SNS COLLEGE OF TECHNOLOGY

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Coimbatore-35

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DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECB212 – DIGITAL SIGNAL PROCESSING

II YEAR/ IV SEMESTER

UNIT 5 – DSP APPLICATIONS

TOPIC – MULTIRATE DSP – UPSAMPLING (INTERPOLATION)

INTRODUCTION

- The processing of a discrete time signal at different sampling rates in different parts of a system is called **multirate DSP**
- The discrete time systems that employ sampling rate conversion while processing the discrete time signals are called **multirate DSP Systems**
- The process of converting a signal from one sampling rate to another sampling rate is called **sampling rate conversion**
- There are two general methods for sampling rate conversion. In the first method, the discrete signal is converted to analog signal using a D/A converter and the analog signal is resampled at the desired rate using an A/D converter

INTRODUCTION

- The advantage in this method is that the new sampling rate need not have any relation to the old sampling rate. The disadvantage of this method is distortion during D/A and A/D process
- In the second method, the sampling rate conversion is entirely performed in the digital domain, using Interpolators and Decimators
- The advantage in rate conversion in the digital domain is that the signal distortion in D/A and A/D process are avoided or eliminated
- There are two ways for sampling rate conversion in the digital domain. They are **1. Downsampling or Decimation & 2. Upsampling or Interpolation**

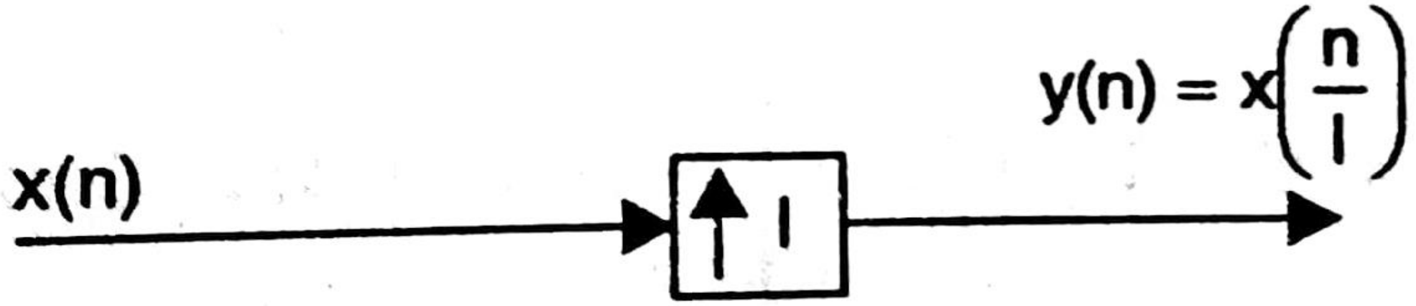
DECIMATION & INTERPOLATION

- **Downsampling or decimation** is the process of reducing the sampling rate by an integer factor D
- **Upsampling or interpolation** is the process of increasing the sampling rate by an integer factor I
- **Advantages of Multirate Processing:**

1. The reduction in number of computations
2. The reduction in memory requirement (or storage) for filter coefficients and intermediate results
3. The reduction in the order of the system
4. The finite word length effects are reduced

UPSAMPLING (OR) INTERPOLATION

- **Upsampling (or Interpolation)** is the process of increasing the samples of the discrete time signal
- Let, x(n) = Discrete time signal
- I = Sampling rate multiplication factor (and I is an integer)
- Now, x(n/I) = Upsampled version of x(n)
- The device which performs the process of upsampling is called a upsampler (or interpolator)
- The upsampler can be represented as



EXAMPLE

Consider the discrete time signal,

$$x(n) = \{1, 2, 3, 4\}$$

Determine the upsampled version of the signals for the sampling rate multiplication factor,
a) $I = 2$ b) $I = 3$ c) $I = 4$

Solution

Given that,

$$x(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$$

$\therefore$ When $n = 0$, $x(n) = x(0) = 1$

When $n = 1$, $x(n) = x(1) = 2$

When $n = 2$, $x(n) = x(2) = 3$

When $n = 3$, $x(n) = x(3) = 4$

EXAMPLE

a) Sampling rate multiplication factor, I = 2.

