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NUMERICAL DIFFERENTIATION & INTEGRATION

NUMERICAL DIFFERENTIATION :

It is the process of computing the value of the desivative dy for some particular value of x, from the given data (x;, y;). If the values of x are equally spaced, we can use Newton's interpolation formula for earnal intervals. If the values of x are unequally spaced, we can use Lagrange's interpolation formula (or) Newton's divided difference interpolation formula. Differentiation Using interpolation formulae: Newton's forward difference formula to compute the derivatives : Let us consider Newton's forward difference formula, $\frac{u(u-1)(u-2)(u-3)}{4!} \Delta^{4} y_{0} + \cdots$ where $U = \frac{\chi - \chi_0}{h}$ i.e., $y = y_0 + u \Delta y_0 + (u^2 - u) \Delta^2 y_0 + (u^3 - 3u^2 + 2u) \Delta^3 y_0$ $+ (u^{4} - 6u^{3} + 11u^{2} - 6u) \Delta^{4} y_{0} + \cdots$ Now, $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$





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$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{h} \quad \frac{dy}{du} \\ \text{i.e.} \quad \frac{dy}{dx} &= \frac{1}{h} \quad \left\{ \Delta y_{0} + \frac{(3u-1)}{2} \quad \Delta^{2} y_{0} + \frac{(3u^{2}-bu+2)}{4} \quad \Delta^{3} y_{0} + \frac{(4u^{3}-18u^{2}+3au-4)}{24} \quad \Delta^{4} y_{0} + \cdots \right\} \\ &= \frac{(4u^{3}-18u^{2}+3au-4)}{24} \quad \Delta^{4} y_{0} + \cdots \right] \\ \frac{d^{2} y}{dx^{2}} &= \frac{1}{h^{2}} \quad \left\{ \quad \Delta^{2} y_{0} + (u-1)\Delta^{3} y_{0} + \frac{(bu^{2}-18u+1)}{12} \quad \Delta^{4} y_{0} + \cdots \right\} \\ &= \sqrt{2} \end{aligned}$$

$$\begin{aligned} \frac{d^{3} y}{dx^{3}} &= \frac{1}{h^{3}} \quad \left\{ \quad \Delta^{3} y_{0} + \frac{12u-18}{12} \quad \Delta^{4} y_{0} + \cdots \right\} \\ &= \sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{Tr particular, at } x = x_{0}, \quad u = 0. \text{ Hence putting} \\ u = 0 \quad \text{in } \quad (a), \quad (b) \quad k \quad (b) \quad we \quad get \quad the \text{ values of } first, \quad \text{Second} \\ and \quad third \quad derivatives \quad at \quad z = x_{0}. \end{aligned}$$

$$\begin{aligned} \left(\frac{d^{4} y}{dx} \right)_{x = x_{0}} &= \left(\frac{d^{4} y}{dx^{2}} \right)_{u = 0} \\ &= \frac{1}{h^{2}} \quad \left\{ \Delta^{2} y_{0} - \frac{1}{2} \quad \Delta^{2} y_{0} + \frac{\Delta^{2} y_{0}}{3} - \frac{\Delta^{4} y_{0} + \cdots + y_{0}}{3} \\ &= \sqrt{2} \end{aligned}$$

$$\begin{aligned} \left(\frac{d^{3} y}{dx^{2}} \right)_{x = x_{0}} &= \left(\frac{d^{2} y}{dx^{2}} \right)_{u = 0} \\ &= \frac{1}{h^{2}} \quad \left\{ \Delta^{2} y_{0} - \Delta^{3} y_{0} + \frac{1}{h^{2}} \Delta^{4} y_{0} - \cdots + y_{0} \\ &= \sqrt{2} \end{aligned}$$



