Subject Code & Name: 19AST203 Aircraft Structural Mechanics

TOPIC: **Bredt – Batho formula**

Bredt-Batho formulae

The 1st Bredt-Batho formula indicates the relationship between the torsional moment MT acting on a thin-walled hollow tube, its enclosed area Am and the resultant shear flow T / shear stress τ. It is calculated as follows: τ=Tt=MT2·Am·t.

The variable t represents the thin-walled component's wall thickness. The enclosed area Am lies within the centre line of the tube and is also called the hollow area. The shear stress τ resulting from the Torsion is constant over the entire wall thickness t, which means that the shear flow T also remains constant in the circumferential direction.

The 2nd Bredt-Batho formula indicates the component's twisting ϑ, which depends on the material's shear modulus G. A component's torsional Resistance I_T can also be determined.
The Bredt-Bredt-Batho formulae apply only to torsion acting on closed hollow tubes with an axis of Rotation that lies on the shear centre.

A beam with a closed section experiencing only a pure torque T and without any axial constraints, does not develop direct stresses, ie s z = 0.

So equations (4.2) and (4.3) become:

$$\frac{\partial q}{\partial s} = \frac{\partial q}{\partial z} = 0$$

The only way to satisfy these equations would be if the shear flow 'q' was constant.

NOTE: Although 'q' is constant, the shear stress 't ' may not be if the wall thickness 't' varied with 's'.

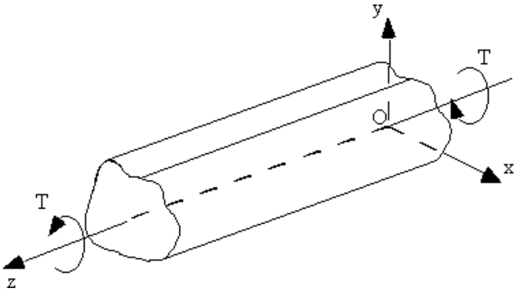


Figure 45: Closed beam with applied torque.

To determine the relationship between applied torque and shear flow,

apply equilibrium to the end of the beam.

In essence the applied Torque T must equal to the torque generated by the shear flow.

Look at the end of beam, and a small section ds.

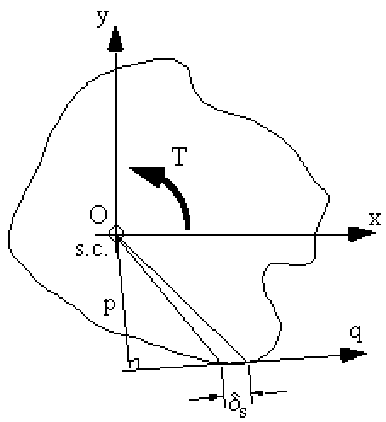


Figure 46: Equating applied torque with moment generated by shear flow.

The torque produced by the shear flow on element ds is pqsd s. Integrating about the whole section gives:

$$T = \oint pqds$$

We have previously defined that:

$$\oint pds = 2A$$

Therefore:

$$q = \frac{T}{2A} \quad (5.1)$$

Often referred to as the 'Bredt-Batho Formula'. Substituting this equation into (4.21) gives the rate of twist due to the Torque 'T':

Let us suppose the origins where the shear flow has unknown values $q_{s,o}$

Then for closed section

$$q_s = \frac{-\bar{S}_x}{I_{yy}} \int x t\, ds - \frac{\bar{S}_y}{I_{xx}} \int y t\, ds + q_{s,o}$$