Now, $x\left(\frac{n}{I}\right) = x\left(\frac{n}{2}\right)$ = Discrete time signal interpolated by multiplication factor 2.

Let, $x\left(\frac{n}{2}\right) = x_{I2}(n)$

$\therefore$ When $n = 0$, $x_{I2}(n) = x_{I2}(0) = x\left(\frac{0}{2}\right) = x(0) = 1$

When $n = 1$, $x_{I2}(n) = x_{I2}(1) = x\left(\frac{1}{2}\right) = x(0.5) = 0$

When $n = 2$, $x_{I2}(n) = x_{I2}(2) = x\left(\frac{2}{2}\right) = x(1) = 2$

When $n = 3$, $x_{I2}(n) = x_{I2}(3) = x\left(\frac{3}{2}\right) = x(1.5) = 0$

When $n = 4$, $x_{I2}(n) = x_{I2}(4) = x\left(\frac{4}{2}\right) = x(2) = 3$

When $n = 5$, $x_{I2}(n) = x_{I2}(5) = x\left(\frac{5}{2}\right) = x(2.5) = 0$

When $n = 6$, $x_{I2}(n) = x_{I2}(6) = x\left(\frac{6}{2}\right) = x(3) = 4$

When $n = 7$, $x_{I2}(n) = x_{I2}(7) = x\left(\frac{7}{2}\right) = x(3.5) = 0$

$$\therefore x\left(\frac{n}{2}\right) = x_{I2}(n) = \{\underset{\uparrow}{1}, 0, 2, 0, 3, 0, 4, 0\}$$

EXAMPLE

b) Sampling rate multiplication factor, I = 3.

Now, $x\left(\frac{n}{I}\right) = x\left(\frac{n}{3}\right)$ = Discrete time signal interpolated by multiplication factor 3.

Let, $x\left(\frac{n}{3}\right) = x_{I3}(n)$

$\therefore$ When $n = 0,\ x_{I3}(n) = x_{I3}(0) = x\left(\frac{0}{3}\right) = x(0) = 1$

When $n = 1,\ x_{I3}(n) = x_{I3}(1) = x\left(\frac{1}{3}\right) = x(0.3) = 0$

When $n = 2,\ x_{I3}(n) = x_{I3}(2) = x\left(\frac{2}{3}\right) = x(0.7) = 0$

When $n = 3,\ x_{I3}(n) = x_{I3}(3) = x\left(\frac{3}{3}\right) = x(1) = 2$

When $n = 4,\ x_{I3}(n) = x_{I3}(4) = x\left(\frac{4}{3}\right) = x(1.3) = 0$

When $n = 5,\ x_{I3}(n) = x_{I3}(5) = x\left(\frac{5}{3}\right) = x(1.7) = 0$

When $n = 6,\ x_{I3}(n) = x_{I3}(6) = x\left(\frac{6}{3}\right) = x(2) = 3$

When $n = 7,\ x_{I3}(n) = x_{I3}(7) = x\left(\frac{7}{3}\right) = x(2.3) = 0$

When $n = 8,\ x_{I3}(n) = x_{I3}(8) = x\left(\frac{8}{3}\right) = x(2.7) = 0$

When $n = 9,\ x_{I3}(n) = x_{I3}(9) = x\left(\frac{9}{3}\right) = x(3) = 4$

When $n = 10,\ x_{I3}(n) = x_{I3}(10) = x\left(\frac{10}{3}\right) = x(3.3) = 0$

When $n = 11,\ x_{I3}(n) = x_{I3}(11) = x\left(\frac{11}{3}\right) = x(3.7) = 0$

$$\therefore x\left(\frac{n}{3}\right) = x_{I3}(n) = \{\underset{\uparrow}{1},\ 0,\ 0,\ 2,\ 0,\ 0,\ 3,\ 0,\ 0,\ 4,\ 0,\ 0\}$$

EXAMPLE

c) Sampling rate multiplication factor, I = 4.

