

# SHEARFLOW IN CLOSED SECTION.

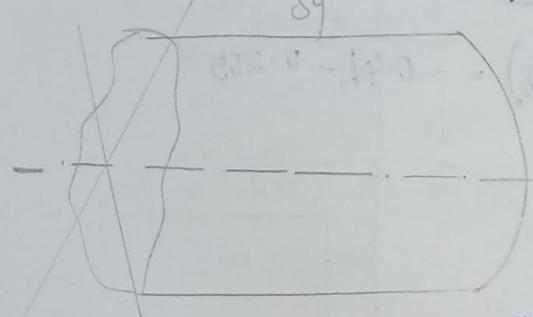
- consider the closed section beam of orbitally section shown in fig.

- The shear load  $\sigma_x$  &  $\sigma_y$  (or)  $V_x$  and  $V_y$  are applied through any point in the cross section. In general direct bending stress & shear flow which are related by equilibrium equation.

$$\frac{\partial \sigma}{\partial s} = t \frac{\partial \sigma_z}{\partial z}$$

we know that

$$\int_0^s \frac{\partial \sigma}{\partial s} ds = - \left[ \frac{8xI_{xx} - 8yI_{xy}}{I_{xx}I_{yy} - I_{xy}^2} \right] \int_0^s x + t ds - \left[ \frac{8yI_{yy} - 8xI_{xy}}{I_{xx}I_{yy} - I_{xy}^2} \right] \int_0^s y + t ds$$



let we choose an object for  $s$  where the shear flow unknown

value  $q_{s,0}$

Integrating the above law

$$q_{s,0} - q_{s,0} = - \left[ \frac{8xI_{xx} - 8yI_{xy}}{I_{xx}I_{yy} - I_{xy}^2} \right] \int_0^s t ds - \left[ \frac{8yI_{yy} - 8xI_{xy}}{I_{xx}I_{yy} - I_{xy}^2} \right] \int_0^s t ds$$

$$q_s = - \left[ \frac{8xI\alpha^2}{\pi} - \dots \right] + q_{so}$$

comparing this with shear flow of open section. The first two term on RHS represent the shear flow distribution in an open sec beam.

load at through initial shear center. This fact indicate the method of solving for the shear loaded closed sect beam.  $q_s = q_{st} + q_{so}$

We obtain  $q_{st}$  by supporting that the closed beam section is cut at same centroidal point thereby producing the open section

### BREDFTH - BATHO formula

$$q = Cxt$$

- let  $q$  be the shear flow per unit length moment of  $q$  about 'O' =  $q ds(OB)$

$$= q (\text{Area of } OAB)$$

Total moment about

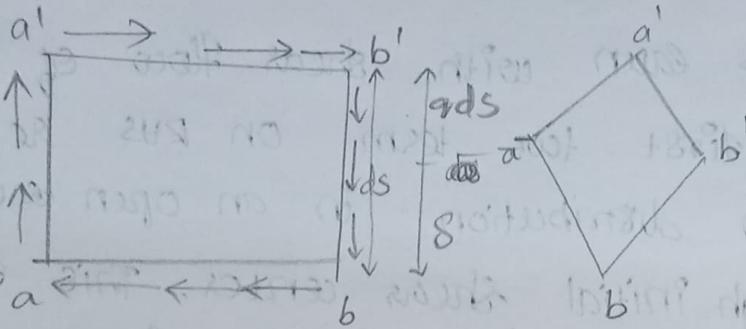
$$T = \frac{1}{2}qa (Oxy)$$

$$T = \frac{1}{2}Aa$$

The theory of torsion of closed beam section is known as breadth-batho theory

$$\frac{\text{abve}}{\text{F.O.A}}$$

## Angle of twist



- Consider a small plate element as shown in fig relatively a free body and length of the plate parallel to z axis and ds is the plane of tube cross section.
- Under the shear force the plate undergoes the deformation as shown in fig
- In phase a, a' move a distance  $\delta$  w.r.t b, b'
- The force of the phase  $aa' = qds$
- elastic strain energy is given by  $du = \frac{1}{2} q^2 ds \cdot \delta$

$$\delta = \frac{T}{GJ} \rightarrow \begin{array}{l} \text{shear stress} \\ \text{modulus of rigidity} \end{array}$$

