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Test 2 :

Test of significance for the difference between two Population means when population standard deviations are not known :

Let $\bar{x}_1$ and $\bar{x}_2$ are the means of two independent samples of sizes $n_1$ and $n_2$ from a normal population with mean $\mu_1$ and $\mu_2$ and standard deviation $s_1$ and $s_2$.

We want to test whether the mean $\mu_1$ and $\mu_2$ of the two populations are equal or not the test-Statistic is defined as, Under $H_0 : \mu_1 = \mu_2$

$$\boxed{t = \frac{\bar{x}_1 - \bar{x}_2}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}}$$

where $$S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}$$

with $v = n_1 + n_2 - 2$ degrees of freedom.

Note :

Suppose the samples sizes are equal. i.e., $n_1 = n_2 = n$. Then we have $n$ pair of values. Further we assume that the $n$ pairs are independent. Then the test-Statistic will be,

$$\boxed{t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2 + s_2^2}{n-1}}}}$$

with $v = n + n - 2 = 2n - 2$ degrees of freedom.

Problems :

(1) Two salesmen A and B are working in a certain district. From a sample survey conducted by the Head office, the following results were obtained. State whether there is any significant difference in the average sales between the two salesmen:

| | A | B |
|---|---|---|
| Number of Sales | 20 | 18 |
| Average Sales (in Rs.) | 170 | 205 |
| Standard Deviation (in Rs.) | 20 | 25 |

Solution :

Given : $n_1 = 20$ , $n_2 = 18$

$\bar{x}_1 = 170$ , $\bar{x}_2 = 205$

$s_1 = 20$ , $s_2 = 25$.

Null hypothesis : $H_0$ : There is no significant difference in the average sales of the two salesmen.

i.e., $H_0 : \mu_1 = \mu_2$

Alternative hypothesis : $H_1 : \mu_1 \neq \mu_2$

Test-Statistic :

$$t = \frac{\bar{x}_1 - \bar{x}_2}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

where $S^2 = \dfrac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}$

$$S^2 = \frac{20 \times 20^2 + 18 \times 25^2}{20 + 18 - 2} = \frac{19250}{36}$$

$$S^2 = 534.72$$

$$\boxed{S = 23.12}$$

$$t = \frac{170 - 205}{23.12\sqrt{\frac{1}{20} + \frac{1}{18}}}$$

$$= \frac{-35}{23.12 \times 0.325} = -4.65$$

$$\boxed{|t| = 4.65}$$

Table value :

At $\alpha = 1\%$ LOS, $V = n_1 + n_2 - 2 = 36$ d.o.f,

the table value of $t$ is given by,

$$\boxed{t_\alpha = 2.58}$$

Decision :

Since $|t| > t_\alpha$, $H_0$ is rejected.

Hence the two salesmen differ significantly with regard to their average sales.

(2) Two independent samples from normal populations with equal variance gave the following:

| Sample | Size | Mean | S.D |
|---|---|---|---|
| 1 | 16 | 23.4 | 2.5 |
| 2 | 12 | 24.9 | 2.8 |

Is the difference between the means significant ?

Solution :

Given : $n_1 = 16$ , $n_2 = 12$

$\bar{x}_1 = 23.4$ , $\bar{x}_2 = 24.9$

$s_1 = 2.5$ , $s_2 = 2.8$

Null hypothesis : $H_0$ : There is no significant difference between the means i.e., $H_0 : \mu_1 = \mu_2$

Alternative hypothesis : $H_1 : \mu_1 \neq \mu_2$ (Two-tailed test)

Test - Statistic :

$$t = \frac{\bar{x}_1 - \bar{x}_2}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

where $$S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}$$

$$S^2 = \frac{16 \times 2.5^2 + 12 \times 2.8^2}{16 + 12 - 2}$$

$S^2 = 7.465$

$\boxed{S = 2.732}$

$$t = \frac{23.4 - 24.9}{2.732\sqrt{\frac{1}{16}+\frac{1}{12}}}$$

$$= \frac{-1.5}{1.0433}$$

$\boxed{|t| = 1.4378}$

Table value :

At $\alpha = 5\%$ Los , $V = n_1 + n_2 - 2 = 16 + 12 - 2$

$V = 26$ d.o.f , the table value of $t$ is given by,

$\boxed{t_\alpha = 2.06}$

Decision :

Since $|t| < t_\alpha$ , $H_0$ is accepted.

$\therefore$ The difference is not significant.

(3) A group of 10 rats fed on diet A and another group of 8 rats ~~on~~ fed on diet B, recorded the following increase in weight (gms).

Diet A : 5, 6, 8, 1, 12, 4, 3, 9, 6, 10

Diet B : 2, 3, 6, 8, 10, 1, 2, 8

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Does it show superiority of diet A over diet B ?

Solution :

Given : $n_1 = 10$

$n_2 = 8$

Null hypothesis : $H_0$ : There is no significant difference in increase of weights . i.e., $H_0 : \mu_1 = \mu_2$

Alternative hypothesis : $H_1 : \mu_1 > \mu_2$ (Right tailed test)

Calculation of sample means and Sample S.D's :

$$\bar{x}_1 = \frac{\Sigma x_1}{n_1} = \frac{64}{10} = 6.4$$

$$\bar{x}_2 = \frac{\Sigma x_2}{n_2} = \frac{40}{8} = 5$$

$$s_1^2 = \frac{\Sigma x_1^2}{n_1} - \left(\frac{\Sigma x_1}{n_1}\right)^2 = \frac{512}{10} - (6.4)^2 = 10.24$$

$$s_2^2 = \frac{\Sigma x_2^2}{n_2} - \left(\frac{\Sigma x_2}{n_2}\right)^2 = \frac{282}{8} - (5)^2 = 10.25$$

Test - Statistic :

$$t = \frac{\bar{x}_1 - \bar{x}_2}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

where $S^2 = \dfrac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}$

$$S^2 = \frac{10 \times 10.24 + 8(10.25)}{10 + 8 - 2}$$

$$= \frac{184.4}{16}$$

$$S^2 = 11.525$$

$$\boxed{S = 3.395}$$

$$t = \frac{6.4 - 5}{3.395\sqrt{\frac{1}{10} + \frac{1}{8}}}$$

$$= \frac{1.4}{1.610}$$

$$\boxed{t = 0.8694}$$

Table value :

At $\alpha = 5\%$ LOS , $v = n_1 + n_2 - 2 = 16$ d.o.f, the table value of $t$ is given by,

$$\boxed{t_\alpha = 1.75}$$

Decision :

Since $t < t_\alpha$ , $H_0$ is accepted. Hence we Cannot Conclude that diet A is superior to diet B.