Now, $x\left(\frac{n}{I}\right) = x\left(\frac{n}{4}\right)$ = Discrete time signal interpolated by multiplication factor 4.

Let, $x\left(\frac{n}{4}\right) = x_{I4}(n)$

$\therefore$ When $n=0$, $x_{I4}(n) = x_{I4}(0) = x\left(\frac{0}{4}\right) = x(0) = 1$

When $n=1$, $x_{I4}(n) = x_{I4}(1) = x\left(\frac{1}{4}\right) = x(0.25) = 0$

When $n=2$, $x_{I4}(n) = x_{I4}(2) = x\left(\frac{2}{4}\right) = x(0.5) = 0$

When $n=3$, $x_{I4}(n) = x_{I4}(3) = x\left(\frac{3}{4}\right) = x(0.75) = 0$

When $n=4$, $x_{I4}(n) = x_{I4}(4) = x\left(\frac{4}{4}\right) = x(1) = 2$

When $n=5$, $x_{I4}(n) = x_{I4}(5) = x\left(\frac{5}{4}\right) = x(1.25) = 0$

When $n=6$, $x_{I4}(n) = x_{I4}(6) = x\left(\frac{6}{4}\right) = x(1.5) = 0$

When $n=7$, $x_{I4}(n) = x_{I4}(7) = x\left(\frac{7}{4}\right) = x(1.75) = 0$

When $n=8$, $x_{I4}(n) = x_{I4}(8) = x\left(\frac{8}{4}\right) = x(2) = 3$

When $n=9$, $x_{I4}(n) = x_{I4}(9) = x\left(\frac{9}{4}\right) = x(2.25) = 0$

When $n=10$, $x_{I4}(n) = x_{I4}(10) = x\left(\frac{10}{4}\right) = x(2.5) = 0$

When $n=11$, $x_{I4}(n) = x_{I4}(11) = x\left(\frac{11}{4}\right) = x(2.75) = 0$

When $n=12$, $x_{I4}(n) = x_{I4}(12) = x\left(\frac{12}{4}\right) = x(3) = 4$

When $n=13$, $x_{I4}(n) = x_{I4}(13) = x\left(\frac{13}{4}\right) = x(3.25) = 0$

When $n=14$, $x_{I4}(n) = x_{I4}(14) = x\left(\frac{14}{4}\right) = x(3.5) = 0$

When $n=15$, $x_{I4}(n) = x_{I4}(15) = x\left(\frac{15}{4}\right) = x(3.75) = 0$

$$\therefore x\left(\frac{n}{4}\right) = x_{I4}(n) = \{\underset{\uparrow}{1}, 0, 0, 0, 2, 0, 0, 0, 3, 0, 0, 0, 4, 0, 0, 0\}$$

EXAMPLE

Consider the discrete time signal shown in fig 1. Sketch the upsampled version of the signals for the sampling rate multiplication factor, **a)** $I = 2$ **b)** $I = 3$.

Solution

From fig 1, we can write the samples of given sequence as shown below.