(1)

$$q = T/t$$

$$\underline{T} = q/t \quad T = \alpha A q$$

$$\delta = \frac{\pi q t}{GJ}$$

$$q = T/\alpha A$$

$$\delta = \frac{T\ell}{GAGt}$$

$$du = \frac{Tqds}{HAGt}$$

$$du = \frac{T^2 ds}{8A^2 G It}$$

$$\alpha = \frac{T}{2A}$$

The above equation gives the elastic strain energy for small element from the entire cube integrating above equation.

$$V = \frac{T^2 ds}{8A^2 G It} \int ds$$

$$\beta = \frac{\partial u}{\partial T}$$

$$\frac{\partial u}{\partial T} = \frac{RT}{8A^2 G It} \int ds$$

$$= \frac{T}{4A^2 G It} \int ds$$

$$T = 2A\alpha$$

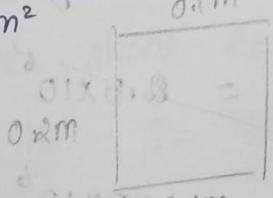
$$\beta = \frac{2A\alpha}{4A^2 G It} \rightarrow \beta = \frac{\alpha}{2AGIt} \int ds$$

2

1. calculate shear flow, shear centre, shear stress and twist per unit length

$$G_I = 25 \times 10^9 \text{ N/m}^2$$

$$T = 1000 \text{ NM}$$



$$A = 0.2 \times 0.1 = 0.02 \text{ m}^2$$

$$T = 2A\alpha$$

$$1000 = 2 \times (0.02) \alpha$$

$$1000 = 0.04 \alpha \quad \alpha = \frac{1000}{0.04} = 25000 \text{ N/m}$$

$$\beta = \frac{2500}{\frac{\alpha}{2AGIt} ds} \left[ 0.2 + 0.1 + 0.2 + 0.1 \right]$$

$$\& (0.02 \times 25 \times 10^9 \times 0.001)$$

$$\beta = 0.015 \text{ rad/sec.}$$

$$\vartheta = \tau/t$$

$$1 \text{ cm} = 10 \text{ mm}$$

$$\tau = \vartheta/t$$

$$\tau = 25000 \times 0.001$$

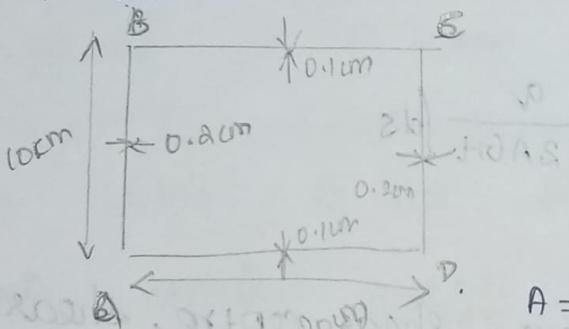
$$= 25 \times 10^6$$

2) calculate the shear flow, shear stress for given section  $20 \text{ cm} = 0.2 \text{ m}$

$$G = 25 \times 10^9 \text{ N/m}^2$$

$$10 \text{ cm} = 0.1 \text{ m}$$

$$T = 100 \text{ Nm}$$



$$\vartheta = \frac{T}{2A}$$

$$= \frac{100}{0.2 \times 0.01}$$

$$= \frac{100}{0.002} = 50000 \text{ N/m}$$

$$A = b \times d$$

$$= 0.20 \times 0.1 = 0.02 \text{ m}^2$$

$$\tau = \vartheta/t$$

$$A = 0.02 \times 0.01 = 0.0002 \text{ m}^2$$

$$\tau_{AB} = \tau_{BC} = \frac{2500}{0.001} = 2.5 \times 10^6 \text{ } (2.5 \times 10^5) \text{ N/m}^2$$

$$\tau_{AB} = \tau_{DC} = \frac{2500}{0.002} = 1.25 \times 10^6 \text{ } (1.25 \times 10^5) \text{ N/m}^2$$

