UNIT-3 Column

1) Strut :

A member of structure which is sub to axial compressive load is called strut.

The strut may have its one or both the ends fixed rigidly (or) hinged (or) pin jointed.

Ex: piston rod , connecting rod

2) column:

* A member of structure which is sub to axial compressive load is called strut. If the strut vertical, inclined at 90° to the horizontal is known as column.

* If the column will have both the ends fixed rigidly Ex. A vertical piller between roof and floor

1) Assumption made in failure column &

1) When the column or strut uniform sectional area A. sub to compressive load. then the compressive stress is given by

$$\sigma_c = P/A$$

2) When the load is gradually increased the column will reach a stage, when it is sub to ultimate crushing stress.

3) Beyond the stage the column will failed by crushing

4) A short column will fail due to crushing

5) sometime the column doesn't failed entirely by crushing but also by bending (or) buckling the load at which a column just buckled is known as buckling load or critical load or crippling load

6) very long column are sub. to buckling only.

2) Assumption made in the Euler's column theory

1) The cross sectional of the column is uniform throughout the length.

2) The material at the column is perfectly elastic homogeneous. and obey hooks law.

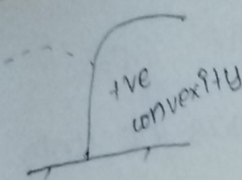
3) The length of the column is very long as compared to its cross sectional dimension

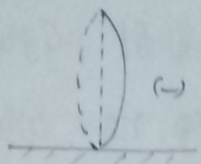
4) The failure of the column occurs due to buckling alone.

5) A self weight of the column is neglegible

3) Sign convention:

A bending moment which bends the column so as to present convexity towards the initial center line of the member will be regarded as positive





A bending Moment which bends columnasto present concavity towards the centreline of the member will be regarded as negative.

4) classification of column:

* short column
* medium column
* long column.

Short: the column which have length less than 8 times their respective dia meters. (or) slenderness ratio less than 32 are called short column.

Medium: The column which have their length varying from 8 times their diameter to 30 time the respective dia meter (or) slenderness ratio lying between 32 & 120 are called medium column.

Long colum: The column which have their length more than 30 times the res dia meter (or) slenderness ratio more than 120 are called long column

5) slenderness ratio (K): It is the ratio of unsupported length of the column to the

minimum radius of gyration. of the cross sectional ends of the column. It has no unit.

6) Buckling factor:- It is the ratio between the equivalent length of the column to the minimum radius of gyration.

7). Buckling load:-

The maximum limiting load at which the column ~~tends~~ to have lateral displacement (or) ~~tends~~ to buckling is called buckling (or) crippling load.

8): Safe load:- It is the load which a column is actually sub to and is well below the buckling load. It is obtained by dividing the buckling load by a suitable Factor of Safety (FOS)

$$\text{Safe load} = \frac{\text{Buckling load}}{\text{Factor of Safety}}$$

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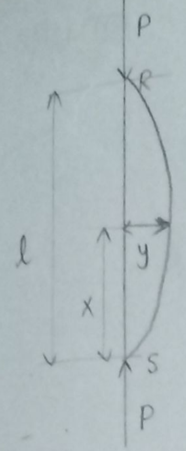
1. column with both ends are pinned (or) hinged explain for critical load (or) Buckling load (or) crippling load.



1) consider a column R.S of length $\Rightarrow l$ and uniform cross sectional area $\Rightarrow A$ carrying load $\Rightarrow P$ at R & S

2) The column is hinged at both of its end R & S.

3) The moment due to the distance x from the end S. let y be the deflection at the section

4) Moment due to crippling load the section is given by.

Moment = load x distance

$$M = -(P \times y)$$

(-) indicate the concavity

General Bending Moment equation

$$EI \frac{d^2y}{dx^2} = M.$$

sub M in general equation

$$EI \frac{d^2y}{dx^2} = -Py.$$

$$EI \frac{d^2y}{dx^2} + Py = 0$$

$\div EI$

$$\frac{d^2y}{dx^2} + \frac{P}{EI} y = 0$$

The sln of above differential eqn is

$$y = A \cos\left(x \sqrt{P/EI}\right) + B \sin\left(x \sqrt{P/EI}\right)$$

At point S,

$x = 0 \; ; \; y = 0$

$0 = A\cos\left(0\sqrt{P/EI}\right) + B\sin\left(0\sqrt{P/EI}\right)$

$0 = A\cos(0) + B\sin(0)$

$0 = A + 0$ $\quad A = 0$

At point 'R'

$x = l \; ; \; y = 0$

$0 = A\cos\left(l\sqrt{P/EI}\right) + B\sin\left(l\sqrt{P/EI}\right)$

$= 0 \times \cos\left(l\sqrt{P/EI}\right) + B\sin\left(l\sqrt{P/EI}\right)$

$0 = 0 + B\sin\left(l\sqrt{P/EI}\right)$

$0 = B\sin\left(l\sqrt{P/EI}\right)$

$\sin\left(l\sqrt{P/EI}\right) = 0$

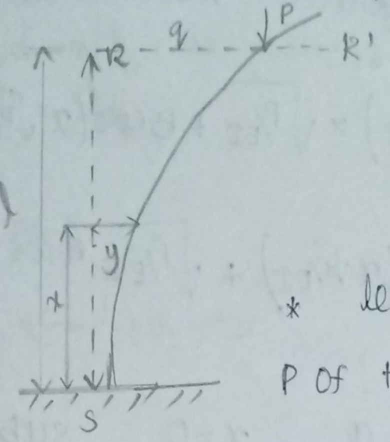
$l\sqrt{P/EI} = \sin^{-1}(0)$

$l\sqrt{P/EI} = \pi$

$l^2\left(P/EI\right) = \pi^2$

$$\boxed{P = \frac{\pi^2 EI}{l^2}} \Rightarrow$$

column with one end fixed other end free



* let due to crippling load P of the column just buckle.

* let a be the deflection of the top end.

