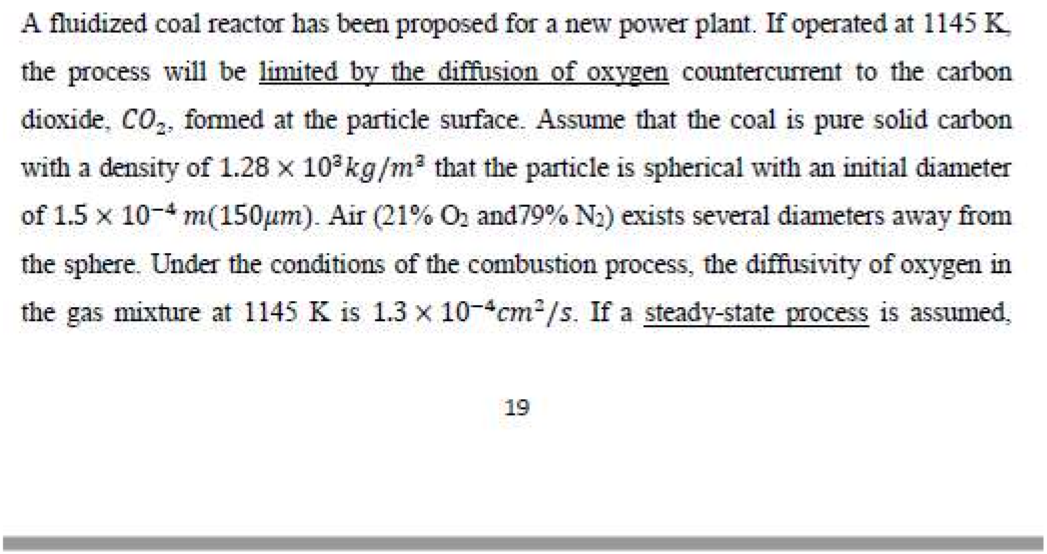
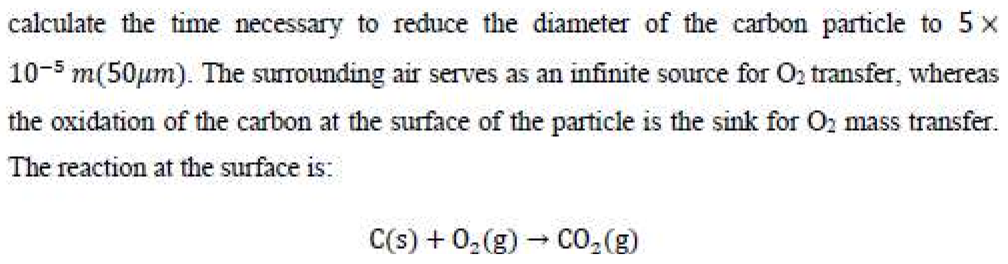
UNIT V - MASS TRANSFER

Topic - Tutorial steady state molecular diffusion



A fluidized coal reactor has been proposed for a new power plant. If operated at 1145 K, the process will be limited by the diffusion of oxygen countercurrent to the carbon dioxide, CO_2, formed at the particle surface. Assume that the coal is pure solid carbon with a density of $1.28 \times 10^3 kg/m^3$ that the particle is spherical with an initial diameter of $1.5 \times 10^{-4}\, m (150 \mu m)$. Air (21% O_2 and79% N_2) exists several diameters away from the sphere. Under the conditions of the combustion process, the diffusivity of oxygen in the gas mixture at 1145 K is $1.3 \times 10^{-4} cm^2/s$. If a steady-state process is assumed,



calculate the time necessary to reduce the diameter of the carbon particle to $5 \times 10^{-5}\, m (50 \mu m)$. The surrounding air serves as an infinite source for O_2 transfer, whereas the oxidation of the carbon at the surface of the particle is the sink for O_2 mass transfer. The reaction at the surface is:

$$C(s) + O_2(g) \rightarrow CO_2(g)$$

At the surface of the coal particle, the reaction is so rapid.

Solution:

The pure carbon particle is the source for the CO_2 flux and the sink for O_2 flux. As the coal particle is oxidized, there will be an output of carbon as stipulated by the stoichiometry of the reaction.

