Derivation of the Thrust Equation

The thrust equation can be derived from fundamental principles of physics, particularly Newton's second and third laws of motion, and the conservation of momentum.

Newton's Third Law of Motion

Newton's third law states that for every action, there is an equal and opposite reaction. When a rocket expels mass in the form of high-speed exhaust gases, the reaction force propels the rocket forward.

Conservation of Momentum

The principle of conservation of momentum states that the total momentum of a closed system remains constant if no external forces act on it.

Assumptions

1. The flow is steady, meaning the mass flow rate and exhaust velocity are constant over time.
2. The exhaust gases are expelled uniformly through the nozzle.
3. The control volume includes only the engine and the immediate exhaust region.

Step-by-Step Derivation

1. Control Volume Analysis

Consider a control volume around the engine where mass enters and exits. For a rocket engine, mass enters as fuel and oxidizer and exits as exhaust gases.

2. Mass Flow Rate ($\dot{m}$)

Let $\dot{m}$ be the mass flow rate of the exhaust gases. This is the rate at which mass is expelled from the rocket.

3. Momentum of Exhaust Gases

The momentum of the exhaust gases leaving the engine per unit time is $\dot{m} \cdot v_e$, where v_e is the exhaust velocity relative to the engine.

4. Change in Momentum

The change in momentum of the rocket (and hence the thrust) is given by the rate of change of momentum of the exhaust gases:

$$\text{Thrust} = \frac{d}{dt}(\text{Momentum})$$

Since the momentum change is due to the mass flow rate of the exhaust gases:

$$F = \dot{m} \cdot v_e$$

This is the primary momentum thrust term.

5. Pressure Thrust

In addition to the momentum thrust, there is also a contribution from the pressure difference between the exit pressure of the exhaust gases P_e and the ambient pressure P_0. This pressure difference acts over the area of the nozzle exit A_e.

The force due to this pressure difference is:

$$F_p = (P_e - P_0) \cdot A_e$$

6. Total Thrust

The total thrust is the sum of the momentum thrust and the pressure thrust:

$$F = \dot{m} \cdot v_e + (P_e - P_0) \cdot A_e$$

Summary

The derived thrust equation combines both the effects of the exhaust velocity and the pressure difference across the nozzle exit:

$$F = \dot{m} \cdot v_e + (P_e - P_0) \cdot A_e$$

- $\dot{m} \cdot v_e$: Momentum thrust, which is the product of the mass flow rate and the exhaust velocity.
- $(P_e - P_0) \cdot A_e$: Pressure thrust, which accounts for the difference in pressure between the exhaust gases at the nozzle exit and the ambient pressure.

This equation accurately describes the thrust produced by a rocket engine, taking into account both the momentum change and the pressure forces involved in the propulsion process.