$$x(n) = \{\underset{\uparrow}{1}, -1, 2, -2\}$$

$\therefore$ When $n = 0$, $x(n) = x(0) = 1$

When $n = 1$, $x(n) = x(1) = -1$

When $n = 2$, $x(n) = x(2) = 2$

When $n = 3$, $x(n) = x(3) = -2$

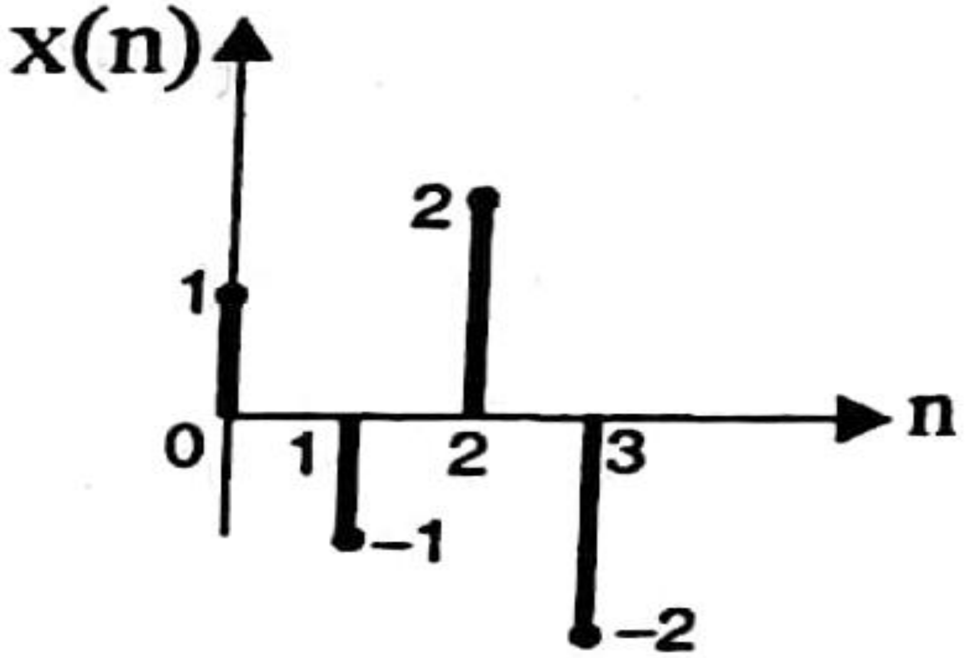


Fig 1.

EXAMPLE

a) Sampling rate multiplication factor, I = 2.

Now, $x\left(\frac{n}{I}\right) = x\left(\frac{n}{2}\right)$ = Discrete time signal interpolated by multiplication factor 2.

Let, $x\left(\frac{n}{2}\right) = x_{I2}(n)$

$\therefore$ When $n = 0$, $x_{I2}(n) = x_{I2}(0) = x\left(\frac{0}{2}\right) = x(0) = 1$

When $n = 1$, $x_{I2}(n) = x_{I2}(1) = x\left(\frac{1}{2}\right) = x(0.5) = 0$

When $n = 2$, $x_{I2}(n) = x_{I2}(2) = x\left(\frac{2}{2}\right) = x(1) = -1$

When $n = 3$, $x_{I2}(n) = x_{I2}(3) = x\left(\frac{3}{2}\right) = x(1.5) = 0$

When $n = 4$, $x_{I2}(n) = x_{I2}(4) = x\left(\frac{4}{2}\right) = x(2) = 2$

When $n = 5$, $x_{I2}(n) = x_{I2}(5) = x\left(\frac{5}{2}\right) = x(2.5) = 0$

When $n = 6$, $x_{I2}(n) = x_{I2}(6) = x\left(\frac{6}{2}\right) = x(3) = -2$

When $n = 7$, $x_{I2}(n) = x_{I2}(7) = x\left(\frac{7}{2}\right) = x(3.5) = 0$

$$\therefore x\left(\frac{n}{2}\right) = x_{I2}(n) = \{\underset{\uparrow}{1},\ 0,\ -1,\ 0,\ 2,\ 0,\ -2,\ 0\} \qquad .....(1)$$

EXAMPLE

b) Sampling rate multiplication factor, I = 3.

Now, $x\left(\frac{n}{I}\right) = x\left(\frac{n}{3}\right)$ = Discrete time signal interpolated by multiplication factor 3.