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Newton's Backward Difference formula to compute the derivatives $\frac{dy}{dz} = \frac{1}{h} \int \nabla y_n + \frac{2v+1}{2} \nabla^2 y_n + \frac{(3v^2+6v+2)}{1} \nabla^3 y_n + \frac{3v+1}{2} \nabla^2 y_n + \frac{3v+1}{2} \nabla$ $\frac{4v^3+18v^2+22v+6}{2^{4}}v^{4}y_{n}+\cdots$ $\frac{d^2 y}{dx^2} = \frac{1}{h^2} \begin{cases} \nabla^2 y_n + (v+1) \nabla^3 y_n + \frac{6v^2 + 18v + 11}{12} \nabla^4 y_n + \dots y_n \end{cases}$ $\frac{d^{3}y}{dr^{3}} = \frac{1}{h^{3}} \begin{cases} \nabla^{3}y_{n} + \frac{12v + 18}{12} \nabla^{4}y_{n} + \cdots \end{cases}$ In particular, at $x = x_n$, V = 0. Then $\left(\frac{dy}{dx}\right)_{\chi=\chi_{n}} = \frac{1}{h} \left\{ \nabla y_{n} + \frac{\nabla^{2}y_{n}}{a} + \frac{\nabla^{3}y_{n}}{3} + \frac{\nabla^{4}y_{n}}{4} + \cdots \right\}$ $\left(\frac{d^2 y}{dx^2}\right)_{\chi=\chi_n} = \frac{1}{h^2} \begin{cases} \nabla^2 y_n + \nabla^3 y_n + \frac{11}{12} \nabla^4 y_n + \cdots \\ \frac{1}{12} \end{cases}$ $\left(\frac{d^3y}{dx^3}\right)_{n=1} = \frac{1}{h^3} \left\{ \nabla^3 y_n + \frac{3}{2} \nabla^4 y_n + \cdots \right\}$

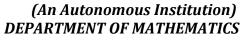


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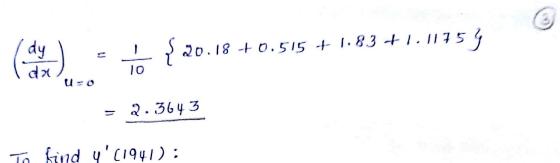


Problems: The population of a certain town is given below. Find the rate of growth of the population in 1931. 1941, 1961 and 1971. 1951 1961 1971 Year : 1931 1941 : 40.62 60.80 79.95 103.56 132.65 Population in thousands y Solution ; $\Delta^4 q$ $\Delta^2 y \quad \Delta^3 y$ Δy y X 40.62 1931 20.18 -1.03 1941 60.80 5.49 19.15 - 4 • 4 7 4.46 79.95 1951 23.61 1-02 5.48. 103.56 1961 29.09 132.65 1971 (i) To get f'(1931) and f'(1941) we use forward formula. $U = \frac{\chi - \chi_0}{h} = \frac{1931 - 1931}{10} = 0$ $\left(\frac{dy}{dx}\right) = \frac{1}{h} \begin{cases} \Delta y_0 - \frac{\Delta^2 y_0}{a} + \frac{\Delta^3 y_0}{a} - \frac{\Delta^4 y_0}{4} + \cdots \right)$ $= \frac{1}{10} \left\{ 20.18 - \frac{(-1.03)}{2} + \frac{(5.49)}{3} - \frac{(-4.47)}{4} \right\}$









(i) To find y'(1941):

$$u = \frac{x - x_{0}}{h} = \frac{1941 - 1931}{10} = 1$$

$$\frac{dy}{dx} = \frac{1}{h} \left\{ \Delta y_{0} + \frac{3u - 1}{2} \Delta^{2} y_{0} + \frac{3u^{2} - bu + 2}{6} \Delta^{3} y_{0} + \frac{4u^{3} - 18u^{2} + 22u - b}{6} \Delta^{4} y_{0} + \cdots \right\}$$

$$= \frac{1}{10} \left\{ 20.18 + \frac{1}{2} (-1.03) - \frac{1}{6} (5.49) + \frac{1}{12} (-4.47) \right\}$$

$$= \frac{1}{10} \left\{ 20.18 - 0.515 - 0.915 - 0.3735 \right\}$$

$$= \frac{1.83775}{10}$$
(ii) To find y'(1961) and y'(1971) we use Newton's backward formula

$$V = \frac{x - x_{n}}{h} = \frac{1961 - 1971}{10} = -1$$

$$\frac{dy}{dx} = \frac{1}{h} \left[\nabla y_{n} + \frac{2v + 1}{2} \nabla^{2} y_{n} + \frac{3v^{2} + bv + 2}{6} \nabla^{3} y_{n} + \frac{4v^{3} + 18v^{2} + 22v + 6}{64} \nabla^{4} y_{n} + \cdots \right]$$

