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2. Testing of Significance for difference of proportions:

Suppose 2 large samples of sizes n_1 and n_2 are taken respectively from 2 different populations. Let x_1 be the number of persons possessing the attribute A in the first sample and let x_2 be the number of persons possessing the same attribute in the second sample. Then the sample proportions are given by,

$$p_1 = \frac{x_1}{n_1}, \quad p_2 = \frac{x_2}{n_2}$$

Under the null hypothesis $H_0 : P_1 = P_2 = P$ (hence $Q_1 = Q_2 = Q$) test statistic will be,

$$\boxed{Z = \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}}$$

Where the population proportion $P = \dfrac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$

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Problems :

(1) A machine produced 20 defective articles in a batch of 400. After overhauling it produced 10 defectives in a batch of 300. Has the machine improved ?

Solution :

Given : $n_1 = 400$, $n_2 = 300$

Proportion of defectives in the first Sample $\}$ $p_1 = \frac{20}{400} = 0.05$

Proportion of defectives in the second Sample $\}$ $p_2 = \frac{10}{300} = 0.0333$

$$P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$$

$$= \frac{400 \times \frac{20}{400} + 300 \times \frac{10}{300}}{400 + 300}$$

$$= \frac{30}{700} = 0.043$$

$$P = 0.043$$

$$Q = 1 - P = 0.957$$

Null hypothesis : H_0 : There is no significant difference in the machine before and after overhauling.

i.e, $H_0 : P_1 = P_2$

Alternative hypothesis : $H_1 : P_1 > P_2$ (Right-tailed test)

Level of Significance : Let $\alpha = 5\%$. Then

$$z_\alpha = 1.645$$

Test Statistic :

$$Z = \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$Z = \frac{0.05 - 0.0333}{\sqrt{0.043 \times 0.957\left(\frac{1}{400} + \frac{1}{300}\right)}}$$

$$= \frac{+\ \cancel{0.283}\ 0.0167}{0.015}$$

$$\boxed{Z = 1.11}$$

Decision :

Since $|Z| < z_\alpha$, H_0 is accepted $\therefore$ The machine has not improved after overhauling.

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(2) In a Sample of 600 men from a certain city, 450 men are found to be Smokers. In a Sample of 900 from another city 450 are found to be smokers. Do the data indicate that the two cities are significantly different w.r.t prevalence of smoking habit among men?

Solution:

Given: $n_1 = 600$, $n_2 = 900$

Proportion of smokers in the first city $\Big\}$ $p_1 = \frac{450}{600} = 0.75$

Proportion of Smokers in the second city $\Big\}$ $p_2 = \frac{450}{900} = 0.5$

$$P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$$

$$= \frac{600 \times 0.75 + 900 \times 0.5}{600 + 900}$$

$$= \frac{900}{1500} = 0.6$$

$P = 0.6$

$Q = 1 - P$

$= 1 - 0.6$

$Q = 0.4$

Null hypothesis : H_0 : There is no significant difference between the two cities w.r.t the prevalence of smoking habit among them.

i.e., $H_0 : P_1 = P_2$

Alt Hyp: $H_1 : P_1 \neq P_2$ (Two-tailed test)

Level of significance : Let $\alpha = 5\%$.

$$z_\alpha = 1.645$$

Test-Statistic :

$$z = \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$z = \frac{0.75 - 0.5}{\sqrt{0.6 \times 0.4\left(\frac{1}{600} + \frac{1}{900}\right)}}$$

$$= \frac{0.25}{0.026} = 9.68$$

$$\boxed{z = 9.68}$$

Decision :

Since $|z| > 3$, H_0 is rejected.

$\therefore$ There is a significant difference between the two cities w.r.t the prevalence of smoking habit among them.