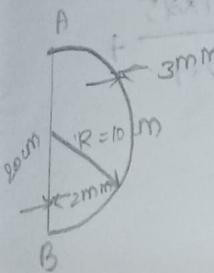
$$\beta = \frac{\vartheta}{2AGIt} ds$$

$$= \frac{2500}{2 \times 0.02 \times 25 \times 10^9 \times} \left[ \frac{0.1}{0.002} + \frac{0.2}{0.001} + \frac{0.1}{0.002} + \frac{0.3}{0.001} \right]$$

$$\beta = 1.85 \times 10^{-3} \text{ rad/sec.}$$

3) Find the shear stress, shear flow,  $\beta$ .

$$G = 30 \times 10^5 \text{ N/cm}^2 ; T = 6000 \text{ Nm}$$



$$A = \frac{\pi r^2}{\theta} = \frac{\pi \times 8^2}{30} = 157.07 \text{ cm}^2$$

$$\tau = \frac{q}{t} \quad q = \frac{T}{\theta A} = \frac{6000}{30 \times 157.07} = 19.09 \text{ N/cm}$$

$$T = q/t$$

$$TA = \frac{19.09}{0.3} = 63.66 \text{ N/m}^2$$

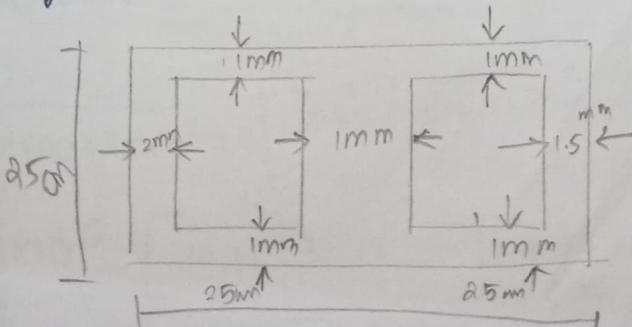
$$TB = \frac{19.09}{0.2} = 95.45 \text{ N/m}^2$$

$$\beta = \frac{q}{\theta A G t} ds$$

$$= \frac{19.09}{2 \times (157.07) (30 \times 10^5)} \left[ \frac{80}{0.2} + \frac{\pi r^2}{0.3} \right]$$

$$= 4.146 \times 10^{-6} \text{ rad/sec}$$

4) Find the shear flow and twist per unit length the  $T = 75000 \text{ Nm}$ .



$$A_1 = b_1 \times d_1 = 25 \times 25 = 625 \text{ cm}^2$$

$$A_2 = b_2 \times d_2 = 25 \times 25 = 625 \text{ cm}^2$$

$$75000 = 2 \times 625 q_1 + (2 \times 625) q_2 \rightarrow ①$$

$$\beta_1 = \frac{1}{1625 \times G} \left[ \frac{q_1 \times 25}{0.2} + \frac{q_1 \times 25}{0.1} + \frac{q_1 \times 25}{0.1} + \right.$$

$$\left. \frac{(q_1 - q_2) \times 25}{0.1} \right]$$

$$= \frac{1}{1250G} [125q_1 + 250q_1 + 250q_1 + 250q_1 - 250q_2]$$

$$\beta_1 = \frac{1}{1250G} [875q_1 - 250q_2] \rightarrow ②$$

$$\beta_2 = \frac{1}{1625 \times G} \left[ \frac{q_2 \times 25}{0.1} + \frac{q_2 \times 25}{0.15} + \frac{q_2 \times 25}{0.1} + \frac{(q_2 - q_1) \times 25}{0.1} \right]$$

