Latin Square Design (LSD) :

It is a three factor experiment

Procedure :

Step 1 : Null hypothesis : H_0 : There is no significant difference between columns, rows and treatments.

Alternative hypothesis : H_1 : There is a significant difference between columns, rows and treatments.

Step 2 :
* Find N
* Find T
* Find $C.F = T^2/N$

Step 3 : Find

* $SST = \Sigma x_1^2 + \Sigma x_2^2 + \Sigma x_3^2 + \cdots - C.F$
* $SSC = \frac{(\Sigma x_1)^2}{c_1} + \frac{(\Sigma x_1)^2}{c_2} + \frac{(\Sigma x_3)^2}{c_3} + \cdots - C.F$
* $SSR = \frac{(\Sigma y_1)^2}{r_1} + \frac{(\Sigma y_2)^2}{r_2} + \frac{(\Sigma y_3)^2}{r_3} + \cdots - C.F$

Step 4 :

Arrange the data by treatment wise.

* $SSK = \frac{(\Sigma z_1)^2}{K} + \frac{(\Sigma z_2)^2}{K} + \frac{(\Sigma z_3)^2}{K} + \cdots - C.F$

where K – Number of treatments (columns).

* $SSE = SST - SSC - SSR - SSK$

Step 5 : ANOVA Table :

Source of Variation	Degree of freedom	Sum of squares	Mean Sum of squares	Variance ratio	Table value
Between columns	$K-1$	SSC	$MSC = \frac{SSC}{K-1}$	$F_C = \frac{MSC}{MSE}$	$F_\alpha(K-1, (K-1)(K-2))$
Between rows	$K-1$	SSR	$MSR = \frac{SSR}{K-1}$	$F_R = \frac{MSR}{MSE}$	$F_\alpha(K-1, (K-1)(K-2))$
Between treatments	$K-1$	SSK	$MSK = \frac{SSK}{K-1}$	$F_K = \frac{MSK}{MSE}$	$F_\alpha(K-1, (K-1)(K-2))$
Between errors	$(K-1)(K-2)$	SSE	$MSE = \frac{SSE}{(K-1)(K-2)}$		

Step 6 : Decision :

If $F_C < F_\alpha$, $F_R < F_\alpha$, $F_K < F_\alpha$ it is accepted otherwise it is rejected.

Problems :

(1) Analyze the variance in the Latin square of yields (in kgs) of paddy where P, Q, R, S denote the different methods of cultivation.

S 122	P 121	R 123	Q 122
Q 124	R 123	P 122	S 125
P 120	Q 119	S 120	R 121
R 122	S 123	Q 121	P 122

Examine whether the different methods of cultivation have given significantly different yields.

Soln :

Step 1 : Null hypothesis : H_0 : There is no significant difference between rows, columns and treatments.

Alternative hypothesis : H_1 : There is a significant difference between rows, columns and treatments.

origin = 120

	x_1	x_2	x_3	x_4	Total	x_1^2	x_2^2	x_3^2	x_4^2
y_1	2	1	3	2	8	4	1	9	4
y_2	4	3	2	5	14	16	9	4	25
y_3	0	-1	0	1	0	0	1	0	1
y_4	2	3	1	2	8	4	9	1	4
Total	8	6	6	10	30	24	20	14	34

Step 2 :

$$N = 16$$

$$T = 30$$

$$C.F = \frac{T^2}{N} = \frac{30^2}{16} = 56.25$$

Step 3 :

$$SST = \Sigma x_1^2 + \Sigma x_2^2 + \Sigma x_3^2 + \Sigma x_4^2 - C.F$$

$$= 24 + 20 + 14 + 34 - 56.25$$

$$\boxed{SST = 35.75}$$

$$SSC = \frac{(\Sigma x_1)^2}{c_1} + \frac{(\Sigma x_2)^2}{c_2} + \frac{(\Sigma x_3)^2}{c_3} + \frac{(\Sigma x_4)^2}{c_4} - C.F$$

$$= \frac{8^2}{4} + \frac{6^2}{4} + \frac{6^2}{4} + \frac{10^2}{4} - 56.25$$

$$\boxed{SSC = 2.75}$$

$$SSR = \frac{(\Sigma y_1)^2}{r_1} + \frac{(\Sigma y_2)^2}{r_2} + \frac{(\Sigma y_3)^2}{r_3} + \frac{(\Sigma y_4)^2}{r_4} - C.F$$

$$= \frac{8^2}{4} + \frac{14^2}{4} + 0 + \frac{8^2}{4} - 56.25$$

$$\boxed{SSR = 24.75}$$

Step 4 : Arrange treatments P, Q, R, S in columnwise.

	z_1 P	z_2 Q	z_3 R	z_4 S
	1	2	3	2
	2	4	3	5
	0	-1	1	0
	2	1	2	3
Total	5	6	9	10

$$SSK = \frac{(\Sigma z_1)^2}{K} + \frac{(\Sigma z_2)^2}{K} + \frac{(\Sigma z_3)^2}{K} + \frac{(\Sigma z_4)^2}{K} - C.F$$

$$= \frac{5^2}{4} + \frac{6^2}{4} + \frac{9^2}{4} + \frac{10^2}{4} - 56.25$$

$\boxed{SSK = 4.25}$ $\quad SSE = SST - SSC - SSR - SSK$

$\boxed{SSE = 4}$

Step 5: ANOVA table:

Source of Variation	Degree of freedom	Sum of Squares	Mean Sum of squares	Variance ratio	Table Value
Between column	$K-1=3$	$SSC = 2.75$	$MSC = \frac{SSC}{K-1} = 0.917$	$F_C = \frac{MSC}{MSE} = 1.375$	$F_\alpha(3,6) = 4.76$.
Between rows	$K-1=3$	$SSR = 24.75$	$MSR = \frac{SSR}{K-1} = 8.25$	$F_R = \frac{MSR}{MSE} = 12.369$	
Between treatments	$K-1=3$	$SSK = 4.25$	$MSK = \frac{SSK}{K-1} = 1.417$	$F_K = \frac{MSK}{MSE} = 2.124$	
Between errors	$(K-1)(K-2) = (4-1)(4-2) = 6$	$SSE = 4$	$MSE = \frac{SSE}{(K-1)(K-2)} = 0.667$		

Step 6: Decision:

Since $F_C < F_\alpha$, $F_K < F_\alpha$ we accept the hypothesis ~~there~~ that there is no significant difference between columns and treatments.

Since $F_R > F_\alpha$, ~~the~~ we reject the hypothesis that there is a significant difference between rows.

Comparison of RBD and CRD

1. RBD is more efficient than CRD for most types of experimental work.
2. In CRD, grouping of the experimental size so as to allocate the treatments at random to the experimental units is not done. But in RBD, treatments are allocated at random within the units of each stratum.
3. RBD is more flexible than CRD since no restrictions are placed on the number of treatments or the number of replications.

Comparison of LSD and RBD

1. In LSD, the number of treatments is equal to the number of replications whereas there is no such restrictions on treatments and replications in RBD.
2. LSD is known to be suitable for a case when the number of treatments is between 5 and 12 since the square becomes large and does not remain homogeneous, whereas RBD can be used for any number of treatments.
3. In the field layout, LSD can be performed on a square field while RBD can be performed either on a square or rectangular field.
4. The main advantage of LSD is that it controls the variations between the rows and columns, whereas RBD controls the effect of one direction and hence the experimental error is reduced to a large extent.