Let, $x\left(\frac{n}{2}\right) = x_{I3}(n)$

$\therefore$ When $n = 0$, $x_{I3}(n) = x_{I3}(0) = x\left(\frac{0}{3}\right) = x(0) = 1$

When $n = 1$, $x_{I3}(n) = x_{I3}(1) = x\left(\frac{1}{3}\right) = x(0.3) = 0$

When $n = 2$, $x_{I3}(n) = x_{I3}(2) = x\left(\frac{2}{3}\right) = x(0.7) = 0$

When $n = 3$, $x_{I3}(n) = x_{I3}(3) = x\left(\frac{3}{3}\right) = x(1) = -1$

When $n = 4$, $x_{I3}(n) = x_{I3}(4) = x\left(\frac{4}{3}\right) = x(1.3) = 0$

When $n = 5$, $x_{I3}(n) = x_{I3}(5) = x\left(\frac{5}{3}\right) = x(1.7) = 0$

When $n = 6$, $x_{I3}(n) = x_{I3}(6) = x\left(\frac{6}{3}\right) = x(2) = 2$

When $n = 7$, $x_{I3}(n) = x_{I3}(7) = x\left(\frac{7}{3}\right) = x(2.3) = 0$

When $n = 8$, $x_{I3}(n) = x_{I3}(8) = x\left(\frac{8}{3}\right) = x(2.7) = 0$

When $n = 9$, $x_{I3}(n) = x_{I3}(9) = x\left(\frac{9}{3}\right) = x(3) = -2$

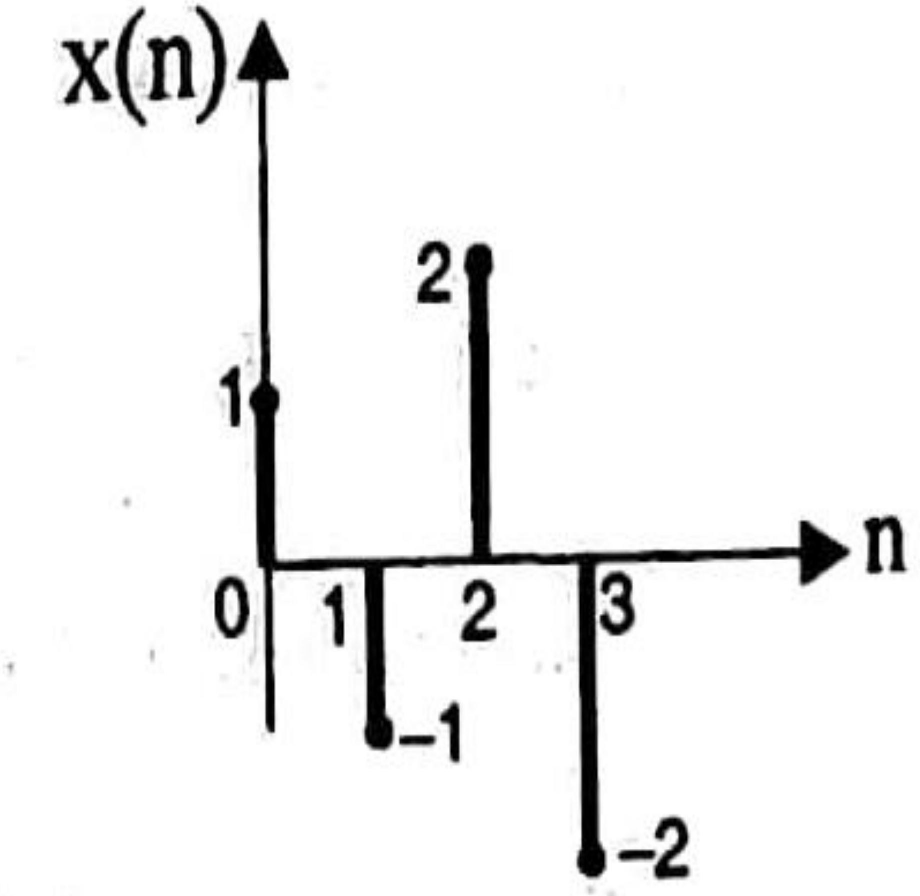
When $n = 10$, $x_{I3}(n) = x_{I3}(10) = x\left(\frac{10}{3}\right) = x(3.3) = 0$