$$= \frac{1}{1250G} [250q_2 + 166.66q_2 + 250q_2 + 250q_2 - 250q_1]$$

$$= \frac{1}{1250G} [916.66q_2 - 250q_1] \quad ③$$

$2=3$

$$\underline{\beta_1 = \beta_2} \quad \underline{916.66q_2 - 250q_1}$$

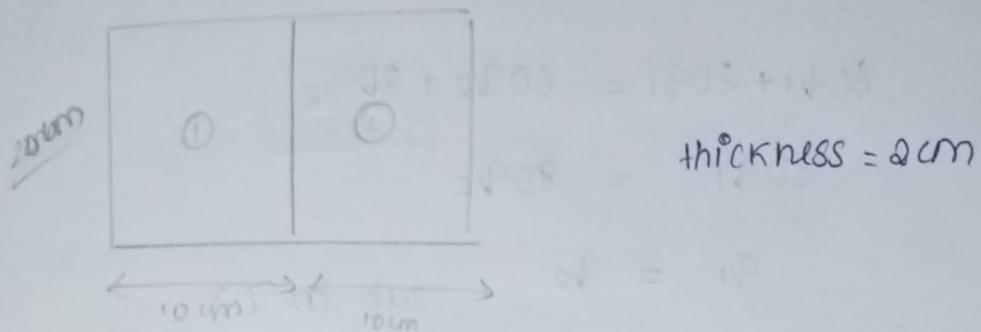
$$\frac{1}{1250G} [875q_1 - 250q_2] = \frac{1}{1250G} [916.66q_2 - 250q_1]$$

$$\underline{1125q_1 - 1766.66q_2 = 0} \quad \checkmark \quad ④$$

$$q_1 = 30.54 \text{ N/mm}$$

$$q_2 = 29.45 \text{ N/mm}$$

calculate the shear flow and it twist per unit length for the given section shown in fig. Assume  $T = 1000 \text{ N cm}$   $G_1 = 25 \times 10^9 \text{ N/m}^2$



$$T = 2Aq_1 + 2Aq_2 \\ = 2A_1 q_1 + 2A_2 q_2 \\ = 2 \times 200 q_1 + 2 \times 200 q_2 \\ 1000 = 400 q_1 + 400 q_2 \rightarrow ①$$

$$\beta_1 = \frac{1}{2A_1 G_1 t} \oint q ds$$

$$= \frac{1}{2 \times 400 \times 25 \times 10^9 \times 2} [80q_1 + 10q_1 + 10q_1 + 20(q_1 - q_2)] \\ = 5 \times 10^{-10} [80q_1 + 10q_1 + 10q_1 + 20q_1 + 20q_2]$$

$$\beta_1 = 5 \times 10^{-10} [60q_1 - 20q_2] \rightarrow ②$$

$$\beta_2 = \frac{1}{2A_2 G_1 t} \oint q ds$$

$$= \frac{1}{2 \times 400 \times 25 \times 10^9 \times 2} [10q_2 + 20q_2 + 10q_2 + 20(q_2 - q_1)] \\ = 5 \times 10^{-10} [10q_2 + 20q_2 + 10q_2 + 20q_2 - 20q_1]$$

$$= 5 \times 10^{-10} [60q_2 - 20q_1] \rightarrow ③$$

$$\beta_1 = \beta_2$$

$$5 \times 10^{-10} [60q_1 - 20q_2] = 5 \times 10^{-10} [60q_2 - 20q_1]$$

$$60q_1 - 20q_2 = 60q_2 - 20q_1$$

$$60q_1 + 20q_1 = 60q_2 + 20q_2$$

$$80q_1 = 80q_2$$

$$q_1 = q_2 \quad \text{sub in (1)}$$

$$1000 = 400q_1 + 400q_2$$

$$1000 = 400q_2 + 400q_2 \geq 800q_2$$

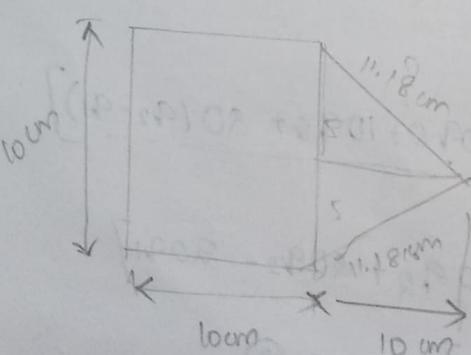
$$q_2 = \frac{1000}{800} = 1.25 \text{ N/cm}$$