Number of moles of oxygen transferred = number of moles of carbon reacted

Number of moles transferred of oxygen = mole flux × area

$$Number\ of\ moles\ transferred\ of\ oxygen\ =\ N_{O_2-mix}\times 4\pi r^2$$

N_{O_2-mix} can be obtained by using the general differential equation with the Fick's equation as follows:

By applying the following assumptions on the general differential equation of mass transfer:

$$\nabla.\vec{N}_A+\frac{\partial c_A}{\partial t}-R_A=0$$

1. Steady state oxygen diffusion
2. One dimensional mass transfer in r direction
3. No homogenous reaction
4. Instantaneous heterogeneous reaction

$$\therefore \nabla.\vec{N}_A=0$$

For diffusion of oxygen in r-direction

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2N_{O_2}\right)=0$$

$$\therefore \frac{d}{dr}(r^2 N_{O_2}) = 0$$

The above equation specifies that $r^2 N_{O_2}$ is constant over the diffusion path in the r direction, so that

$$r^2 N_{O_2}\Big|_r = R^2 N_{O_2}\Big|_R$$

Form Fick's equation:

$$N_{O_2} = -cD_{O_2-mix}\frac{dy_{O_2}}{dr} + y_{O_2}(N_{O_2} + N_{CO_2})$$

But from the stoichiometry of the reaction

$$N_{O_2} = -N_{CO_2}$$

i.e. equimolar counter diffusion

$$\therefore N_{O_2} = -cD_{O_2-mix}\frac{dy_{O_2}}{dr}$$

$$N_{O_2}\int_R^{\infty} dr = -cD_{O_2-mix}\int_0^{y_{O_2,\infty}} dy_{O_2}$$

$$N_{O_2} r^2 \int_R^{\infty} \frac{dr}{r^2} = -cD_{O_2-mix}\int_0^{y_{O_2,\infty}} dy_{O_2}$$

$$N_{O_2} r^2 \left(\frac{1}{R}\right) = -cD_{O_2-mix}(y_{O_2,\infty} - 0)$$

$$N_{O_2} r^2 = -RcD_{O_2-mix}\, y_{O_2,\infty}$$

$$number\ of\ moles\ of\ oxygen\ transfered\ per\ unit\ time = N_{O_2} \times 4\pi r^2$$

$$number\ of\ moles\ of\ oxygen\ transfered\ per\ unit\ time = -4\pi R\ cD_{O_2-mix}\, y_{O_2,\infty}$$

The negative sign because the transfer of oxygen is in the opposite direction of r

$$\therefore number\ of\ moles\ of\ carbon\ consumed\ \ per\ unit\ time = 4\pi R\ cD_{O_2-mix}\, y_{O_2,\infty}$$

By applying the law of conservation of mass on the carbon:

$$\text{Input} - \text{output} + \text{generation} - \text{consumption} = \text{accumulation(rate of change)}$$

$$-\text{consumption} = \text{accumulation(rate of change)}$$

$$-4\pi R\, cD_{O_2-mix}\, y_{O_2,\infty} = \frac{dN}{dt} = \frac{\rho_c}{M.wt}\frac{dV}{dt}$$

$$V = \frac{4}{3}\pi R^3$$

$$-4\pi R\, cD_{O_2-mix}\, y_{O_2,\infty} = \frac{\rho_c}{M.wt}4\pi R^2\frac{dR}{dt}$$

$$dt = -\frac{\rho_c}{M.wt}\frac{R\,dR}{cD_{O_2-mix}\, y_{O_2,\infty}}$$

by integrating the above equation between the limits:

$$\text{at } t = 0 \qquad R = R_i = 7.5\times10^{-5}\ m$$

$$\text{at } t = t \qquad R = R_f = 2.5\times10^{-5}\ m$$

$$\int_0^t dt = -\frac{\rho_c}{M.wt}\frac{1}{cD_{O_2-mix}\, y_{O_2,\infty}}\int_{R_i}^{R_f} R\,dR$$

$$t = \frac{\rho_c}{2\,M.wt}\frac{(R_i^2 - R_f^2)}{cD_{O_2-mix}\, y_{O_2,\infty}}$$

$$c = \frac{P}{RT} = 0.0106\ kmol/m^3$$

$$y_{O_2,\infty} = 0.21$$

$$\therefore t = 0.92\ s$$

References:

1. Kothandaraman C.P “Fundamentals of Heat and Mass Transfer” New Age International, New Delhi,4th Edition 2012 (Unit I, II, III, IV, V).
2. Frank P. Incropera and David P. DeWitt, “Fundamentals of Heat and Mass Transfer”, John Wiley and Sons, New Jersey,6th Edition1998(Unit I,II,III,IV, V)
3. MIT open courseware – https://ocw.mit.edu/courses/mechanical-engineering

Other web sources