Taking Moment = load x distance

$$M = P \times (a - y)$$

$$M = Pa - Py$$

$$M = EI \frac{d^2y}{dx^2}$$

$$EI \frac{d^2y}{dx^2} = Pa - Py$$

$\div EI$

$$\frac{d^2y}{dx^2} = \frac{P}{EI} a - \frac{P}{EI} y$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI} y = \frac{P}{EI} a$$

The solution of above differential eqn is

$$y = A \cos\left(x\sqrt{P/EI}\right) + B \sin\left(x\sqrt{P/EI}\right) + a$$

Applying the endi condition at point "S"

$x = 0$; $y = 0$

$$y = A\cos\left(x\sqrt{P/EI}\right) + B\sin\left(x\sqrt{P/EI}\right) + a \rightarrow (1)$$

Orenaal eaun

$$0 = A\cos\left(0\sqrt{P/EI}\right) + B\sin\left(x\sqrt{P/EI}\right) + a$$

$$= A\cos 0 + B(0) + a \Rightarrow 0 = A + 0 + a \Rightarrow \boxed{A = -a}$$

differentiate the general eqn(1) w.r.t. to 'x'

the slope at any section is given by

$$\frac{dy}{dx} = -A \sin\left(x\sqrt{P/EI}\right) \times \sqrt{P/EI} + B\cos\left(x\sqrt{P/EI}\right) \times \sqrt{P/EI}$$

$$\frac{dy}{dx} = -\sqrt{P/EI}\, A \sin\left(x\sqrt{P/EI}\right) + \sqrt{P/EI}\, B\cos\left(x\sqrt{P/EI}\right) \longrightarrow (2)$$

$\frac{dy}{dx} = 0$; $A = -a$; $x = 0$ sub in eqn (2)

$$0 = \sqrt{P/EI}\, a \cancel{\sin\left(0\sqrt{P/EI}\right)} + \sqrt{P/EI}\, B\cos\left(0\sqrt{P/EI}\right)$$

$$0 = \sqrt{P/EI}\, B$$

$B = 0$ $\sqrt{P/EI} \neq 0$

$A = -a$; $B = 0$ sub in general eqn

$$y = -a\cos\left(x\sqrt{P/EI}\right) + \cancel{B \sin\left(x\sqrt{P/EI}\right)} + a$$

$$y = -a\cos\left(x\sqrt{P/EI}\right) + a \longrightarrow (3)$$

At a point 'R' $x = l$; $y = a$ → sub in $(3)^{rd}$ eqn

$$a = -a \cos\left(l\sqrt{P/EI}\right) + a$$

$$a + a\cos\left(l\sqrt{P/EI}\right) = a$$

$$a\cos\left(l\sqrt{P/EI}\right) = a - a$$

$$a\cos\left(l\sqrt{P/EI}\right) = 0$$

$a \neq 0$ apply the condition in above eqn

$$\cos\left(l\sqrt{P/EI}\right) = 0$$

$l\sqrt{P/EI} = \cos^{-1}(0)$

$\left(l\sqrt{P/EI}\right) = \pi/2$

$\sqrt{P/EI} = \pi/2l$

Squaring on both side

$$P/EI = \frac{\pi^2}{4l^2}$$

$$P = \frac{\pi^2 E.I}{4l^2}$$

column with both ends are fixed:-

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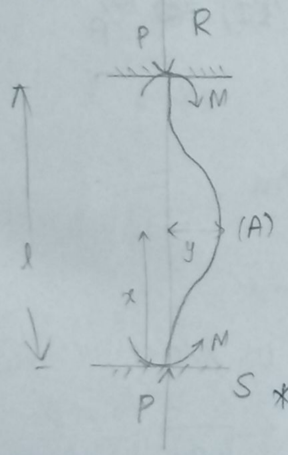


consider a column R,S of length l and uniform cross sectional area a carrying a vertical load P at both ends R&S the column is fixed at both of its ends R&S due to application of load P the column will deflect as shown in figure.

Taking moment at point A

$M = M - Py \rightarrow (1)$ (-ve indicate concavity)

$$M = EI\frac{d^2y}{dx^2} \rightarrow (2)$$

equating (1) and (2)

$$EI\frac{d^2y}{dx^2} = M - Py$$

dividing the equation by EI

$$\frac{d^2y}{dx^2} = M/EI - P/EI\, y$$

$$\frac{d^2y}{dx^2} + P/EI\, y = M/EI \longrightarrow (3)$$

÷ & × with (P) on RHS

$$\frac{d^2y}{dx^2} + P/EI\, y = M/EI \times P/P$$

$$\frac{d^2y}{dx^2} + P/EI = P/EI \times M/P$$

The general equation

$$y = A\cos\left(x\sqrt{P/EI}\right) + B\sin\left(x\sqrt{P/EI}\right) + M/P$$

At a point "s" $x = 0$; $y = 0$; $\frac{dy}{dx} = 0$;

$$0 = A\left(\cos 1 0\sqrt{P/EI}\right) + B\sin \cancel{\left(0\sqrt{P/EI}\right)}^{0}\ M/P$$

$$0 = A\cos(0) + M/P$$

$$A = -M/P$$

differentiate the general eqn w. r to x;

$$\frac{dy}{dx} = -A\sin\left(x\sqrt{P/EI}\right)\sqrt{P/EI} + B\cos\left(x\sqrt{P/EI}\right)\sqrt{P/EI}$$

$x=0$,

$$\frac{dy}{dx} = -\sqrt{P/EI}\,A\sin\left(x\sqrt{P/EI}\right) + \sqrt{P/EI}\,B\cos\left(x\sqrt{P/EI}\right)$$

$$0 = +\sqrt{P/EI}\,\cancel{M/P\left(\sin(0)\sqrt{P/EI}\right)}^{0} + \sqrt{P/EI}\,B\cos\left((0)\sqrt{P/EI}\right)$$

$$0 = \sqrt{P/EI}\,B$$

$B = 0$ $\sqrt{P/EI} \neq 0$

$$B\sqrt{P/EI} = 0$$

apply this condition in ...

