



SNS COLLEGE OF TECHNOLOGY
An Autonomous Institution
Coimbatore-35



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Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECB212 – DIGITAL SIGNAL PROCESSING

II YEAR/ IV SEMESTER

UNIT 5 – DSP APPLICATIONS

TOPIC – MULTIRATE DSP – DOWNSAMPLING (OR) DECIMATION



INTRODUCTION



- The processing of a discrete time signal at different sampling rates in different parts of a system is called **multirate DSP**
- The discrete time systems that employ sampling rate conversion while processing the discrete time signals are called **multirate DSP Systems**
- The process of converting a signal from one sampling rate to another sampling rate is called **sampling rate conversion**
- There are two general methods for sampling rate conversion. In the first method, the discrete signal is converted to analog signal using a D/A converter and the analog signal is resampled at the desired rate using an A/D converter



INTRODUCTION



- The advantage in this method is that the new sampling rate need not have any relation to the old sampling rate. The disadvantage of this method is distortion during D/A and A/D process
- In the second method, the sampling rate conversion is entirely performed in the digital domain, using Interpolators and Decimators
- The advantage in rate conversion in the digital domain is that the signal distortion in D/A and A/D process are avoided or eliminated
- There are two ways for sampling rate conversion in the digital domain. They are **1. Downsampling or Decimation & 2. Upsampling or Interpolation**



INTRODUCTION



- **Downsampling or decimation** is the process of reducing the sampling rate by an integer factor D
- **Upsampling or interpolation** is the process of increasing the sampling rate by an integer factor I
- **Advantages of Multirate Processing:**
 1. The reduction in number of computations
 2. The reduction in memory requirement (or storage) for filter coefficients and intermediate results
 3. The reduction in the order of the system
 4. The finite word length effects are reduced



APPLICATIONS OF MULTIRATE DSP SYSTEMS



- Sub-band coding of speech signals and image compression
- QMF(Quadrature Mirror Filters) for realizing alias-free LTI multirate systems
- Narrowband FIR and IIR filters for various applications
- Digital transmultiplexers for converting TDM signals to FDM signals and vice versa
- Oversampling A/D and D/A converters for high quality digital audio systems and data loggers (or digital storage systems)
- In digital audio systems the sampling rates of broadcasted signal, CD (Compact Disc), MPEG (Motion Picture Expert Group) Standard CD, etc., are different. Hence to access signals from all these devices, sampling rate converters are needed in digital audio systems



DOWN SAMPLING (OR) DECIMATION



- **Downsampling (or decimation)** is the process of reducing the samples of the discrete time signal
- Let, $x(n)$ = Discrete time signal
- D = Sampling rate reduction factor (and D is an integer)
- Now, $x(Dn)$ = Downsampled version of $x(n)$
- The device which performs the process of downsampling is called a downsampler (or decimator)
- The downsampler can be represented as





EXAMPLE

Consider the discrete time signal,

$$x(n) = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

Determine the downsampled version of the signals for the sampling rate reduction factors.

a) $D = 2$ b) $D = 3$ c) $D = 4$.

Solution

Given that,

$$x(n) = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$



$$\therefore \text{When } n = 0, x(n) = x(0) = 1$$

$$\text{When } n = 1, x(n) = x(1) = 2$$

$$\text{When } n = 2, x(n) = x(2) = 3$$

$$\text{When } n = 3, x(n) = x(3) = 4$$

$$\text{When } n = 4, x(n) = x(4) = 5$$

$$\text{When } n = 5, x(n) = x(5) = 6$$

$$\text{When } n = 6, x(n) = x(6) = 7$$

$$\text{When } n = 7, x(n) = x(7) = 8$$

$$\text{When } n = 8, x(n) = x(8) = 9$$

$$\text{When } n = 9, x(n) = x(9) = 10$$

$$\text{When } n = 10, x(n) = x(10) = 11$$

$$\text{When } n = 11, x(n) = x(11) = 12$$



EXAMPLE



a) Sampling rate reduction factor, $D = 2$.

Now, $x(Dn) = x(2n)$ = Discrete time signal decimated by reduction factor 2.

