



DEPARTMENT OF MATHEMATICS

Randomized block Design (RBD) (or Two way Classification)

It is a two factor experiment.

Procedure :

Step 1 : Null hypothesis : H_0 : There is no significant difference between columns and rows.

Alternative hypothesis : H_1 : There is a significant difference between columns and rows.

Step 2 : * Find N
* Find T
* Find C.F = T^2/N

Step 3 : * Find $SST = \sum x_1^2 + \sum x_2^2 + \dots - C.F$
* Find $SSC = \frac{(\sum x_1)^2}{c_1} + \frac{(\sum x_2)^2}{c_2} + \dots - C.F$
* Find $SSR = \frac{(\sum y_1)^2}{r_1} + \frac{(\sum y_2)^2}{r_2} + \dots - C.F$
* Find $SSE = SST - SSC - SSR$

Step 4 : ANOVA table :

Source of Variation	Degree of freedom	Sum of Squares	Mean sum of squares	Variance ratio	Table value
Between Columns	$(c-1)$	SSC	$MSC = \frac{SSC}{c-1}$	$F_c = \frac{MSC}{MSE}$	$F_{\alpha}(c-1, (c-1)(r-1))$
Between Rows	$(r-1)$	SSR	$MSR = \frac{SSR}{r-1}$	$F_r = \frac{MSR}{MSE}$	$F_{\alpha}(r-1, (c-1)(r-1))$
Between Errors	$(c-1) \times (r-1)$	SSE	$MSE = \frac{SSE}{(c-1)(r-1)}$		

Step 5 : Decision : If $F_c < F_{\alpha}$, $F_r < F_{\alpha}$, H_0 is accepted otherwise it is rejected.



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Problem :

- ① An experiment was designed to study the performance of 4 different detergents for cleaning of injectors. The following "cleanliness" readings were obtained with specially designed equipment for 12 tanks of gas distributed over 3 different models of engines.

Detergent	Engine 1	Engine 2	Engine 3	Total
A	45	43	51	139
B	47	46	52	145
C	48	50	55	153
D	42	37	49	128
Total	182	176	207	565

Perform the ANOVA test at 0.01 level of significance whether there are differences in the detergents or in the engines.

Solution:

Fix origin = 50. Subtract each element from 50.

Engine \ Detergent	x_1	x_2	x_3	Total	x_1^2	x_2^2	x_3^2
A (y_1)	-5	-7	+1	-11	25	49	1
B (y_2)	-3	-4	2	-5	9	16	4
C (y_3)	-2	0	5	3	4	0	25
D (y_4)	-8	-13	-1	-22	64	169	1
Total	-18	-24	7	-35	102	234	31



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Step 1 : Null hypothesis H_0 : There is no significant difference between engines and detergents.

Alternative hypothesis H_1 : There is a significant difference between engines and detergents.

Step 2 : * $N = 12$

$$T = -35$$

$$C.F = T^2/N = 102.08$$

Step 3 : $SST = \sum x_1^2 + \sum x_2^2 + \sum x_3^2 - C.F$

$$= 102 + 234 + 31 - 102.08$$

$$\boxed{SST = 264.92}$$

$$SSC = \frac{(\sum x_1)^2}{c_1} + \frac{(\sum x_2)^2}{c_2} + \frac{(\sum x_3)^2}{c_3} - C.F$$

$$= \frac{(-18)^2}{4} + \frac{(-24)^2}{4} + \frac{7^2}{4} - 102.08$$

$$\boxed{SSC = 135.17}$$

$$SSR = \frac{(\sum y_1)^2}{r_1} + \frac{(\sum y_2)^2}{r_2} + \frac{(\sum y_3)^2}{r_3} + \frac{(\sum y_4)^2}{r_4} - C.F$$

$$= \frac{(-11)^2}{3} + \frac{(-5)^2}{3} + \frac{3^2}{3} + \frac{(-22)^2}{3} - 102.08$$

$$\boxed{SSR = 110.91}$$

$$SSE = TSS - SSC - SSR = 264.92 - 135.17 - 110.91$$

$$\boxed{SSE = 18.84}$$

Step 4 : ANOVA table :

Source of Variation	Degree of freedom	Sum of Squares	Mean Sum of Squares	Variance Ratio	Table Value at 1% level
Between Columns	$C-1 = 3-1 = 2$	$SSC = 135.17$	$MSC = \frac{SSC}{C-1} = 67.585$	$F_C = \frac{MSC}{MSE} = 21.52$	$F_{\alpha}(2, 6) = 10.92$
Between rows	$r-1 = 4-1 = 3$	$SSR = 110.91$	$MSR = \frac{SSR}{r-1} = 36.97$	$F_R = \frac{MSR}{MSE} = 11.77$	$F_{\alpha}(3, 6) = 9.78$
Between errors	$(C-1)(r-1) = 6$	$SSE = 18.84$	$MSE = \frac{SSE}{(C-1)(r-1)} = 3.14$		

Steps : Decision : Since $F_C > F_{\alpha}$ and $F_R > F_{\alpha}$, H_0 is rejected.

\therefore There is a significant difference between engines and detergents.