$$y = -M/P \cos\left(x\sqrt{P/EI}\right) + 0 \times \sin\left(x\sqrt{P/EI}\right) + M/P$$

$$y = -M/P \cos\left(x\sqrt{P/EI}\right) + M/P \rightarrow (4)$$

considering the point R hence the value of x & y is $x = l$; $y = 0$

sub in eqn (4)

$$0 = -M/P \cos\left(l\sqrt{P/EI}\right) + M/P$$

$$\cancel{M/P} \cos\left(l\sqrt{P/EI}\right) = \cancel{M/P}$$

$$\cos l\left(\sqrt{P/EI}\right) = 1$$

$$l\sqrt{P/EI} = \cos^{-1}(1)$$

$$l\sqrt{P/EI} = 2\pi$$

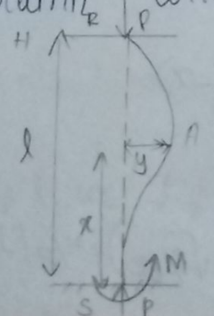
$$\sqrt{P/EI} = 2\pi/l$$

squaring on both side to cancell the root

$$P/EI = \frac{4\pi^2}{l^2}$$

$$P = \frac{4\pi^2 EI}{l^2}$$

4) column with one end fixed other end hinged



consider a column R,S of length l and uniform cross sectional area a carrying a critical load P at both ends R&S

* the column is fixed at end's and hinged at end R

* due to application of load P, the column will deflection as shown in fig.

* There will be horizontal reaction H at R as shown in fig

Moment at a point A = $\underline{-Py + H(l-x)}$ $\rightarrow (1)$

moment due to critical load + Moment due to horizontal reaction at 'R'

(-) the deflection distance

$$M = -Py + H(l-x)$$

$$M = EI \frac{d^2y}{dx^2} \rightarrow (2)$$

equating (1) & (2)

$$EI \frac{d^2y}{dx^2} = -Py + H(l-x)$$

÷ EI the above eqn

$$\frac{d^2y}{dx^2} = \frac{-P}{EI} y + \frac{H}{EI}(l-x)$$

× & ÷ by P/P on RHS

$$\frac{d^2y}{dx^2} + P/EI\, y = H/EI\,(l-x) \times P/P$$

$$\frac{d^2y}{dx^2} + P/EI\, y = P/EI \times H/P\,(l-x)$$

The general eqn is

$$y = A \cos(x\sqrt{P/EI}) + B \sin(x\sqrt{P/EI}) + H/P(l-x)$$

At a point S the values of x & y are

$x = 0;\ y = 0$ sub the values in general eqn.

$$0 = A\cos\left(0\sqrt{P/EI}\right) + B\cancel{\sin\left(0\sqrt{P/EI}\right)}^{\to 0} + H/P\,(l-0)$$

$$0 = A\cos(0) + H/P(l)$$

$$0 = A + H/P\,(l)$$

$$A = -H/P\,(l)$$

$H/P(l) - H/P(x)$

$H/P(l-x)$ (0) (1)

H/P

Diff the general eqn w.r.to x.

$$\frac{dy}{dx} = -A\sin\left(x\sqrt{P/EI}\right) \times \sqrt{P/EI} + B\cos\left(x\sqrt{P/EI}\right)\left(\sqrt{P/EI}\right) - H/P$$

$$\frac{dy}{dx} = -\sqrt{P/EI}\,A\sin\left(x\sqrt{P/EI}\right) + \sqrt{P/EI}\,B\cos\left(x\sqrt{P/EI}\right) - H/P$$

$$0 = \sqrt{P/EI} \times \cancel{H/P \sin\left(0\sqrt{P/EI}\right)}^{\to 0} + \sqrt{P/EI}\,B\cos\left(0\sqrt{P/EI}\right) - H/P$$

(0) (1)

$$0 = \sqrt{P/EI}\,B - H/P$$

$$H/P = B\sqrt{P/EI}$$

$$B = H/P\sqrt{EI/P}$$

sub A & b value in general eqn

$$y = -H/P\,l\cos\left(x\sqrt{P/EI}\right) + H/P\sqrt{EI/P}\sin\left(x\sqrt{P/EI} + H/P(l-x)\right)$$

at a point R $x = l;\ y = 0$ sub in above eqn

$$0 = -H/P\,l\cos\left(l\sqrt{P/EI}\right) + H/P\sqrt{EI/P}\sin\left(l\sqrt{P/EI}\right) + H/P(l-l)$$

$$0 = -H/P\,l\cos\left(l\sqrt{P/EI}\right) + H/P\sqrt{EI/P}\sin\left(l\sqrt{P/EI}\right)$$

$$\cancel{H/P}\,l\cos\left(l\sqrt{P/EI}\right) = \cancel{H/P}\sqrt{EI/P}\sin\left(l\sqrt{P/EI}\right)$$

$$l \cos\left(l\sqrt{P/EI}\right) = \sqrt{\frac{EI}{P}} \sin\left(l\sqrt{P/EI}\right)$$

$$\frac{l}{\sqrt{EI/P}} = \frac{\sin}{\cos}\left(l\sqrt{P/EI}\right)$$

$$l \times \sqrt{P/EI} = \tan\left(l\sqrt{P/EI}\right)$$

$$l \times \sqrt{P/EI} = 4.5 \text{ radia}$$

$$\sqrt{P/EI} = \frac{4.5}{l}$$

squaring on both side

$$P/EI = \frac{20.25}{l^2}$$

$$\boxed{P = \frac{2\pi^2 EI}{l^2}}$$

1. calculate the safe compressive load on hollow cas iron column (one end rigidy fixed & other hinged of 150 mm external dia meter, 100 mm internal dia and 10 m length. use euler's formula with a FOS of 5 and $E = 95\ GN/m^2$