Let, $x(2n) = x_{D_2}(n)$

$$\therefore \text{When } n = 0, x_{D_2}(n) = x_{D_2}(0) = x(2 \times 0) = x(0) = 1$$

$$\text{When } n = 1, x_{D_2}(n) = x_{D_2}(1) = x(2 \times 1) = x(2) = 3$$

$$\text{When } n = 2, x_{D_2}(n) = x_{D_2}(2) = x(2 \times 2) = x(4) = 5$$

$$\therefore x(2n) = x_{D_2}(n) = \{1, 3, 5, 7, 9, 11\}$$



$$\text{When } n = 3, x_{D_2}(n) = x_{D_2}(3) = x(2 \times 3) = x(6) = 7$$

$$\text{When } n = 4, x_{D_2}(n) = x_{D_2}(4) = x(2 \times 4) = x(8) = 9$$

$$\text{When } n = 5, x_{D_2}(n) = x_{D_2}(5) = x(2 \times 5) = x(10) = 11$$



EXAMPLE



b) Sampling rate reduction factor, $D = 3$.

Now, $x(Dn) = x(3n) =$ Discrete time signal decimated by reduction factor 3.

Let, $x(3n) = x_{D3}(n)$

∴ When $n = 0$, $x_{D3}(n) = x_{D3}(0) = x(3 \times 0) = x(0) = 1$

When $n = 1$, $x_{D3}(n) = x_{D3}(1) = x(3 \times 1) = x(3) = 4$

When $n = 2$, $x_{D3}(n) = x_{D3}(2) = x(3 \times 2) = x(6) = 7$

When $n = 3$, $x_{D3}(n) = x_{D3}(3) = x(3 \times 3) = x(9) = 10$

∴ $x(3n) = x_{D3}(n) = \{1, 4, 7, 10\}$
↑



EXAMPLE

c) Sampling rate reduction factor, $D = 4$.

Now, $x(Dn) = x(4n)$ = Discrete time signal decimated by reduction factor 4.

$$\text{Let, } x(4n) = x_{D4}(n)$$

$$\therefore \text{ When } n = 0, x_{D4}(n) = x_{D4}(0) = x(4 \times 0) = x(0) = 1$$

$$\text{When } n = 1, x_{D4}(n) = x_{D4}(1) = x(4 \times 1) = x(4) = 5$$

$$\text{When } n = 2, x_{D4}(n) = x_{D4}(2) = x(4 \times 2) = x(8) = 9$$

$$\therefore x(4n) = x_{D4}(n) = \{1, 5, 9\}$$

↑



EXAMPLE

Consider the discrete time signal shown in fig 1.

Sketch the downsampled version of the signals for the sampling rate reduction factors, **a) $D = 2$** **b) $D = 3$** .

Solution

From fig 1, we can write the samples of the given sequence as shown below.