$$q_s = q_b + q_{s,o}$$

$$q_{s,o} = -\frac{M}{2A}$$

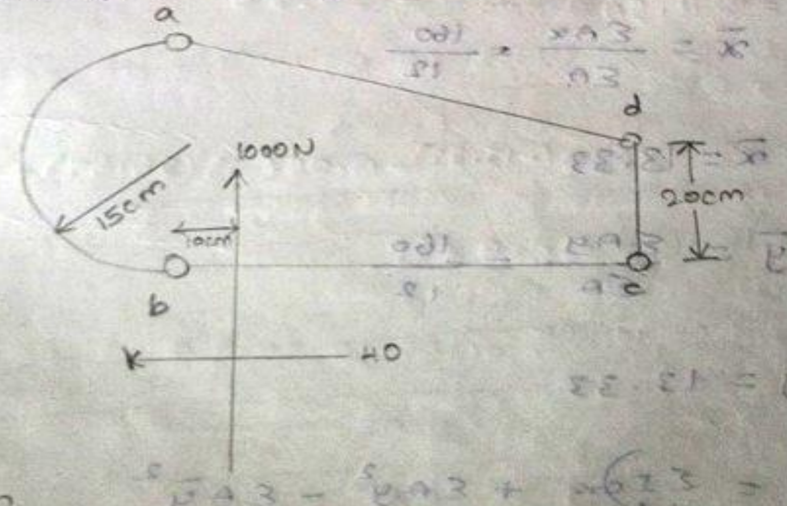
M → Unbalanced Moment

q_b → basic shear flow equation for open tube

$q_{s,o}$ → unknown shearflow equation at origin of 's' co-ordinates.

The value of shear flow at origin of 's' is found by making a cut at that point and equating applied the initial moments taken abo some convinience point.

) plot shear flow for given closed section.

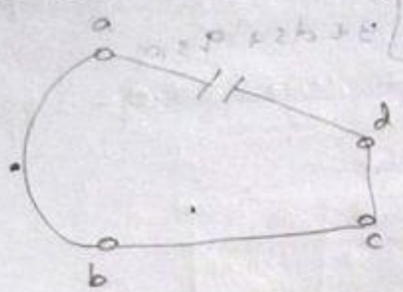


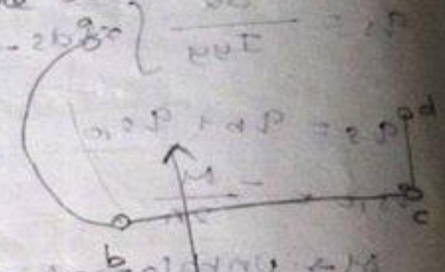
given

$a = b = 4cm^2$

Soln:-

To Solve the above problem; make a cut between Stringes a and d to make it as open section. The section becomes





Boom	Area	x	y	Ax	Ay	Ax^2	Ay^2	Axy
a	4	0	30	0	120	0	3600	0
b	4	0	0	0	0	0	0	0
c	2	40	0	80	0	3200	0	0
d	2	40	20	80	40	3200	800	1600
Σ	12	80	50	160	160	6400	4400	1600

$$\bar{x} = \frac{\Sigma Ax}{\Sigma A} = \frac{160}{12}$$

$$\bar{x} = 13.33$$

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = \frac{160}{12}$$

$$\bar{y} = 13.33$$

$$I_{xx} = \Sigma I_{xx} + \Sigma Ay^2 - \Sigma A\bar{y}^2$$

$I_{xx} = 4400 - 12(13.33)^2$

$I_{xx} = 2267.74 \text{ cm}^4$

$I_{yy} = \Sigma I_{y_0} + \Sigma A x^2 - \Sigma A \bar{x}^2$

$I_{yy} = 6400 - 12(13.33)^2$

$I_{yy} = 4267.73 \text{ cm}^4$

$I_{xy} = \Sigma A x y - \Sigma A \bar{x}\bar{y}$

$I_{xy} = 1600 - 12(13.33)(13.33)$

$I_{xy} = -532.26 \text{ cm}^4$

$S_x = 0 \; ; \; S_y = 1000$

$$\bar{S}_x = \frac{S_x - S_y \dfrac{I_{xy}}{I_{xx}}}{1 - \dfrac{I^2_{xy}}{I_{xx} I_{yy}}}$$

$$\bar{S}_x = \frac{0 - \left[1000 \times \dfrac{(-532.26)}{2267.74}\right]}{1 - \dfrac{(-532.26)^2}{2267.74 \times 4267.73}}$$

$\bar{S}_x = 241.71$ N

$$\bar{S}_y = \frac{S_y - S_x \dfrac{I_{xy}}{I_{yy}}}{1 - \dfrac{I^2_{xy}}{I_{xx} I_{yy}}}$$

$$\bar{S}_y = \frac{1000 - \left[0 \times \dfrac{(-532.26)}{2267.74}\right]}{1 - \dfrac{(-532.26)^2}{2267.74 \times 4267.73}}$$

