Test for Single proportion :   (17)

Problems in the formula $\boxed{Z = \dfrac{p - P}{\sqrt{PQ/n}}}$ where

P is the population proportion and p is the Sample

Proportion , $p = \dfrac{x}{n}$:

(1) A manufacturer claimed that atleast 95 % of the equipment which he supplied to a factory confirmed to specifications. An examination of a sample of 200 pieces of equipment revealed that 18 were faulty. Test his claim at a significance level of (i) 5 % (ii) 1 %.

Solution :

Given :   n = 200

Sample proportion of defectives $p = \dfrac{18}{200} = 0.09$

Population proportion of defectives $P = \dfrac{5}{100} = 0.05$

$Q = 1 - P = 1 - 0.05 = 0.95$

Null hypothesis : $H_0 : P = 0.05$

Alternative hypothesis : $H_1 : P > 0.05$ (Right - tailed test)

Level of Significance:

(i) The value of Z_α at 5% LOS for right-tail test is given by,

$$Z_\alpha = 1.645$$

(ii) The value of Z_α at 1% LOS for right-tail test is given by,

$$Z_\alpha = 2.33$$

Test-Statistic:

$$Z = \frac{p - P}{\sqrt{\dfrac{PQ}{n}}}$$

$$= \frac{0.09 - 0.05}{\sqrt{\dfrac{0.05 \times 0.95}{200}}}$$

$$= \frac{0.04 \sqrt{200}}{\sqrt{0.05 \times 0.95}}$$

$$= \frac{0.5657}{0.2179}$$

$$\boxed{Z = 2.596}$$

Decision :

(i) At 5% LOS :

Since $Z > Z_\alpha$, we reject the null hypothesis

∴ The manufacturer's claim is not acceptable

(ii) At 1% LOS :

Since $Z > Z_\alpha$, we reject the null hypothesis

∴ The manufacturer's claim is not acceptable.

② In a sample of 500 people in Kerala, 280 are tea drinkers and the rest are coffee drinkers. Can we assume that both coffee and tea are equally popular in the state at 5% LOS ?

Solution :

Given : $n = 500$

Proportion of tea drinkers in the state $\} p = \dfrac{280}{500} = 0.56$

Proportion of tea drinkers in the population $\} P = \dfrac{1}{2}$

$Q = 1 - P = \dfrac{1}{2}$

Null hypothesis : $H_0 : P = \dfrac{1}{2}$

Alternative hypothesis : $H_1 : P \neq \dfrac{1}{2}$ (Two-tailed test)

Level of Significance:

At $\alpha = 5\%$ LOS, the value of Z_α for two-tailed test is given by,

$$Z_\alpha = 1.96$$

Test-Statistic:

$$Z = \frac{P - p}{\sqrt{\frac{PQ}{n}}}$$

$$= \frac{0.56 - 0.5}{\sqrt{\frac{0.5 \times 0.5}{500}}}$$

$$= \frac{0.06}{0.02236}$$

$$\boxed{Z = 2.68}$$

Decision:

At Since $|Z| > Z_\alpha$, we reject our null hypothesis. Therefore Coffee and tea are not equally popular in the state.