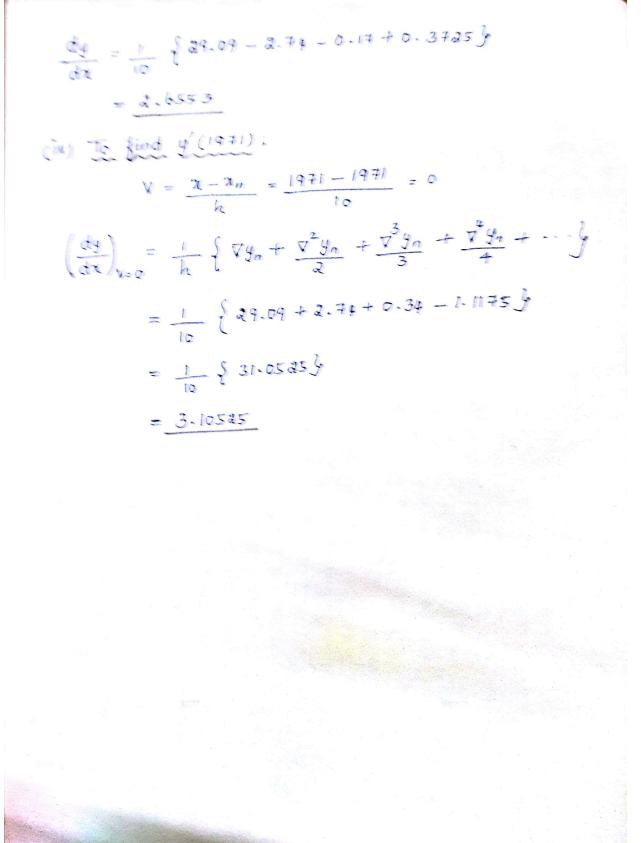
$$\frac{1}{10} \left[\frac{29.09}{2} - \frac{1}{2} (5.48) - \frac{1}{6} (1.02) - \frac{1}{2} (-4) \right]$$

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| (4) |
|---|
| (R) A jet fighter's position on an aircraft carrier's |
| upos timed during landing. |
| 1.2 1.3 1.4 1.3 1.4 |
| Y(m): 7.989 8.403 8.781 9.129 9.451 9.750 10.031 |
| where y is the distance from the end of the carrier. |
| Where y is the distance in acceleration (d ² y) at |
| Estimate velocity $\left(\frac{dy}{dt}\right)$ and acceleration $\left(\frac{d^2y}{dt^2}\right)$ at |
| (i) t = 1.1 (ii) t = 1.6 Using numerical differentiation. |
| |
| χ γ Δy $\Delta^2 y$ $\Delta^3 y$ $\Delta^4 y$ $\Delta^5 y$ $\Delta^6 y$ |
| |
| 1.0 7.989 0.414 -0.036 |
| 0.378 -0.002 |
| 0.348 0.004 0.002 |
| $\begin{array}{cccccccccccccccccccccccccccccccccccc$ |
| 1.4 9.451 -0.023 0.002 |
| 0.299 0.005 |
| 0.281 |
| 1.6 10.031 |
| (i) To find $t=1.1$: |
| Here $h = 0.1$ |
| $u = \frac{\chi - \chi_0}{h} = \frac{1 \cdot 1 - 1 \cdot 0}{0 \cdot 1} = 1$ |
| h 0,1 |
| $\left(\frac{dy}{dx}\right)_{t=1,1} = \left(\frac{dy}{dx}\right)_{u=1} = \frac{1}{h} \left[\Delta y_0 + \left(\frac{2u-1}{2}\right)\Delta^2 y_0 + \frac{u}{2}\right]$ |
| $\left(\frac{3u^2-6u+2}{6}\right)\Delta^3 y_0 + \left(\frac{4u^3-18u^2+22u-6}{24}\right)$ |
| $\left($ |