$$x(n) = \{1, -1, 1, -1, 2, -2, 2, -2, 3, -3, 3, -3\}$$

↑

$$\therefore \text{When } n = 0, x(n) = x(0) = 1$$

$$\text{When } n = 1, x(n) = x(1) = -1$$

$$\text{When } n = 2, x(n) = x(2) = 1$$

$$\text{When } n = 3, x(n) = x(3) = -1$$

$$\text{When } n = 4, x(n) = x(4) = 2$$

$$\text{When } n = 5, x(n) = x(5) = -2$$

$$\text{When } n = 6, x(n) = x(6) = 2$$

$$\text{When } n = 7, x(n) = x(7) = -2$$

$$\text{When } n = 8, x(n) = x(8) = 3$$

$$\text{When } n = 9, x(n) = x(9) = -3$$

$$\text{When } n = 10, x(n) = x(10) = 3$$

$$\text{When } n = 11, x(n) = x(11) = -3$$

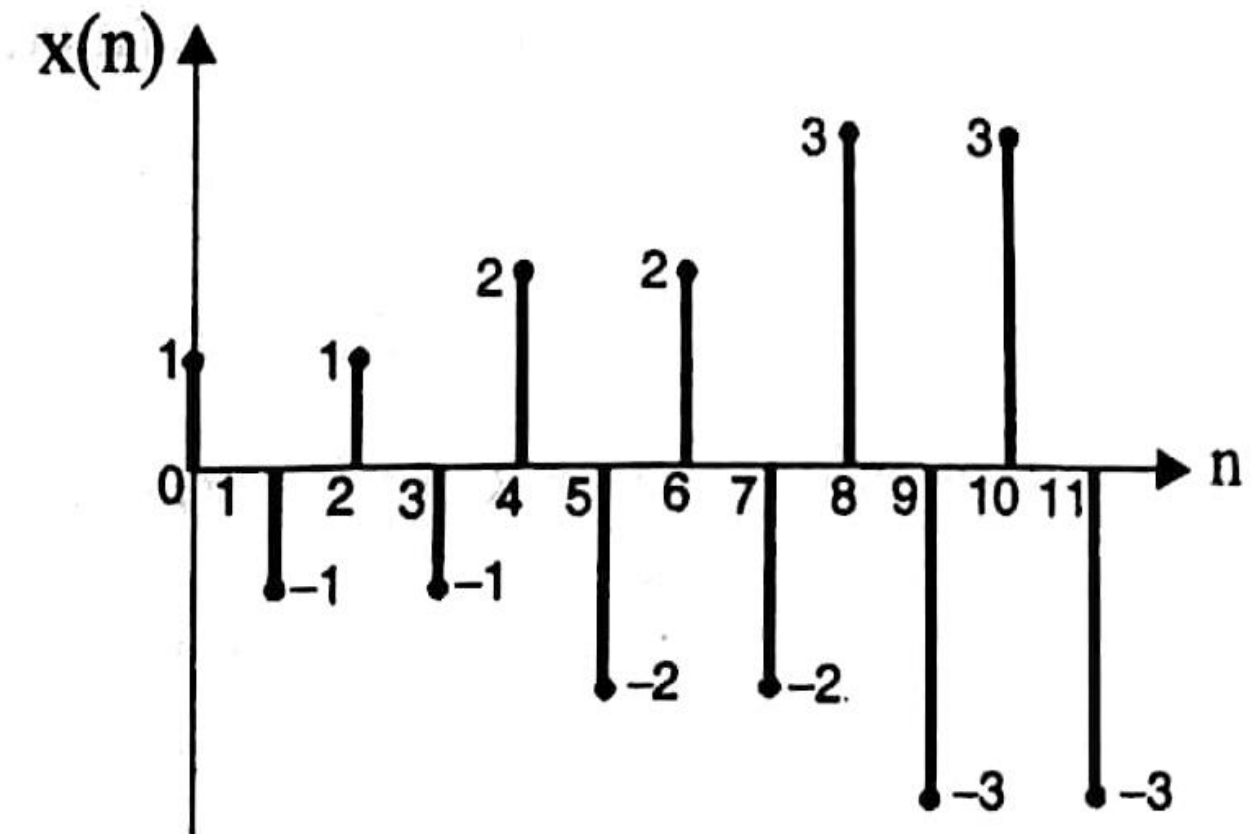


Fig 1.



EXAMPLE



a) Sampling rate reduction factor, $D = 2$.

Let, $x_{D_2}(n)$ = Discrete time signal decimated by reduction factor 2.

Now, $x_{D_2}(n) = x(Dn) = x(2n)$

$$\therefore \text{When } n = 0, x_{D_2}(n) = x_{D_2}(0) = x(2 \times 0) = x(0) = 1$$

$$\text{When } n = 1, x_{D_2}(n) = x_{D_2}(1) = x(2 \times 1) = x(2) = 1$$

$$\text{When } n = 2, x_{D_2}(n) = x_{D_2}(2) = x(2 \times 2) = x(4) = 2$$

$$\text{When } n = 3, x_{D_2}(n) = x_{D_2}(3) = x(2 \times 3) = x(6) = 2$$

$$\text{When } n = 4, x_{D_2}(n) = x_{D_2}(4) = x(2 \times 4) = x(8) = 3$$

$$\text{When } n = 5, x_{D_2}(n) = x_{D_2}(5) = x(2 \times 5) = x(10) = 3$$

$$\therefore x(2n) = x_{D_2}(n) = \{1, 1, 2, 2, 3, 3\} \quad \dots(1)$$





EXAMPLE

b) Sampling rate reduction factor, $D = 3$.