$\bar{S}_y = 1030.18$ N

$$q = \frac{-S_y}{I_{xx}} \Sigma A_i y_i - \frac{S_x}{I_{yy}} \Sigma A_i x_i$$

$$q = -\frac{1030.18}{2267.74} \Sigma A_i y_i - \frac{241.71}{4267.73} \Sigma A_i x_i$$

$$\boxed{q = -0.45 \Sigma A_i y_i - 0.05 \Sigma A_i x_i}$$

	A	B	C	D
A	4	4	2	2
x	-13.33	-13.33	26.67	26.67
y	16.67	-13.33	-13.33	6.67

$$q_{ab} = -0.45(4)(16.67) - 0.05(4)(-13.33)$$

$$q_{ab} = -30.27 + 3.017$$

$$q_{ab} = -27.26$$

$$q_{bc} = -0.45(4)(-13.33) - 0.05(4)(-13.33) + q_{ab}$$

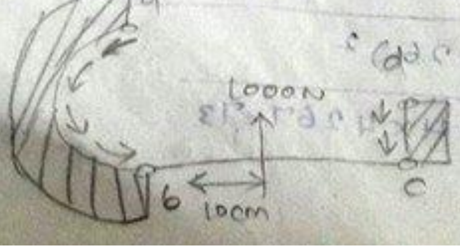
$$q_{bc} = 24.20 + 3.017 - 27.26$$

$$q_{bc} = 0$$

$$q_{cd} = -0.45(2)(-13.33) - 0.05(2)(26.67) + q_{bc}$$

$$q_{cd} = 12.10 - 3.019 + 0$$

$$q_{cd} = 9.05 \text{ N/cm}$$



$q_s = q_b + q_{s,0}$

$q_{s,0} = \frac{-M}{2A}$

$A = \frac{1}{2}\pi r^2 + \frac{1}{2} bh$

$A = \left[\frac{1}{2} \times \pi \times 15^2 + \frac{1}{2} \times (40 \times 10) + (40 \times 20)\right]$

$A = 1353.42 \text{ cm}^2$

Moment about 'b' (unbalanced Moment) $\rightarrow 2Aq$

$M = (-1000 \times 10 - 2\left(\frac{\pi r^2}{2}\right) \times 27.26)$

$+ 9.08 \times 40 \times 20$

$= 10000 - 27.26 \times \pi \times 15^2 + 9.08 \times 800$

$= -2736 - 27.26 \times \pi \times 15^2$

$M = -22004.95 \text{ N/cm}$

$q_{s,0} = 8.129 \text{ N/cm}$

$q_{s,0} = q_{ad}$

Shear flow of closed section

$q_s = q_b + q_{s,0}$

$q_{ab} = -27.26 + 8.13$

$q_{ab} = -19.13$

$q_{bc} = 0 + 8.13$

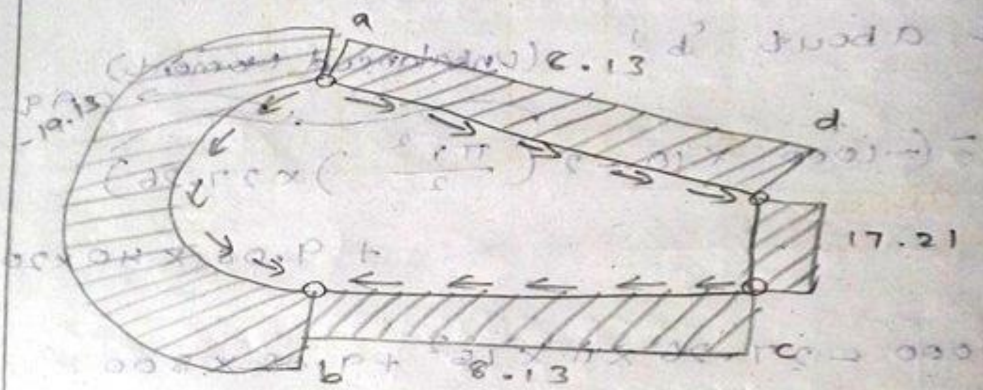
$q_{bc} = 8.13 \text{ N/cm}$

$q_{cd} = 9.08 + 8.13$

$q_{cd} = 17.21 \text{ N/cm}$

$$q_{s,0} = \frac{-(-22004.95)}{2 \times 1353.45}$$

$$\boxed{q_{s,0} = 8.13 \text{ N/cm}}$$



3-14 Torsional effect of Multicell tube:-

Assumption:-

1. Angle of twist is equal for all cells.

$$\theta_1 = \theta_2 = \theta_3 = \theta.$$

2. Material is homogeneous.

3. It obeys hooks law.

4. Beam is subjected to torque alone.

for single cell

$T = 2Aq$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