

(An Autonomous Institution) **DEPARTMENT OF MATHEMATICS**

(6)

NUMERICAL INTEGRATION

TRAPEROIDAL RULL:

$$\begin{bmatrix}
x_n \\
j \\
+ f(x) dx = \frac{h}{a} \begin{bmatrix} (y_n + y_n) + a (y_1 + y_2 + \dots + y_{n-1}) \end{bmatrix}$$

$$= \frac{h}{a} \begin{bmatrix} (sum of the first and the last ordinates) \\
+ a (sum of the remaining ordinates) \end{bmatrix}$$
Where $h = \frac{b-a}{n}$
SIMPSON'S ONE-THIED RULE:

$$\begin{bmatrix} x_n \\
j \\
+ (y_n + y_n) + a (y_a + y_4 + \dots) + y_n \\
- y_n (y_1 + y_2 + \dots) \end{bmatrix}$$

Note

- 1. Even though y, has suffix even, it is the third ordinate (odd).
- 2. This formula can be applied only if the number of intervals is even. (or) the number of ordinates is odd.

SIMPSON'S THREE - EIGHTH RULE :

$$\begin{aligned} \chi_{0} + nh \\ \int f(\chi) d\chi &= \frac{3h}{8} \left[(y_{0} + y_{n}) + 3 (y_{1} + y_{2} + y_{4} + y_{5} \\ &+ \cdots + y_{n-1} \right) \\ &+ 2 (y_{1} + y_{6} + y_{9} + \cdots + y_{n}) \right] \end{aligned}$$

which is applicable only when n is a multiple of 3.

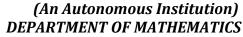


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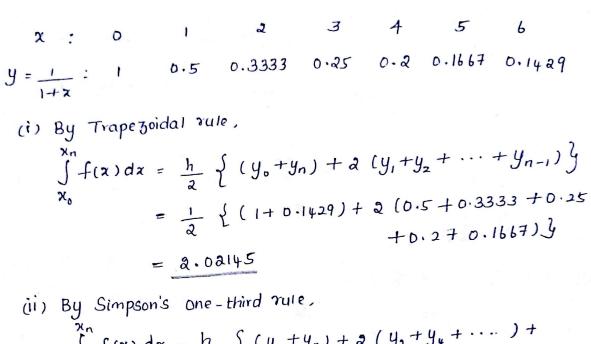


(7)I Evaluate $\int \frac{dx}{1+x^2}$, using Trapezoidal rule with h= 0.2. Hence Obtain an approximate value of TT. x : 0 0.2 0.4 0.6 0.8 1 $y = \frac{1}{1+x^2} : 1 0.9615 0.8621 0.7353 0.6098 0.5$ Solution : By Trapezoidal rule, $\int f(x) dx = \frac{h}{a} \left[(y_0 + y_n) + a(y_1 + y_2 + \dots + y_{n-1}) \right]$ $= \frac{0.2}{2} \left[(1+0.5) + 2 (0.9615 + 0.8621 + 0.7353 + 0.6098) \right]$ = 0.7837 To find the value of T: By actual integration, $\int \frac{dx}{1+x^2} = \left[\tan^2(x) \right]_0^1 = \tan^2(0) - \tan^2(0)$ = $\frac{\pi}{4}$ -0 = $\frac{\pi}{4}$ $\frac{\pi}{4} = 0.7837 \implies \pi = 3.1348 \text{ approximately}$ (a) Evaluate $I = \int \frac{1}{1+\chi} dx$ using (i) Trapezoidal rule (i) Simpson's rule. Also, Check up & by direct integration. Solution : Take the number of intervals n as b. $h = \frac{b-a}{a} = \frac{b-a}{a} = 1$









$$\int_{x_0} f(x) dx = \frac{h}{3} \left\{ (y_0 + y_1) + 2 (y_2 + y_4 + \cdots) \right]$$

= $\frac{1}{3} \left\{ (1 + 0 \cdot 14 \cdot 29) + 2 (0 \cdot 33 \cdot 33 + 0 \cdot 2) + (0 \cdot 5 + 0 \cdot 25 + 0 \cdot 166 \cdot 7) \right\}$
= $\frac{1}{3} \left\{ (1 + 0 \cdot 14 \cdot 29) + 2 (0 \cdot 33 \cdot 33 + 0 \cdot 2) + (0 \cdot 5 + 0 \cdot 25 + 0 \cdot 166 \cdot 7) \right\}$
= $\frac{1}{3} \left\{ (1 + 0 \cdot 14 \cdot 29) + 2 (0 \cdot 33 \cdot 33 + 0 \cdot 2) + (0 \cdot 5 + 0 \cdot 25 + 0 \cdot 166 \cdot 7) \right\}$