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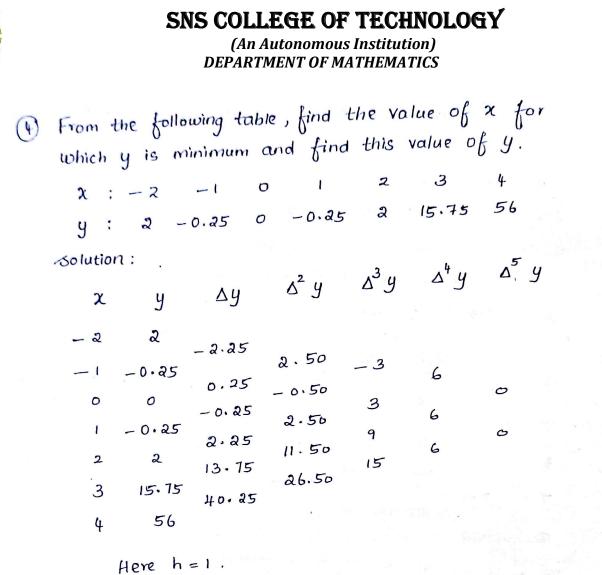
$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{0.1} \left[0.444 + \left(\frac{2-1}{2}\right) \left(-0.036\right) + \left(\frac{3-6-2}{6}\right) \left(0.006\right) \\ &+ \left(\frac{4-18+22-6}{24}\right) \left(-0.002\right) + \cdots \right] \\ &= \frac{3.9483}{12} \left[-\frac{3}{2} \left[-\frac{3}{2} y_0 + (u-1) \frac{3}{2} y_0 + \left(\frac{6u^2-18u+11}{12}\right) \frac{3}{2} y_0 + \cdots \right] \\ &= \frac{1}{(0.1)^2} \left[-\frac{3.036}{2} + (1-1) \left(0.006\right) + \left(\frac{6-18+11}{12}\right) \left(-0.002\right) \\ &= -3.5833 \\ (i) To find f = 1.6 : \\ &V = \frac{t-t_n}{h} = \frac{1.6-1.6}{0.1} = 0 \\ &\frac{dy}{dx} = \frac{1}{h} \left[\nabla y_n + \frac{1}{2} \nabla^2 y_n + \frac{1}{3} \nabla^3 y_n + \frac{\nabla^4 y_n}{4} + \cdots \right] \\ &= \frac{1}{0.1} \left[0.281 + \frac{1}{2} \left(-0.018 \right) + \frac{1}{3} \left(0.005 \right) + \frac{1}{4} \left(0.002 \right) + \frac{1}{5} \left(0.003 \right) + \frac{1}{6} \left(0.002 \right) \right] \\ &= \frac{2.751}{dx^2} \left[\frac{1}{h^2} \left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12} \nabla^4 y_n + \cdots \right] \\ &= \frac{1}{(0.1)^2} \left[-0.018 + 0.005 + \frac{11}{12} \left(0.002 \right) + \cdots \right] \\ &= \frac{-1.1167}{4} \end{aligned}$$



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| (3) Using the following data, find f'(5), f"(5) and (5) |
|--|
| the maximum value of $f(x)$ |
| 2:023479 |
| f(x): 4 26 58 112 466 922 |
| Solution: Since the values of x are not equally spaced, |
| use use Neutron's divided difference formula |
| χ f(x) Δ f(x) Δ^{2} f(x) Δ^{3} f(x) Δ^{4} f(x) |
| 0 + $3b-4 = 11$ $2a-11 = 7$ |
| $2 - 2b = \frac{-1}{2} - \frac{-1}{3-0} - \frac{11-7}{4-0} = 1 - 0$ |
| $\begin{array}{cccccccccccccccccccccccccccccccccccc$ |
| $\frac{112 - 38}{112 - 3} = 54$ |
| 466-112 = 118 228-118 = 22 |
| 7 466 7-4 400 -4 9aa-166 = aab 9-4 |
| 9 7da |
| By Newton's divided difference formula, |
| $f(x) = f(x) + (2 - \chi_0) \wedge f(\chi_0) + (\chi - \chi_0) (\chi - \chi_0) + \chi_0$ |
| +(2-20)(2-2) (2-2) 1 + (20) |
| = 4 + (x - 0) 11 + (x - 0)(x - 2)(7) + (x - 0) (x - 2)(x - 3)(1) |
| $f(x) = \chi^3 + 2\chi^2 + 3\chi + 4$ |
| $f'(x) = 3x^2 + 4x + 3$ |
| f''(x) = 6x + 4 |
| $f'(5) = 3(5^2) + 4(5) + 3 = 98$ |
| f''(5) = 6(5) + 4 = 3f |
| f(x) is maximum if $f'(x) = 0$ |
| i.e., 3x2+4x+3=0 |
| But the roots of this equation are imaginary. Hence there |
| is no extremum value. |



For minimum value of y, $\frac{dy}{dx} = 0$

$$i \cdot e \cdot , \quad \frac{1}{h} \left[\Delta y_0 + \left(\frac{2u-1}{2} \right) \Delta^2 y_0 + \cdots \right] = 0$$

$$i \cdot e \cdot , \quad \frac{1}{1} \left[-2 \cdot 25 + \left(\frac{2u-1}{2} \right) (2 \cdot 5) \right] = 0$$

$$u = 1 \cdot 4$$

$$\frac{\chi - \chi_0}{h} = 1 \cdot 4 \implies \chi = 1 \cdot 4h + \chi_0 = -0 \cdot 6$$

$$y (\chi = -0 \cdot 6) = y_0 + u \Delta y_0 + u (u-1) \Delta^2 y_0 + \cdots$$

$$= 2 + (1 \cdot 4) (-2 \cdot 25) + (1 \cdot 4) (0 \cdot 4) (2 \cdot 5) + (1 \cdot 4) [0 \cdot 4)$$

$$= -0 \cdot 1476 + (1 \cdot 4) (0 \cdot 4) (-2 \cdot 6) (-1 \cdot 6) \frac{6}{2}$$