$$q_1 = 1.25 \text{ N/cm}$$

$$\beta_1 = 5 \times 10^{-10} [60(1.25) + 20(1.25)]$$

$$2.5 \times 10^{-8} = \beta_2 = 2.5 \times 10^{-8}$$

6. Determine the shear flow & shear center for the section shown in fig. Assume  $T = 10000 \text{ N cm}$  &  $G = 2 \text{ GPa}$ ;  $t = 2 \text{ mm}$



$$1 \text{ GPa} = 1 \text{ N/mm}^2 = 2 \times 10^9 \text{ N/m}^2$$

$$A_1 = b \times d =$$

$$= 10 \times 10 = 100 \text{ cm}^2$$

$$A_2 = \frac{1}{2} \times b \times h + \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times 10 \times 5 + \frac{1}{2} \times 10 \times 5$$

$$AC = \sqrt{AB^2 + BC^2}$$

$$AC = \sqrt{5^2 + 10^2} = \sqrt{125} = 11.18 \text{ cm}$$

$$T = 2A_1q_1 + 2A_2q_2$$

$$10000 = 2(100q_1) + 2(50q_2)$$

$$10000 = 200q_1 + 100q_2 \rightarrow ①$$

$$\beta_1 = \frac{1}{2 \times A_1 \times G_I t} \oint q ds$$

$$= \frac{1}{2 \times 100 \times 2 \times 10^5 \times 0.2} [10q_1 + 10q_1 + 10q_1 + 10(q_1 - q_2)]$$

$$= 1.25 \times 10^{-7} [40q_1 - 10q_2] \rightarrow ②$$

$$\beta_2 = \frac{1}{2 \times A_2 \times G_I t} \oint q ds$$

$$= \frac{1}{2 \times 50 \times 2 \times 10^5 \times 0.2} [11.18q_2 + 11.18q_2 + 10[q_2 - q_1]]$$

$$= 2.5 \times 10^{-7} [32.36q_2 + 10q_2 - 10q_1]$$

$$= 2.5 \times 10^{-7} [32.36q_2 - 10q_1] \rightarrow ③$$

$$\beta_1 = \beta_2$$

$$1.25 \times 10^{-7} [40q_1 - 10q_2] = 2.5 \times 10^{-7} [32.36q_2 - 10q_1]$$

$$50q_1 - 12.5q_2 = 80.9q_2 - 25q_1$$

$$50q_1 + 25q_1 = 80.9 + 12.5q_2$$

$$75q_1 = 93.4q_2$$

$$q_1 = \frac{93.4}{75} q_2$$

$$q_1 = 1.24 q_2 \quad \text{N/cm}$$

$$10000 = 200(1.24)q_2 + 100q_2$$

$$10000 = 348 q_2 \Rightarrow \frac{10000}{348}$$

$$q_2 = 28.73 \quad \text{N/cm} \quad q_1$$

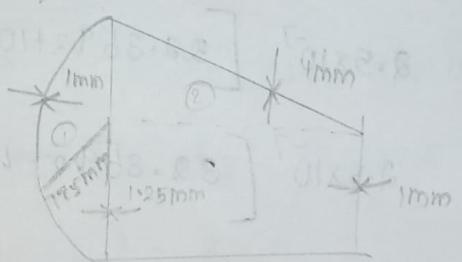
$$q_1 = 1.24 \times 28.73 = 35.62 \quad \text{N/cm}$$

$$\beta_1 = 1.25 \times 10^{-7} [40(35.62) - 10(28.73)] = 1.421 \times 10^{-4}$$

$$\begin{aligned} \beta_2 &= 2.5 \times 10^{-7} [82.36(28.73) - 10(35.62)] \\ &= 1.433 \times 10^{-4} \end{aligned}$$

