = \frac{2z+3}{(z+2)^2} = (2z+3)(z+2)^{-2}$

[since principal part is negative powers]

$(z+2)^{-2} \Rightarrow z = -2$  is a singular pole of order 2

$$\text{Res}\{f(z)\}_{z=-2} = \lim_{z \rightarrow -2} \frac{d}{dz} \left[ (z+2)^2 \cdot \frac{(2z+3)}{(z+2)^2} \right] = 2$$

10. Evaluate  $\int_c \frac{dz}{z-2}$  where C is the square with vertices (0,0),(1,0),(1,1) and (0,1).

$$\text{Given } \int_c \frac{dz}{z-2}$$

Here  $f(z) = 1$

$a = 2$  lies outside

$\therefore$  By Cauchy's integral formula  $\int_c \frac{dz}{z-2} = 0$ .