Let, $x_{D3}(n)$ = Discrete time signal decimated by reduction factor 3.

$$\text{Now, } x_{D3}(n) = x(Dn) = x(3n)$$

$$\therefore \text{ When } n = 0, x_{D3}(n) = x_{D3}(0) = x(3 \times 0) = x(0) = 1$$

$$\text{When } n = 1, x_{D3}(n) = x_{D3}(1) = x(3 \times 1) = x(3) = -1$$

$$\text{When } n = 2, x_{D3}(n) = x_{D3}(2) = x(3 \times 2) = x(6) = 2$$

$$\text{When } n = 3, x_{D3}(n) = x_{D3}(3) = x(3 \times 3) = x(9) = -3$$

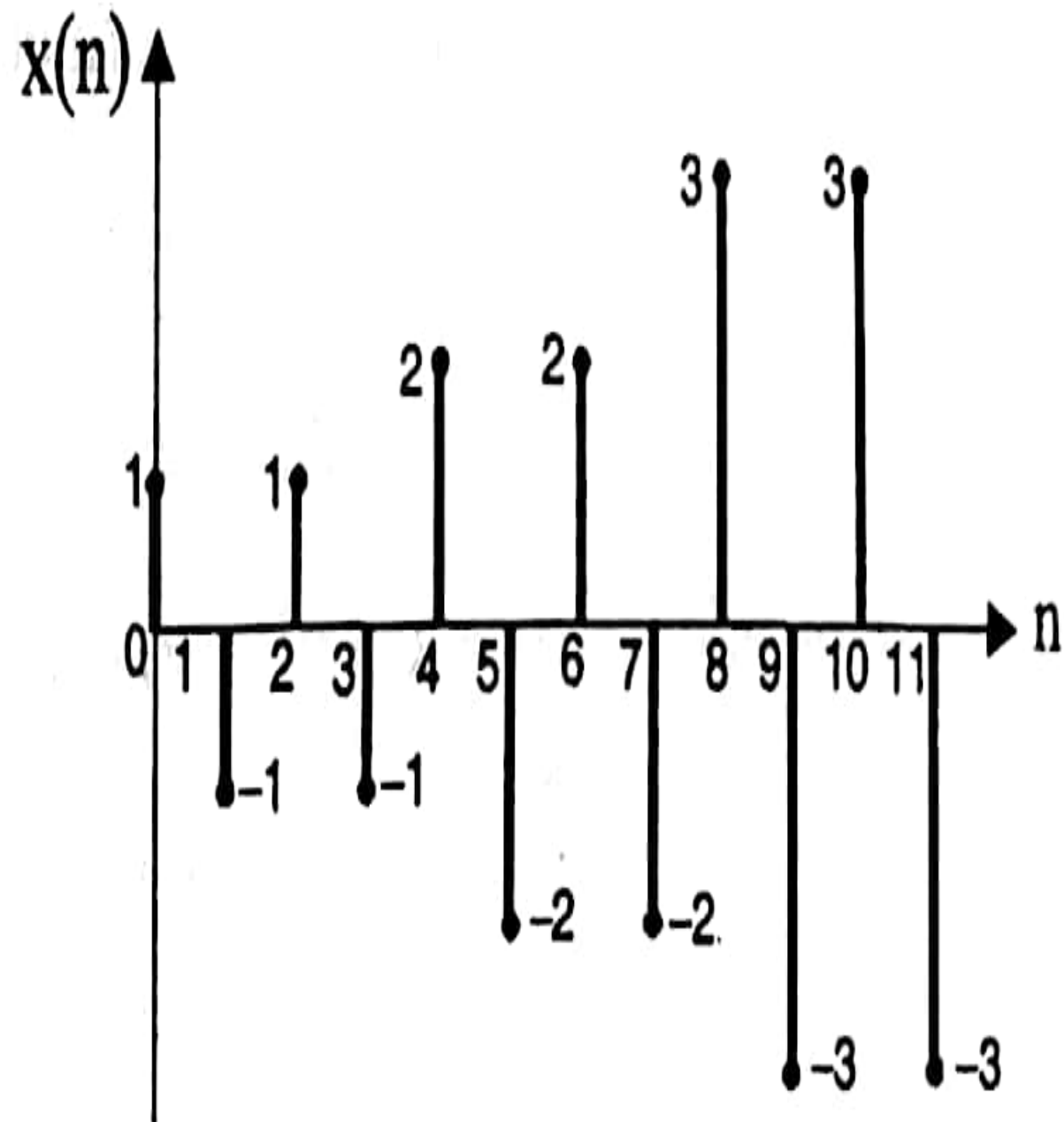
$$\therefore x(3n) = x_{D3}(n) = \{1, -1, 2, -3\} \quad \dots(2)$$