$D = 150mm \Rightarrow 0.15\ m$

$d = 100mm \Rightarrow 0.1\ m$

$l = 10m$

$F.O.S = 5$

$E = 95\ GN/m^2 \Rightarrow 95 \times 10^9$

$$P = \frac{2\pi^2 EI}{l^2}$$

$74.8\ KN$

$$I = \pi/64\ (D^4 - d^4)$$

$$I = \pi/64\ (0.15^4 - 0.1^4) \Rightarrow 1.994 \times 10^{-5} \Rightarrow$$

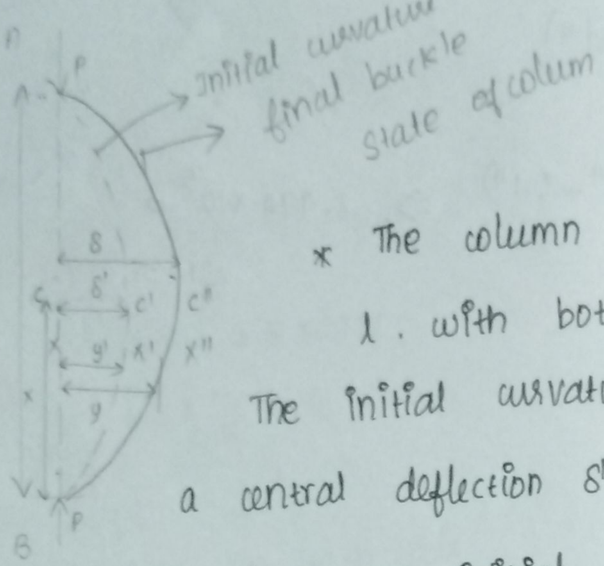
$P_{cr} \Rightarrow$

$$S = \frac{P_{cr}}{FOS}$$

Q. A slender pin ended aluminium column 1.8 m long and of circular cross section is to have an out dia 50 mm calculate the necessary internal dia to prevent failure by buckling of the actual load is applied 13.6 KN and P_{cr} applied is twice the actual load take E = for alumi ⇒ 70 GPa

$P_{cr} = 2 \times \text{actual}$

Column with initial curvature (Axial loading):



* The column AB with length l. with both ends pinned. The initial curvature AC'B having a central deflection δ'

* let y' be the initial deflection at a distance x from the end B.

* Assume the sine curve for initial profile of the centre of the column.

$$y' = \delta' \sin \frac{\pi x}{l} \longrightarrow (1)$$

diff w.r.to 'x'

$$\frac{dy'}{dx} = \delta' \cos \frac{\pi x}{l} \cdot \pi/l$$

$$\frac{dy'}{dx} = \pi/l \cdot \delta' \cos \frac{\pi x}{l}$$

Again diff w.r.to 'x'

$$\frac{d^2y'}{dx^2} = \pi/l \cdot \delta' - \sin \pi x/l \cdot \pi/l \qquad 1/C_1$$

$$\frac{d^2y'}{dx^2} = -\pi^2/l^2 \cdot \delta' \sin \frac{\pi x}{l} \longrightarrow (2)$$

The load of the column reaches the

critical value P. The column will deflect

$AC''B''$ so that the deflection x changes from

y' to y due to BM Py

~~Where~~

Moment at a point R

$M = -Py$

$M = EI \frac{d^2y}{dx^2}$

$EI \frac{d^2y}{dx^2} = -Py$

$EI \frac{d^2y}{dx^2} + Py = 0$

$\div EI$ both side ($\div$ EI)

$\frac{d^2y}{dx^2} + P/EI \; y = 0$

where $(y-y')$ is the initial & final condition

$\frac{d^2(y-y')}{dx^2} + P/EI \; y = 0$

$\frac{d^2y}{dx^2} - \frac{d^2y'}{dx^2} + P/EI \; y = 0$ → (3)

$\frac{d^2y}{dx^2} + P/EI \; y = \frac{d^2y'}{dx^2}$

$\frac{d^2y}{dx^2} + P/EI \; y = -\pi^2/l^2 \, \delta' \sin \pi x/l$

The sln above diff eqn form

$y = c \, \delta' \sin \pi x/l$

Diff w.r.to x

$$\frac{dy}{dx} = c\,\delta' \cos\frac{\pi x}{l}\cdot \pi/l$$

$$\frac{dy}{dx} = \pi/l\; c\,\delta' \cos \pi x/l$$

Again diff. w. r. to x

$$\frac{d^2y}{dx^2} = -\pi/l \cdot c\,\delta' \sin \pi x/l \cdot \pi/l$$

$$\frac{d^2y}{dx^2} = -\pi^2/l^2\; c\,\delta' \sin\frac{\pi x}{l}$$

$$-\pi^2/l^2\, c\,\delta' \sin\frac{\pi x}{l} + P/EI\, y = -\pi^2/l^2\,\delta' \sin \pi x/l$$

$$P/EI\, y = -\pi^2/l^2\,\delta' \sin\frac{\pi x}{l} + \pi^2/l^2\, c\,\delta' \sin\frac{\pi x}{l}$$

$$P/EI\,(c\,\cancel{\delta' \sin \pi x/l}) = \pi^2/l^2\,\cancel{\delta' \sin\frac{\pi x}{l}}\,(c-1)$$

$$P/EI\; c = \pi^2/l^2\,(c-1)$$

$$P_c = \frac{\pi^2 EI}{l^2}\,(c-1) \Rightarrow \underline{\text{Peuler's}} \text{ for both end hinged}$$

$$P \times C = \frac{Pe}{}\,(c-1)$$

$$P\times C = Pe\,C - Pe$$

$$Pe = Pe\,C - Pc$$

$$Pe = (Pe - P)\,C$$

$$C = \frac{Pe}{Pe - P}$$

$$y = \frac{P_e}{P_e - P} \delta' \sin \pi x/l \rightarrow (4)$$

The maximum deflection at the mid point (centre)

