(55)

II. (B). (i) Testing of Significance for equality of means of 2 normal populations with known S.D

i.e., $H_0 : \mu_1 = \mu_2$; σ_1, σ_2 are known.

Let $\bar{x}_1$ be the mean of a sample of size n_1 from a population with mean μ_1 and variance σ_1^2 and let $\bar{x}_2$ be the mean of an independent sample of size n_2 from another population with mean μ_2 and variance σ_2^2. Then under the null hypothesis $H_0 : \mu_1 = \mu_2$ the test statistic will be,

$$\boxed{Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}}$$

Note:

1. If σ_1 and σ_2 are not known and $\sigma_1 \neq \sigma_2$, σ_1 and σ_2 can be approximated by the sample S.D's σ_1 and σ_2. Hence we have,

$$\boxed{Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}}$$

(The samples are taken from different population)

2. If σ_1 and σ_2 are equal and not known, then $\sigma_1 = \sigma_2 = \sigma$ is approximated by $\sigma^2 = \dfrac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2}$.

Hence the test-statistic is given by,

$$\boxed{Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_2} + \frac{s_2^2}{n_1}}}}$$

The samples are taken from same population

(5) A sample of heights of 6400 Englishmen has a mean of 170 cms and a SD of 6.4 cms, while a sample of heights of 1600 Australians has a mean of 172 cm and a standard deviation of 6.3 cm. Do the data indicate that the Australians are on the average taller than the English men?

Solution:

Given: Englishmen - $n_1 = 6400$

$\bar{x}_1 = 170$

$s_1 = 6.4$ cm

Australians : $n_2 = 1600$

$\bar{x}_2 = 172$

$s_2 = 6.3$ cm

Null hypothesis : H_0 : Australians and Englishmen have the same mean height.

i.e., $H_0 : \mu_1 = \mu_2$

Alternative hypothesis : $H_1 : \mu_2 > \mu_1$ (Right-tailed test)

Level of significance : At $\alpha = 5\%$, $Z_\alpha = 1.645$

Test Statistic :

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$z = \frac{170 - 172}{\sqrt{\frac{6.4^2}{6400} + \frac{6.3^2}{1600}}}$$

$$= \frac{-2 \times 40}{\sqrt{40.93}} = -11.3$$

$$\boxed{|z| = 11.3}$$

Decision :

Since $|z| > 3$, H_0 is rejected. $\therefore$ The data indicate that the Australians are on the average taller than the Englishmen.

(12) In a certain factory there are two independent processes manufacturing the same item. The average waight in a sample of 250 items produced from one process is found to be 120 ozs with a SD of 12 ozs while the corresponding figures in a sample of 400 items from the other process are 124 and 14. Obtain the standard error of difference between the two sample means. Is this difference significant?

Solution :

Given : $n_1 = 250$, $\bar{x}_1 = 120$, $s_1 = 12$

$n_2 = 400$, $\bar{x}_2 = 124$, $s_2 = 14$

Null hypothesis : H_0 : The sample means do not differ significantly . i.e., $H_0 : \mu_1 = \mu_2$.

Alternative hypothesis : $H_1 : \mu_1 \neq \mu_2$ (Two-tailed test)

Level of significance : At $\alpha = 5\%$, $Z_\alpha = 1.96$

Standard Error :

$$S.E(\bar{x}_1 - \bar{x}_2) = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$= \sqrt{\frac{144}{250} + \frac{196}{400}}$$

$$\boxed{S.E = 1.034}$$

Test Statistic :

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$= \frac{120 - 124}{1.034}$$

$$= -3.87$$

$$\boxed{|Z| = 3.87}$$

Decision:

Since $|z| > 3$, H_0 is rejected. Hence we conclude that there is significant difference between the sample means.

(13) Test the significance of the difference between the means of the samples, drawn from two normal populations with the same SD from the following data:

	Size	Mean	S.D
Sample 1	100	61	4
Sample 2	200	63	6

Solution:

Given: $n_1 = 100$, $\bar{x}_1 = 61$, $s_1 = 4$

$n_2 = 200$, $\bar{x}_2 = 63$, $s_2 = 6$

Null hypothesis: H_0: The samples drawn from the two normal populations have the same mean with the same SD. i.e., $H_0 : \mu_1 = \mu_2$

Alternative hypothesis: $H_1 : \mu_1 \neq \mu_2$ (Two-tailed test)

Level of Significance: At $\alpha = 5\%$, $Z_\alpha = 1.96$

(65)

Test Statistic:

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_2} + \frac{s_2^2}{n_1}}}$$

$$= \frac{61 - 63}{\sqrt{\frac{4^2}{200} + \frac{6^2}{100}}} = -3.02$$

$$\boxed{|z| = 3.02}$$

Decision:

Since $|z| > 3$, H_0 is rejected. Therefore the samples drawn from the 2 normal populations do not have the same mean though they may have the same S.D.