When $n = 11$, $x_{I3}(n) = x_{I3}(11) = x\left(\frac{11}{3}\right) = x(3.7) = 0$

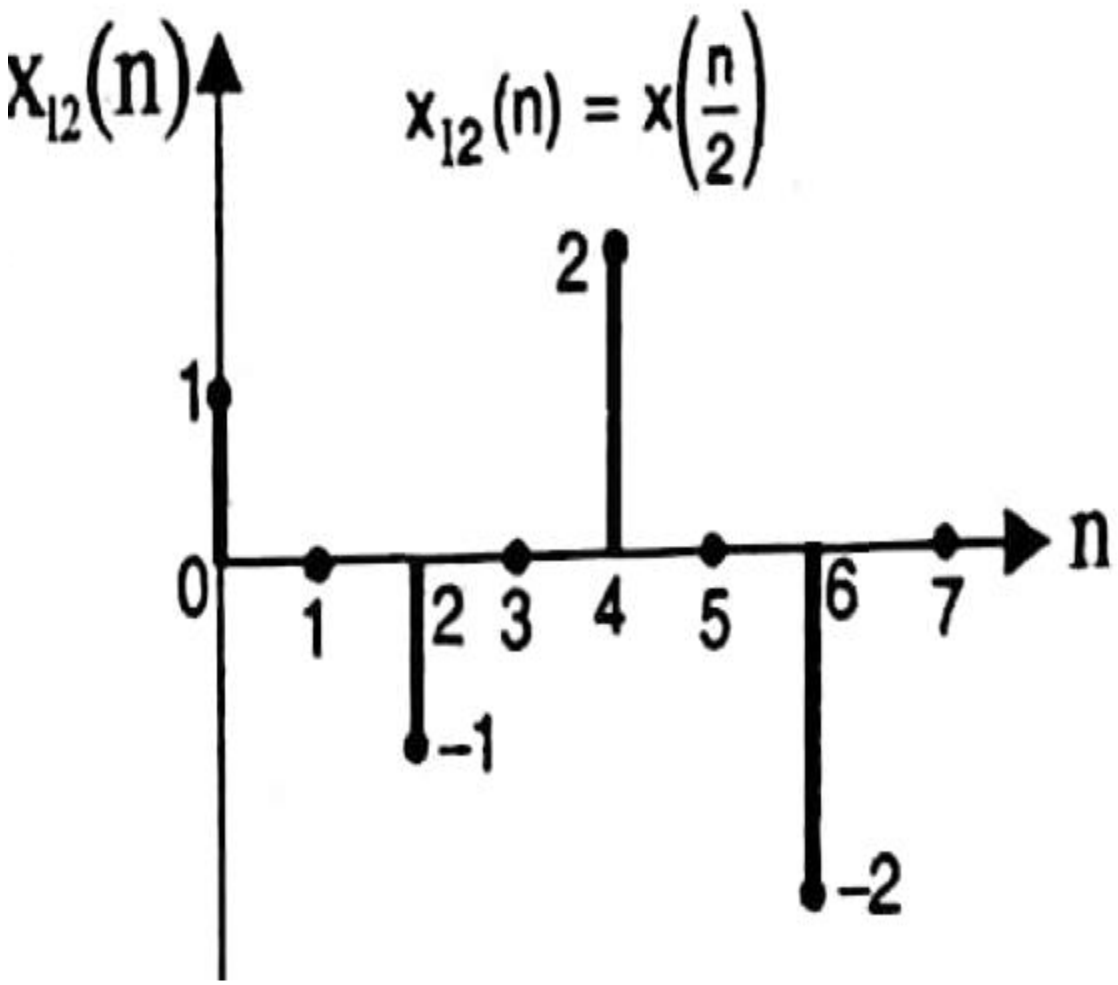
$$\therefore x\left(\frac{n}{3}\right) = x_{I3}(n) = \{\underset{\uparrow}{1}, 0, 0, -1, 0, 0, 2, 0, 0, -2, 0, 0\} \quad .....(2)$$