$x = l/2 \quad y = \delta$ (at centre point)

$$\delta = \frac{P_e}{P_e - P} \delta' \frac{\sin \pi (\cancel{l}/2)}{\cancel{2l}}$$

$$\delta = \frac{P_e}{P_e - P} \delta' \sin \pi/2 \qquad \sin \pi/2 (1)$$

$$\delta = \frac{P_e}{P_e - P} \delta'$$

maximum deflection at mid point

$$M = P\delta$$

$$M = P\left(\frac{P_e}{P_e - P}\right)\delta'$$

Maximum compressive stress

$$\sigma_{max} = \sigma_d + \sigma_b$$

$K = \sqrt{I/A}$

$I = AK^2$

$\sigma_d \Rightarrow$ direct stress $\Rightarrow P/A$

$\sigma_b \Rightarrow$ bending stress $M/I \, y$

$$\sigma_{max} = P/A + \frac{P\delta' \left(\frac{P_e}{P_e - P}\right) y}{I \; (AK^2)}$$

$$= P/A\left[1 + \frac{\delta'\left(\frac{P_e}{P_e - P}\right) y}{I}\right]$$

=

1) A Steel strud as on out dia 180 mm inside dia 120 mm is 6m long. It is hinged at both ends and is initially bend. Assuming the centre line of strud sinusoidal with max deviation of 9mm, det the max stress developed due to an axial load of 150 KN. Take $E = 208G\ N/m^2$

Given:-

$D = 150\ mm = 0.15m$

$d = 120mm = 0.12m$

$l = 6m$

$\delta' = 9mm = 0.09$

$P = 150KN = 150 \times 10^3\ N$

$E = 208 \times 10^9\ N/m^2$

$\sigma_{max} = P/A + M/I\, y$

$A = \pi/4\ (D^4 - d^4) = \pi/4\ (0.18^2 - 0.12^2) = 0.014 m^2$

$I = \pi/64\ (D^4 - d^4) = 4.135 \times 10^{-5}\ m^4$

$P_e = \frac{\pi^2 EI}{l^2} = 2.357 \times 10^6 N$ (or) 2.357×10^3 KN

$y_c = D/2 = 0.18/2 = 0.09m$

$\sigma_{max} = P/A + P\delta' \left(\frac{P_e}{P_e - P} / I \right) y_c$

$= 10714.28 + 3138.036$

$= 13852.32\ KN/m^2$

$= 13.85\ MN/m^2$

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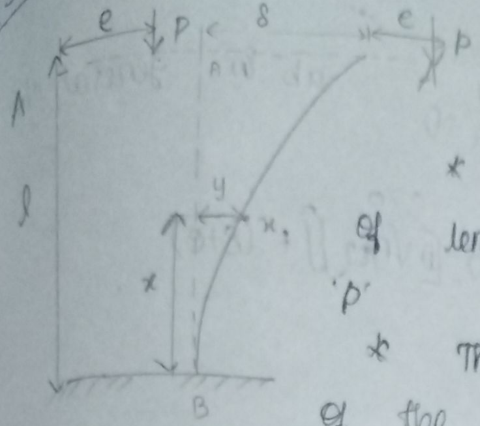
column with eccentric



* consider a column AB of length l. sub to eccentric load 'P'

* The eccentricity e as assume of the column is one end is fixed other end free.

$y \rightarrow$ deflection at any section x distance x from B (fixed end)

$\delta \rightarrow$ deflection at a, The BM at the section is given by

$$M = P(\delta + e - y)$$

$$M = P\delta + Pe - Py$$

$$M = P(\delta + e) - Py$$

$$M = EI \frac{d^2y}{dx^2}$$

$$EI \frac{d^2y}{dx^2} = P(\delta + e) - Py$$

$$EI \frac{d^2y}{dx^2} + Py = P(\delta + e)$$

$\div EI$

$$\frac{d^2y}{dx^2} + \frac{P}{EI} y = \frac{P}{EI}(\delta + e)$$

The general sln is

$$y = A\cos\left[x\sqrt{P/EI}\right] + B\sin\left[x\sqrt{P/EI}\right] + (\delta + e)$$

At point B

$x=0$; $y=0$; $\frac{dy}{dx}=0$ sub in general s/n

$0 = A\cos[0\sqrt{P/EI}] + B[\cancel{\sin[0\sqrt{P/EI}]}] + (\delta+e)$

$0 = A\cos(0) + \delta+e$

$A = -(\delta+e)$

Differentiate w.r to x (general eqn)

$\frac{dy}{dx} = -A\sin(x\sqrt{P/EI})\times(\sqrt{P/EI}) + B\cos(x\sqrt{P/EI})\times\sqrt{P/EI}$

$0 = -\sqrt{P/EI}\,A\sin(x\sqrt{P/EI}) + \sqrt{P/EI}\,B\cos(x\sqrt{P/EI})$

$0 = \sqrt{P/EI}\,(\delta+e)\cancel{\sin(0\sqrt{P/EI})} + \sqrt{P/EI}\cos(0\sqrt{P/EI})$

$0 = \sqrt{P/EI}\,B\cos(0)$

$0 = \sqrt{P/EI}\,B$ $\qquad B=0;\ \sqrt{P/EI}\neq 0$

At a point A

$x=l$; $y=\delta$ sub in General eqn

$y = -(\delta+e)\cos(x\sqrt{P/EI}) + 0\times\cancel{\sin(x\sqrt{P/EI})} + (\delta+e)$

$\delta = -(\delta+e)\cos(l\sqrt{P/EI}) + (\delta+e)$

$(\delta+e)\cos(l\sqrt{P/EI}) = \delta+e-\cancel{\delta}$

$(\delta+e)\cos(l\sqrt{P/EI}) = e$

$(\delta+e) = \frac{e}{\cos(l\sqrt{P/EI})} \Rightarrow (\delta+e) = e.\sec(l\sqrt{P/EI})$

$e/\delta+e = \cos l\sqrt{P/EI}$

$1/\cos$

The general sln is

$$y = A\cos\left(x\sqrt{P/EI}\right) + B\sin\left(x\sqrt{P/EI}\right) + (\delta+e)$$

