SNS COLLEGE OF TECHNOLOGY

(AN AUTONOMOUS INSTITUTION)

COIMBATORE-35

DEPARTMENT OF MATHEMATICS

UNIT II

PART – A

1. Solve : $\left(D^2 - 2D + 2\right)y = 0$.
2. Solve : $\left(D^2 - 6D + 13\right)y = 0$.
3. Solve : $\left(D^2 + 4\right)y = 0$.
4. Solve : $\left(D^2 + 1\right)y = e^{-x}$.
5. Solve : $\dfrac{d^4y}{dx^4} = 16y$.
6. Solve : $\dfrac{d^2y}{dx^2} + 4y = e^{-2x}$.
7. Find the particular integral of $\dfrac{d^2y}{dx^2} + 4y = \sin 2x$.
8. Find the particular integral of $\left(D^4 + D^2\right)y = \cos x$.
9. Find the particular integral of $\left(D^4 + D^2\right)y = \sin x$.
10. Solve : $\left(D^3 + 3D^2 + 3D + 1\right)y = 0$.
11. Solve : $\left(D^4 - 2D^3 + D^2\right)y = 0$.
12. Solve : $\left(D^4 + 2D^2 + 1\right)y = 0$.
13. Solve : $\left(4D^2 - 4D + 1\right)y = 4$.
14. Solve : $(D-2)^2 y = e^{2x}$.

Answers

1) *The Auxillary Equation is* $m^2 - 2m + 2 = 0$

$$m = \frac{2 \pm \sqrt{4-8}}{2} = \frac{2 \pm 2i}{2} = 1 \pm i$$

The Complete Solution is $y = e^{x}\left(A\cos x + B\sin x\right)$

2) The Auxillary Equation is $m^2 - 6m + 13 = 0$

$m = \frac{6 \pm \sqrt{36-52}}{2} = \frac{6 \pm 4i}{2} = 3 \pm 2i$

The Complete Solution is $y = e^{3x}\left(A\cos 2x + B\sin 2x\right)$

3) The Auxillary Equation is $m^2 + 4 = 0$

$m = \pm 2i$

The Complete Solution is $y = A\cos 2x + B\sin 2x$

4) The Auxillary Equation is $m^2 + 1 = 0$

$m = \pm i$

The Complementary Function $= A\cos x + B\sin x$

$P.I = \frac{1}{D^2+1}e^{-x} = \frac{1}{2}e^{-x}$

The Complete Solution is $y = A\cos x + B\sin x + \frac{1}{2}e^{-x}$

5) The Auxillary Equation is $m^4 - 2^4 = 0$

$\left(m^2 - 2^2\right)\left(m^2 + 2^2\right) = 0$

$(m-2)(m+2)\left(m^2+4\right) = 0$

$m = -2, 2, \pm 2i$

The Complementary Function $= Ae^{-2x} + Be^{2x}\left(C\cos 2x + D\sin 2x\right)$

6) The Auxillary Equation is $m^2 + 4 = 0$

$\left(m^2 + 2^2\right) = 0$

$m = \pm 2i$

The Complementary Function $= A\cos 2x + B\sin 2x$

$P.I = \frac{1}{D^2+4}e^{-2x} = \frac{1}{8}e^{-2x}$

The Complete Solution is $y = A\cos 2x + B\sin 2x + \frac{1}{8}e^{-2x}$

7) $P.I = \frac{1}{D^2+4}\sin 2x = \frac{x}{2D}\sin 2x = \frac{-x\cos 2x}{4}$

8) $P.I = \frac{1}{D^4+D^2}\cos x = \frac{x}{4D^3+2D}\cos x = \frac{x}{-4D+2D}\cos x = \frac{x}{-2D}\cos x = \frac{-x\sin x}{2}$

9) $P.I = \frac{1}{D^4 + D^2}\sin x = \frac{x}{4D^3 + 2D}\sin x = \frac{x}{-4D + 2D}\sin x = \frac{x}{-2D}\sin x = \frac{x\cos x}{2}$

10) $The\ Auxillary\ Equation\ is\ m^3 + 3m^2 + 3m + 1 = 0$

$$\begin{array}{r|rrrr} -1 & 1 & 3 & 3 & 1 \\ & 0 & -1 & -2 & -1 \\ \hline & 1 & 2 & 1 & \lfloor 0 \end{array}$$

$m = -1,\ m^2 + 2m + 1 = 0$

$m = -1,\ m = -1, m = -1$

$The\ Complementary\ Function = \left(Ax^2 + Bx + C\right)e^{-x}$

11) $The\ Auxillary\ Equation\ is\ m^4 - m^3 + m^2 = 0$

$m^2\left(m^2 - m + 1\right) = 0$

$m = 0, 0, m = \frac{1 \pm \sqrt{1-4}}{2} = \frac{1 \pm \sqrt{3}i}{2}$

$The\ Complete\ Solution\ is\ y = (Ax + B) + e^{\frac{x}{2}}\left(C\cos\frac{\sqrt{3}}{2}x + D\sin\frac{\sqrt{3}}{2}x\right)$

12) $The\ Auxillary\ Equation\ is\ m^4 + 2m^2 + 1 = 0$

$Take,\ t = m^2, Then\ t^2 + 2t + 1 = 0$

$t = -1, -1$

$m^2 = -1, m^2 = -1$

$m = \pm i, \pm i$

$The\ Complete\ Solution\ is\ y = (Ax + B)\cos x + (Cx + D)\sin x$

13) $The\ Auxillary\ Equation\ is\ 4m^2 - 4m + 1 = 0$

$m = \frac{4 \pm \sqrt{16-16}}{8} = \frac{1}{2}(twice)$

$The\ Complementary\ function\ is = (Ax + B)e^{\frac{x}{2}}$

$P.I = \frac{1}{4D^2 - 4D + 1}4 = 4$

$The\ Complete\ Solution\ is\ y = (Ax + B)e^{\frac{x}{2}} + 4$

$The\ Auxillary\ Equation\ is\ (m-2)^2 = 0$

$m = 2\,(twice)$

14) $The\ Complementary\ funtion\ is = (Ax+B)e^{2x}$

$P.I = \frac{1}{(D-2)^2}e^{2x} = \frac{x}{2(D-2)}e^{2x} = \frac{x^2}{2D}e^{2x} = \frac{x^2}{4}e^{2x}$