EXAMPLE

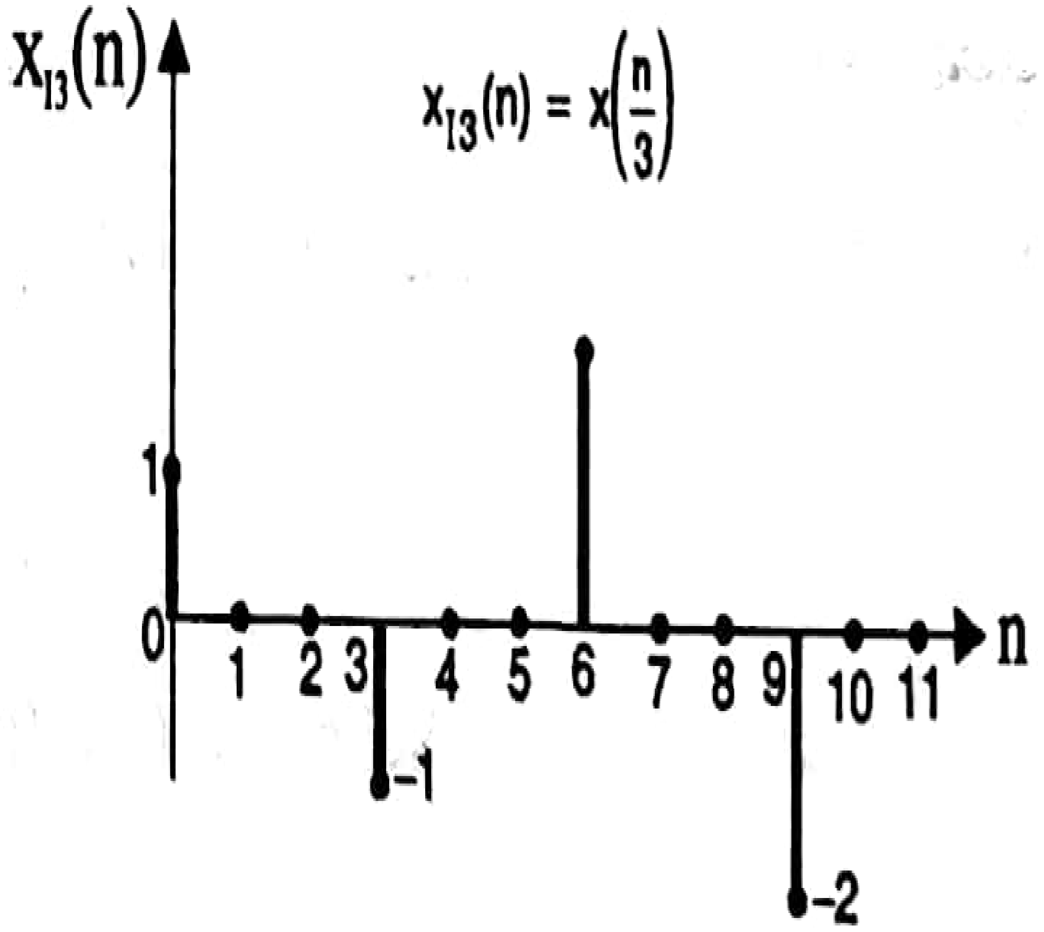
Samples of sequence



x(n) interpolated by 2



x(n) interpolated by 3



ASSESSMENT

1. Define multirate DSP.
2. The discrete time systems that employ sampling rate conversion while processing the discrete time signals are called --------------------
3. What is meant by sampling rate conversion.
4. List the two ways for sampling rate conversion in the digital domain
5. What is meant by downsampling and upsampling?
6. What are the advantages of multirate Processing?

THANK YOU