$y=0;\ x=0$

$$0 = A\cos\left(0\sqrt{P/EI}\right) + B\cancel{\sin\left(0\sqrt{P/EI}\right)}^{\to 0} + (\delta+e)$$

$$0 = A\underset{(1)}{\cos(0)} + (\delta+e)$$

$$A = -(\delta+e)$$

Diff the general eqn w.r.to x

$$\frac{dy}{dx} = -A\sin\left(x\sqrt{P/EI}\right)\left(\sqrt{P/EI}\right) + B\cos\left(x\sqrt{P/EI}\right)$$

$$0 = \sqrt{P/EI}\left(\cancel{(\delta+e)\sin(0)}^{\to 0}\right) + B\cos\left(0\sqrt{P/EI}\right)$$

$$0 = B\sqrt{P/EI}$$

$B = 0 \qquad \sqrt{P/EI} \neq 0$

At a point A $\qquad x=l;\ y=\delta \qquad$ sub in gnrl eqn

$$y = -(\delta+e)\cos\left(x\sqrt{P/EI}\right) + 0\times\cancel{\sin\left(x\sqrt{P/EI}\right)} + (\delta+e)$$

$x = l;\qquad y=\delta$

$$\delta = -(\delta+e)\cos\left(l\sqrt{P/EI}\right) + 0 + (\delta+e)$$

$$\delta = -(\delta+e)\cos\left(l\sqrt{P/EI}\right) + (\delta+e)$$

$$(\delta+e)\cos\left(l\sqrt{P/EI}\right) = \cancel{\delta}+e-\cancel{\delta}$$

$$(\delta+e)\cos\left(l\sqrt{P/EI}\right) = e$$

$$\cos l\sqrt{P/EI} = e/\delta+e \qquad \boxed{\delta = e}$$

$$\cos l\sqrt{P/EI} = e/\delta+e \qquad \Rightarrow \cos l\sqrt{P/EI} = \frac{e}{2e}$$

$\cos l\sqrt{P/EI} = 1/2$

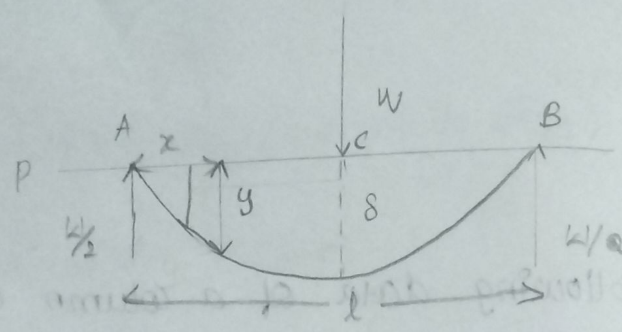
$l\sqrt{P/EI} = \cos^{-1}(1/2)$

$l\sqrt{P/EI} = \pi/3$

$l = \pi/3\sqrt{EI/P}$

2) From the following data of a column of circular section calculate the extreme stress on the column section. Also find the max eccentricity in order that there may be no tension anywhere on the section. exter dia = 20 cm. I, d = 16 cm Length 4 m load carried by the column 200 KN eccentricity of load 2.5 cm (from the axis of the column) end condition both end is fixed. $E = 94\ GN/m^2$

Beam Column:-



column having transverse load in addition to the axial compressive load are termed as b.c

case: 1: strut pinned at both ends and sub to axial thrust P, and a transverse point load W at the center.

Euler's Bernoulli's Bending equation:-

$$\frac{d^2y}{dx^2} + \frac{M}{EI}y = 0$$

$$\frac{d^2y}{dx^2} = -\frac{M}{EI}$$

$$EI\frac{d^2y}{dx^2} = -M \quad (-\text{ concavity})$$

$$(-Py + R_A x)$$

$$M = -\left(Py + \frac{W}{2}x\right)$$

$$EI\frac{d^2y}{dx^2} = -Py - \frac{W}{2}x$$

$\div EI$

$$\frac{d^2y}{dx^2} = -\frac{P}{EI}y - \frac{W}{2EI}x$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI} \cdot y = -\frac{W}{2EI} x \times \frac{P}{P}$$ mult P/P in RHS.

$$\frac{d^2y}{dx^2} + \frac{P}{EI} y = -\frac{P}{EI} \times \frac{W}{2P} x$$

$\frac{-W}{2EI}(x) \times \frac{P}{P}$

The general sln:-

$\Rightarrow -\frac{P}{EI} \times \frac{W}{2P} x$

$$y = A \cos\left(x\sqrt{P/EI}\right) + B \sin\left(x\sqrt{P/EI}\right) - \frac{W}{2P}(x) \rightarrow (1)$$

At point 'A' $x=0$; $y=0$

$$0 = A\left(\cos\left(0\sqrt{P/EI}\right)\right) + B \sin\left(0\sqrt{P/EI}\right) - \frac{W}{2P}x$$

$$0 = A + B(0) - \frac{W}{2P}(0)$$

$\boxed{A=0}$

diff the general eqn w.r.to 'x'

$$\frac{dy}{dx} = -A \sin\left(x\sqrt{P/EI}\right)\sqrt{\frac{P}{EI}} + B\cos\left(x\sqrt{P/EI}\right)\left(\sqrt{P/EI}\right) - \frac{W}{2P} \rightarrow (2)$$

$$\frac{dy}{dx} = -\sqrt{\frac{P}{EI}}\, A \sin\left(x\sqrt{P/EI}\right) + \sqrt{\frac{P}{EI}}\, B \cos\left(x\sqrt{P/EI}\right) - \frac{W}{2P}$$

sub $A=0$ $\quad dy/dx = 0 \quad x = l/2$ in eqn (2)

$$0 = \cancel{-\sqrt{P/E}\,(0) \cdot \sin(l/2)\sqrt{P/EI}} + \sqrt{P/EI}\, B \cos l/2 \sqrt{P/EI} - \frac{W}{2P}$$

$$0 = B\sqrt{P/EI} \cos l/2 \sqrt{P/EI} - \frac{W}{2P}$$

$$\frac{W}{2P} = B\sqrt{P/EI} \cos\left(l/2 \sqrt{P/EI}\right)$$

$$B = \frac{W}{2P}\sqrt{\frac{EI}{P}} \sec\left(l/2 \sqrt{P/EI}\right)$$

$$y = \cancel{0 \cos\left(x\sqrt{P/EI}\right)} + \frac{W}{2P}\sqrt{\frac{EI}{P}} \sec(l/2)\sqrt{P/EI}) \sin\left(x\sqrt{P/EI}\right) - \frac{W}{2P}x$$

$$y = \frac{w}{2P}\sqrt{\frac{EI}{P}}\ \sec\left(\frac{l}{2}\sqrt{P/EI}\right) \cdot \sin\left(x\sqrt{P/EI}\right) - \frac{w}{2P}x$$

