TEST OF SIGNIFICANCE FOR LARGE SAMPLES:

If the Sample Size n is greater than 30, we usually take Sample as large Sample. For large Samples, we apply normal test assuming the Population is normal.

II. TESTS FOR MEANS. :

(A). 1. Testing of Significance for single mean if the population standard deviation is known.

i.e., $H_0 : \mu = \mu_0$, σ is known :

Suppose we want to test whether the given sample of size n has been drawn from a population with mean μ. We set up null hypothesis that there is no difference between $\bar{x}$ and μ where $\bar{x}$ is the sample mean. Then the test statistic will be,

$$\boxed{z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}}$$

Where σ is the S.D of the population.

(44)

(4) A sample of 100 students is taken from a large population. The mean height of the students in this sample is 160 cm. Can it be reasonably regarded that, in the population, the mean height is 165 cm, and the S.D is 10 cm.

Solution:

Given: $n = 100$

$\bar{x} = 160$ cm

$\mu = 165$ cm

$\sigma = 10$ cm.

Null hypothesis: H_0: The sample is drawn from a normal population with mean $\mu = 165$ cm and S.D $\sigma = 10$ cm.

i.e., $H_0 : \mu = 165$

Alternative hypothesis: $H_1 : \mu \neq 165$ (Two-tailed test)

Level of significance: At $\alpha = 5\%$, $Z_\alpha = 1.96$

Test-Statistic:

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$= \frac{160 - 165}{10/\sqrt{100}} = -5$$

$$\boxed{|Z| = 5}$$

Decision :

Since $|z| > 3$, H_0 is rejected.

$\therefore$ The sample cannot be drawn from the normal Population with mean $\mu = 165$ and SD $\sigma = 10$

(5) The mean breaking strength of the cables supplied by a manufacturer is 1800 with a S.D of 100. By a new technique in the manufacturing process, it is claimed that the breaking strength of the cable has increased. In order to test this claim, a sample of 50 cables is tested and it is found that the mean breaking strength is 1850. Can we support the claim at 5% LOS?

Solution :

Given : $\mu = 1800$

$\sigma = 100$

$\bar{x} = 1850$

$n = 50$

Null hypothesis : $H_0 : \mu = 1800$

Alternative hypothesis : $H_0 : \mu \neq 1800$ (Two-tailed test)

Level of Significance : At $\alpha = 5\%$,

$Z_\alpha = 1.96$

Test Statistic :

$$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

$$= \frac{1850 - 1800}{100 / \sqrt{50}}$$

$$\boxed{Z = 3.54}$$

Decision :

Since $Z > 3$, H_0 is rejected. ∴ The mean breaking strength of the cable has not increased.

(5) An automatic machine fills in tea in sealed tins with mean weight of tea 1 kg and S.D 1 gm. A random sample of 50 tins was examined and it was found that their mean weight was 999.50 gms. Is the machine working properly ?

Solution :

Given : $n = 50$

$\bar{x} = 999.50$ gms

$\mu = 1000$ gms

$\sigma = 1$ gm

Null hypothesis : H_0 : The machine is working properly.

i.e., $H_0 : \mu = 1000$ gms

Alternative hypothesis :

$H_1 : \mu \neq 1000$ gms.

Level of significance :

At $\alpha = 5\%$, $Z_\alpha = 1.96$

Test-Statistic :

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$= \frac{999.50 - 1000}{1/\sqrt{50}}$$

$$= -0.5 \times \sqrt{50}$$

$$= -3.54$$

$$\boxed{|Z| = 3.54}$$

Decision :

Since $|Z| > 3$, H_0 is Rejected.

Therefore the machine is not working properly.

(7) The mean life time of samples of 100 fluorescent light tubes produced by a Company is computed to be 1570 hours with a SD of 120 hours. The company claims that the average life of the tubes produced by the company is 1600 hours. Using the level of significance of 0.05 is the claim acceptable.

Solution :

Given :

$$n = 100$$

$$\bar{x} = 1570$$

$$\mu = 1600 \quad , \sigma = 120$$

Null hypothesis : H_0 : The claim is acceptable at 5 % LOS. i.e., $H_0 : \mu = 1600$

Alternative hypothesis : $H_1 : \mu \neq 1600$ (Two-tailed test)

Level of Significance : At $\alpha = 5\%$, $Z_\alpha = 1.96$

Test - Statistic :

$$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

$$= \frac{1570 - 1600}{120 / \sqrt{100}}$$

$$Z = -2.5$$

$$\boxed{|Z| = 2.5}$$

Decision :

Since $|Z| > Z_\alpha$, H_0 is rejected. Hence the Company's claim is not acceptable at 5 % LOS.