(iii) By Simpson's
$$3 - 8^{m}$$
 rule,
 $x_{0} + nh$

$$\int f(x) dx = \frac{3h}{8} \left\{ (y_{0} + y_{n}) + 3 (y_{1} + y_{2} + y_{4} + y_{5} + \cdots) + 2 (y_{3} + y_{6} + y_{9} + \cdots) \right\}$$

$$= \frac{3}{8} \begin{cases} (0-1+0.1429) + 3(0.5+0.3333+ 0.2+0.1667) \\ = 1.9661 + 2(0.25) \end{cases}$$

(iv) By actual integration.

$$\int_{0}^{6} \frac{dx}{1+x} = \left[\log\left(1+x\right)\right]_{0}^{6} = \log 7 - \log 1 = \log 7 - 0$$

$$= 1.9459$$



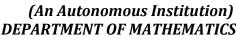
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(8) 3 The velocity v of a particle at distance 's' from a point On its linear path is given in the following table: S(m) : 0 2.5 5.0 7.5 10.0 12.5 15.0 17.5 20.0 v (m/sec): 16 19 21 22 20 9 11 13 17 Estimate the time taken by the posticle to traverse the distance of 20 meters, using Simpson's one third rule. $V = \frac{ds}{dt} \Rightarrow \frac{dt}{v} = \frac{ds}{v} \Rightarrow t = \int \frac{1}{v} ds$ Solution: Time required = $\int \frac{1}{v} ds$ 2.5 5.0 7.5 10.0 12.5 15.0 17.5 20.0 $S = \frac{1}{V}$: 0.0625 0.0526 0.0476 0.0455 0.05 0.0588 0.0769 0.0909 0.111 By simpson's one-third formula, $\int_{x_0}^{x_0} f(x) dx = \int_{y_0}^{y_0} \frac{1}{y_0} ds = \frac{h}{3} \left[(y_0 + y_0) + \lambda (y_1 + y_1 + \cdots) + 4 (y_1 + y_2 + \cdots) \right]$ $= \frac{2.5}{.3} \left[(0.0625 + 0.1111) + 2 (0.0476 + 0.05 + 0.05) \right]$ 0.0769) +4 (0.0526+0.0455+0.0588+0.0909)] = 1.2615 seconds

19MAT202 & Statistics and Numerical Methods







Compute $\int \log_e x \, dx$, Using simpson's $\frac{1}{3}$ and $\frac{3}{8}$ rule 3 Solution : Here $f(x) = \log_0 x$ $h = \frac{b-a}{n} = \frac{5 \cdot 2 - 4}{4} = \frac{1 \cdot 2}{6} = 0 \cdot 2$ 4 4.2 4.4 4.6 4.8 5.0 5.2 y = log = 1.386 1.435 1.482 1.526 1.569 1.609 1.649 By Simpson's 1/3rd rule, $I = \frac{h}{3} \left[(y_{0} + y_{n}) + a (y_{2} + y_{4} + \dots) + 4 (y_{1} + y_{3} + \dots) \right]$ $= \frac{0.2}{3} \left[(1.386 + 1.649) + 2 (1.482 + 1.569) + 4 \right]$ (1.435+1.526+1.609)] I = 1.828 By Simpson's 3/8" TUR, $I = \frac{3h}{8} \left[(9_0 + 9_n) + 3 (9_1 + 9_2 + 9_4 + \dots + 9_{n-1}) + \frac{3h}{8} \right]$ $2(y_3+y_1+y_9+\cdots)$ $= \frac{3 \times 0.2}{8} \left[(1.386 + 1.649) + 3 (1.435 + 1.482 + 1.569) + 1.699 \right]$ + 2 (1.526 + 44649)] I = 1.888(2) S is the Specific heat of a body at temperature O C. Find the total heat required to raise the temperature of the body of weight I gram from O'c to la'c, using the following data of values and simpson's 3/8" rule. 4 0 1.00564 1.00543 1.00435 1.00331 1.00233 1.00149 1.00078 S



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