Maximum deflection :-

Maximum deflection at the center. $x = l/2$ $y = \delta$

$$\delta = \frac{w}{2P}\sqrt{EI/P}\ \sec\left(\frac{l}{2}\sqrt{P/EI}\right) \cdot \sin\left(\frac{l}{2}\sqrt{P/EI}\right) - \frac{w}{2P}\left(\frac{l}{2}\right)$$

$$\delta = \frac{w}{2P}\sqrt{EI/P} \cdot \frac{1}{\cos}\left(\frac{l}{2}\sqrt{P/EI}\right) \cdot \sin\left(\frac{l}{2}\sqrt{P/EI}\right) - \frac{w}{4P}(l)$$

$$\frac{\sin \frac{l}{2}\sqrt{P/EI}}{\cos \frac{l}{2}\sqrt{P/EI}}$$

$$\delta = \frac{w}{2P}\sqrt{EI/P}\ \tan\left(\frac{l}{2}\sqrt{P/EI}\right) - \frac{w}{4P}(l)$$

Maximum Bending Moment:-

$$M_{max} = P \times \delta + \frac{w}{2} \times \frac{l}{2}$$

$$= P\delta + wl/4$$

$$= P\left[\frac{w}{2P}\sqrt{EI/P} \cdot \tan\left(\frac{l}{2}\sqrt{P/EI}\right) - \frac{wl}{4P}\right] + \frac{wl}{4}$$

$$= \frac{w}{2}\sqrt{EI/P}\ \tan\left(\frac{l}{2}\sqrt{P/EI}\right) - \cancel{\frac{wl}{4}} + \cancel{\frac{wl}{4}}$$

$$= \frac{w}{2}\sqrt{EI/P}\ \tan\left(\frac{l}{2}\sqrt{P/EI}\right)$$

$$\sigma_{max} = \sigma_d + \sigma_b$$

$$= P/A + M/I\ y$$

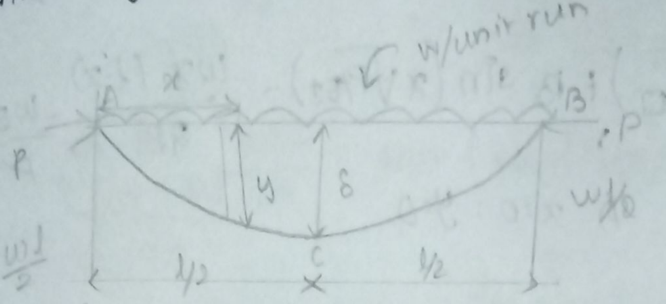
$$= P/A + \frac{\frac{w}{2}\sqrt{EI/P}\ \tan\left(\frac{l}{2}\sqrt{P/EI}\right)}{I}\ y$$

$I = AK^2$

$K = \sqrt{I/A}$

$K^2 = I/A$

Case ii) Strut pinned at both ends and sub to an axial thrust P, and a lateral uniformly distributed load of intensity w/unit run



Euler's Bernoulli's Bending equation:-

$$\frac{d^2y}{dx^2} + M/EI = 0$$

$$\frac{d^2y}{dx^2} = -M/EI.$$

$$EI\frac{d^2y}{dx^2} = -M$$

$$EI\frac{d^2y}{dx^2} = -\left[Py + \frac{wl}{2}x - wx.x/2\right]$$

$$EI\frac{d^2y}{dx^2} = -Py - \frac{wlx}{2} + wx^2/2$$

$$= -Py - \frac{wx}{2}(l-x)$$

$$EI\frac{d^2y}{dx^2} = -Py - \frac{wx}{2}(l-x).$$

÷EI on both side

$$\frac{d^2y}{dx^2} = -P/EI\, y - \frac{wx}{2EI}(l-x)$$

P/P on RHS:-

$$\frac{d^2y}{dx^2} + P/EI\ y = \frac{-Pw x}{2PEI}(l-x) \rightarrow \frac{wEI}{P} \quad \frac{wxl}{2P} - \frac{wx^2}{2P}$$

The general sln of above equation is

$$y = A\cos\left(x\sqrt{P/EI}\right) + B\sin\left(x\sqrt{P/EI}\right) - \frac{wx(l-x)}{2P} - \frac{wEI}{P^2}$$

$$\frac{wlx}{2P} - \frac{wx^2}{2P}$$

At point A $x=0;\ y=0$

$$y = A\cos\left(0\sqrt{P/EI}\right) + B\sin\left(0\sqrt{P/EI}\right) - \frac{w(0)(l-0)}{2P} - \frac{wEI}{P^2}$$

$$y = A(1) + 0 - 0 - wEI/P^2.$$

$$A = wEI/P^2$$

Diff the gen equan w.r.to x

$$\frac{dy}{dx} = -A\sin\left(x\sqrt{P/EI}\right)\left(\sqrt{P/EI}\right) + B\cos\left(x\sqrt{P/EI}\right)\left(\sqrt{P/EI}\right) - \frac{wl}{2P} + \frac{\cancel{2}wx}{\cancel{2}P} = 0$$

$$\frac{dy}{dx} = -A\left(\sqrt{P/EI}\right)\sin\left(x\sqrt{P/EI}\right) + B\sqrt{P/EI}\cos\left(x\sqrt{P/EI}\right) - \frac{wl}{2P} + \frac{wx}{P}.$$