↑

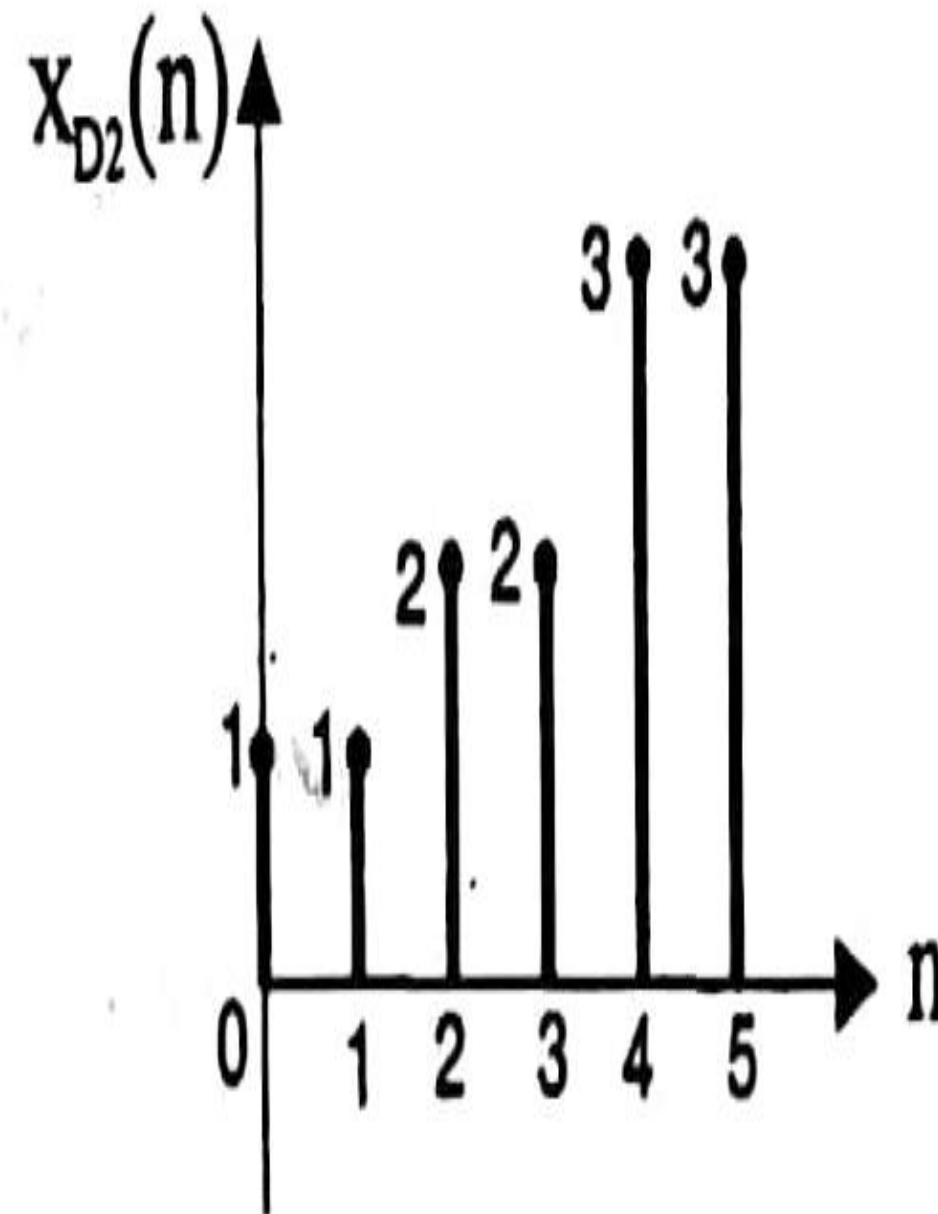


EXAMPLE

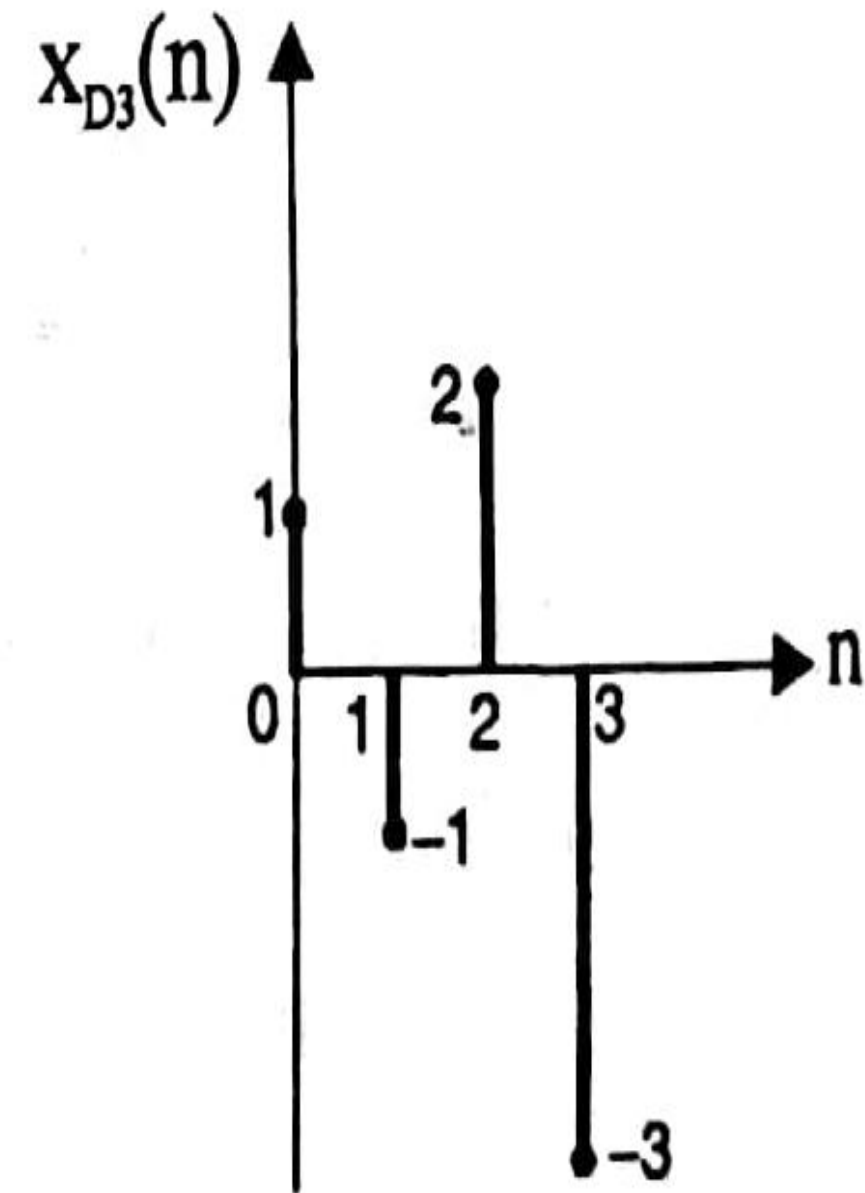
Samples of sequence



$x(n)$ decimated by 2



$x(n)$ decimated by 3





ASSESSMENT



1. Define multirate DSP.
2. The discrete time systems that employ sampling rate conversion while processing the discrete time signals are called -----
3. What is meant by sampling rate conversion.
4. List the two ways for sampling rate conversion in the digital domain
5. What is meant by downsampling and upsampling?
6. What are the advantages of multirate Processing?
7. Summarize the applications of multirate DSP Systems.



THANK YOU