$The\ Complete\ Solution\ is\ y = (Ax+B)e^{2x} + \frac{x^2}{4}e^{2x}$

LEVEL-2, QUESTIONS

1. Find the particular integral of $\left(D^2+1\right)y = \cos(2x-1)$.
2. Find the particular integral of $y'' + 2y' + 5y = e^{-x}\cos 2x$.
3. Find the particular integral of $\left(D^2+4D+5\right)y = e^{-2x}\cos x$.
4. Find the particular integral of $\left(D^2+1\right)y = \cosh 2x$.
5. Find the particular integral of $(D-1)^2 y = \sinh 2x$.
6. Find the particular integral of $(D+1)^2 y = e^{-x}\cos x$.
7. Find the particular integral of $\left(D^2+4\right)y = \cos^2 x$.
8. Find the particular integral of $\left(D^3-7D-6\right)y = x$.
9. Find the particular integral of $\left(D^2+4D+3\right)y = 2e^{-x}\left(x^2+2\right)$.
10. Find the particular integral of $\left(D^2+6D+8\right)y = \cos^2 x$.
11. Find the particular integral of $(D-3)^2 y = xe^{-2x}$.
12. Find the particular integral of $\left(D^2+4D+4\right)y = xe^{-2x}$.

Answers

1) $P.I = \frac{1}{\left(D^2+1\right)}\cos(2x-1) = \frac{1}{(-4+1)}\cos(2x-1) = -\frac{1}{3}\cos(2x-1)$

2)
$$P.I = \frac{1}{\left(D^2+2D+5\right)}e^{-x}\cos 2x = e^{-x}\frac{1}{\left((D-1)^2+2(D-1)+5\right)}\cos 2x$$
$$= e^{-x}\frac{1}{\left(D^2+4\right)}\cos 2x = e^{-x}\frac{x}{2D}\cos 2x = \frac{xe^{-x}\sin 2x}{4}$$

3)
$$P.I=\frac{1}{\left(D^2+4D+5\right)}e^{-2x}\cos x=e^{-2x}\frac{1}{\left((D-2)^2+4(D-2)+5\right)}\cos x$$
$$=e^{-2x}\frac{1}{D^2+1}\cos x=e^{-2x}\frac{1}{-1+1}\cos x=\frac{xe^{-2x}\sin x}{2}$$

4)
$$P.I=\frac{1}{\left(D^2+1\right)}\cosh 2x=\frac{1}{\left(D^2+1\right)}\left(\frac{e^{-2x}+e^{2x}}{2}\right)$$
$$=\frac{1}{2}\left\{\frac{1}{D^2+1}e^{-2x}+\frac{1}{D^2+1}e^{2x}\right\}=\frac{1}{2}\left\{\frac{1}{-4+1}e^{-2x}+\frac{1}{-4+1}e^{2x}\right\}$$
$$=\frac{1}{2}\left\{\frac{1}{-3}e^{-2x}+\frac{1}{-3}e^{2x}\right\}=\frac{1}{-3}\left(\frac{e^{-2x}+e^{2x}}{2}\right)=\frac{\cosh 2x}{-3}$$

5)
$$P.I=\frac{1}{(D-1)^2}\sinh 2x=\frac{1}{\left(D^2-2D+1\right)}\left(\frac{e^{-2x}-e^{2x}}{2}\right)$$
$$=\frac{1}{2}\left\{\frac{1}{D^2-2D+1}e^{-2x}-\frac{1}{D^2-2D+1}e^{2x}\right\}=\frac{1}{2}\left\{\frac{1}{-4-4+1}e^{-2x}-\frac{1}{-4-4+1}e^{2x}\right\}$$
$$=\frac{1}{2}\left\{\frac{1}{-7}e^{-2x}-\frac{1}{-7}e^{2x}\right\}=-\frac{1}{7}\left(\frac{e^{-2x}-e^{2x}}{2}\right)=-\frac{1}{7}\sinh 2x$$

6)
$$P.I=\frac{1}{(D+1)^2}e^{-x}\cos x=e^{-x}\frac{1}{((D-1)+1)^2}\cos x$$
$$=e^{-x}\frac{1}{D^2}\cos x=e^{-x}\frac{1}{-1}\cos x=-e^{-x}\cos x$$

7)
$$P.I=\frac{1}{\left(D^2+4\right)}\cos^2 x=\frac{1}{\left(D^2+4\right)}\left(\frac{1+\cos 2x}{2}\right)$$
$$=\frac{1}{2}\frac{1}{\left(D^2+4\right)}(1+\cos 2x)=\frac{1}{2}\left\{\frac{1}{\left(D^2+4\right)}(1)+\frac{1}{\left(D^2+4\right)}\cos 2x\right\}$$
$$=\frac{1}{2}\left\{\frac{1}{4}+\frac{1}{(-4+4)}\cos 2x\right\}=\frac{1}{2}\left\{\frac{1}{4}+\frac{x}{2D}\cos 2x\right\}=\frac{1}{2}\left\{\frac{1}{4}+\frac{x}{2}\frac{\sin 2x}{2}\right\}=\frac{1}{8}\{1+x\cos 2x\}$$