$$\frac{dy}{dx} = 0;\quad x = l/2;\quad A = wEI/P^2$$

$$0 = -wEI/P^2\left(\sqrt{P/EI}\right)\sin\left(l/2\sqrt{P/EI}\right) + B\sqrt{P/EI}\cos\left(l/2\sqrt{P/EI}\right) - \cancel{\frac{wl}{2P}} + \cancel{\frac{wl}{2P}}$$

$$\frac{WEI}{P^2}\cancel{\sqrt{P/EI}}\sin(l/2\sqrt{P/EI}) = B\cancel{\sqrt{P/EI}}\cos(l/2\sqrt{P/EI}).$$

$$B = \frac{WEI}{P2}\cdot\frac{\sin(l/2\sqrt{P/EI})}{\cos(l/2(\sqrt{P/EI}))}$$

$$B = \frac{WEI}{P^2}\tan(l/2\sqrt{P/EI})$$

$$y = WEI/P^2\cos(x\sqrt{P/EI}) + \frac{WEI}{P^2}\tan(l/2\sqrt{P/EI})$$

$$\sin(x\sqrt{P/EI}) - \frac{wx(l-x)}{2P} - WEI/P^2$$

$$x = l/2 \quad y = \delta$$

$$\delta = \frac{WEI}{P^2}\cos(l/2\sqrt{P/EI}) + \frac{WEI}{P^2}\tan(l/2\sqrt{P/EI})$$

$$\sin(l/2\sqrt{P/EI}) - \frac{w(l/2)(l-l/2)}{2P} - \frac{WEI}{P^2}$$

$$\delta = \frac{WEI}{P^2}\left[\cos(l/2\sqrt{P/EI}) + \tan(l/2)(\sqrt{P/EI})\sin(l/2\sqrt{P/EI})\right]$$

$$- wl^2/8P - \frac{WEI}{P^2}$$

$$\delta = \frac{WEI}{P^2}\left[\frac{\cos(l/2)\sqrt{P/EI}}{1} + \frac{\sin(l/2)(\sqrt{P/EI})}{\cos(l/2)\sqrt{P/EI}}\sin(l/2)(\sqrt{P/EI}) - wl^2/8P - WEI/P^2\right]$$

$$\delta = \frac{WEI}{P^2}\left[\frac{\cos^2(l/2)(\sqrt{P/EI}) + \sin^2(l/2)(\sqrt{P/EI})}{\cos(l/2)(\sqrt{P/EI}}\right] - \frac{wl^2}{8P} - \frac{WEI}{P^2}$$

$$\delta = \frac{wEI}{P^2}\left[\frac{1}{\cos(l/2)(\sqrt{P/EI})} - 1\right] - \frac{wl^2}{8P}$$

$$\delta = \frac{wEI}{P^2}\left(\sec\left(l/2\sqrt{P/EI}\right) - 1\right] - \frac{wl^2}{8P}$$

Maximum Bending Moment:

(w) udl

$$M_{max} = P \times \delta + \frac{wl}{2} \times l/2 - \frac{wl}{2}(l/4)$$

$$= P\delta + \frac{wl^2}{4} - \frac{wl^2}{8}$$

$$\cancel{= P\left[\frac{wEI}{P^2}\left(\sec\left(l/2\sqrt{P/EI}\right)-1\right] - \frac{wl^2}{8P} + \frac{wl^2}{4}\right]}$$

$$\cancel{= \frac{wEI}{P}\left(\sec\left(l/2\sqrt{P/EI}\right) - 1\right) \frac{wl^2}{8}}$$

$$= P\delta + \frac{8wl^2 - 4wl^2}{32}$$

$$= P\delta + 4wl^2/32$$

$$= P\delta + \frac{wl^2}{8}$$

$$= P\left[\frac{wEI}{P^2}\left(\sec l/2\sqrt{P/EI} - 1\right) - \frac{wl^2}{8P}\right] + \frac{wl^2}{8}$$

$$= \frac{wEI}{P}\left(\sec\left(l/2\sqrt{P/EI} - 1\right) - \cancel{\frac{wl^2}{8}} + \cancel{\frac{wl^2}{8}}\right.$$

$$M = \frac{wEI}{P}\left(\sec\left(l/2\sqrt{P/EI}\right) - 1\right)$$

$$\sigma_{max} = \sigma_d + \sigma_b$$

$$= P/A + M/I\, y$$

$$= P/A + \frac{\frac{wEI}{P}\left(\sec\left(l/2\sqrt{P/EI}\right) - 1\right) y_c}{I}$$

—— x ——

1) A rod 2m in length and of rectangular cross-section 88mm to 44mm is supported horizontally through pin joints. it carries a vertical load of 3.3KN/m length and axial thrust of 110 KN. If E = 208 KN/mm², calculate the maximum shear stress induced.

117.64 N/mm²

length = 2m

$A = b \times d$

$= 88 \times 44 = 3872 \text{ mm}^2$

$P =$ ~~[illegible]~~ 110 KN

$E = 208 \text{ KN/mm}^2$

$w = 3.3 \text{ KN/m}$

$$\sigma_{max} = P/A + \frac{wEI}{P} \frac{(\sec(l/2\sqrt{P/EI}) - 1)}{I} y_c$$

$\sec(l/2\sqrt{P/EI}) - 1$

$$\Rightarrow l/2\sqrt{P/EI} = \frac{2000}{2}\sqrt{\frac{110 \times 1000}{208 \times 10^3 \times 624682.67}}$$

$$= 0.9201 \times \frac{180}{\pi} = 52.7178$$

$\sec(52.7178) = 1.6509$

$$P/A = \frac{110 \times 100}{3872} = 28.4091$$

$$wE/P = \frac{3.3 \times 208 \times 10^3}{110 \times 10^3} = 6.2400$$

$$\sigma_{max} = 28.4091 + 6.24\,(1.6509 - 1)\,22$$

$= 117.7647 \text{ N/mm}^2$