7. Determine the SF & shear center for the section shown in Fig. Assume  $G = 25 \text{ GPa}$   
thickness:  $T = 100 \text{ kNm}$

$$\begin{aligned} A_1 &= \frac{\pi r^2}{2} = \frac{\pi \times (175)^2}{2} \\ &= 48.10 \times 10^3 \text{ mm}^2 \end{aligned}$$



$$A_2 = b \times d + \frac{1}{2} \times b \times h$$

$$= 350 \times 175 + \frac{1}{2} (350 \times 175)$$

$$= 61250 + 30625$$

$$= 91875 \text{ mm}^2$$

$$175 \quad ? \quad \sqrt{350^2 + 175^2} = \sqrt{122500 + 30625} = 391.81$$

$$T = 100 \text{ KNm} \Rightarrow 100 \times 10^3 \text{ N.m}$$

$$100 \times 10^6 \text{ Nmm}$$

$$G_1 = 25 \text{ GPa} = 25 \times 10^9 \text{ N/mm}^2$$

$$= 25 \times 10^9 \text{ N/mm}^2 \Rightarrow 25 \times 10^9 - 10^6 = 25 \times 10^3 \text{ N/mm}^2$$

$$T = 2A_1 q_1 + 2A_2 q_2$$

$$100 \times 10^6 = 2(48.10 \times 10^3) q_1 + 2(91875 q_2)$$

$$100 \times 10^6 = 96260 q_1 + 183750 q_2 \rightarrow ①$$

$$\beta_1 = \frac{1}{2A_1 G t} \oint q ds$$

$$= \frac{1}{2 \times 48.10 \times 10^3 \times 25 \times 10^3} \left[ \frac{\pi \times 175}{1} q_1 + \frac{350}{1.25} (q_1 - q_2) \right]$$

$$= 4.15 \times 10^{-10} (549.77 q_1 + 280 q_1 - 280 q_2)$$

$$\beta_2 = \frac{1}{2A_2 G t} \oint q ds = (829.77 q_1 - 280 q_2) \rightarrow ②$$

$$= \frac{1}{2 \times 91875 \times 25 \times 10^3} \left[ \frac{391.3}{1} q_1 + 175 q_2 + 350 q_2 + \frac{350}{1.25} (q_2 - q_1) \right]$$

$$= 2.017 \times 10^{-10} [1196.39 q_2 - 280 q_1] \rightarrow ③$$

$$\beta_1 = \beta_2$$

$$4.15 \times 10^{-10} [829.77 q_1 - 280 q_2] = 2.017 \times 10^{-10} [1196.39 q_2 - 280 q_1]$$

$$3443.5 q_1 - 1162 q_2 = 2595.9 q_2 - 607.6 q_1$$

$$3443.5q_1 + 607.6q_1 = 2598.9q_2 + 1629q_2$$

$$4051.1q_1 = 3757.9q_2$$

$$q_1 = \frac{3757.9}{4051.1} q_2$$

$$q_1 = 0.927q_2$$

$$100 \times 10^6 = 96200 (0.927)q_2 + 183750q_2$$

$$= 272927.4q_2$$

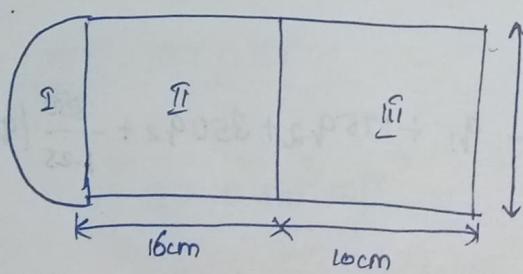
$$q_2 = 366.39 \text{ Nmm}; q_1 = 0.927(366.39)$$

$$q_1 = 339.65 \text{ Nmm}$$

$$\beta_1 = 7.513 \times 10^{-5} [7.438 \times 10^{-5} \text{ N}]$$

$$\beta_2 = 7.44 \times 10^{-5} [7.44 \times 10^{-5}]$$

8. A multi stage shown in Fig is subjected to a clockwise torque of 1000Nm. Compute the shear flow in the shell structure and the associated twist.



$$t = 3 \text{ mm} = 0.3 \text{ cm}$$

everywhere

$$T = 1000 \text{ Nm}$$

$$\Rightarrow 100000 \text{ Ncm}$$

$$A_1 = \frac{\pi r^2}{2}$$

$$A_2 = b \times d$$

$$A_3 = b \times d$$

$$T = qAq$$

$$A_1 = \frac{\pi \times 100^2}{2}$$

$$20 \times 16 \\ = 320.0 \text{ cm}^2$$

$$= 20 \times 10 \\ = 200 \text{ cm}^2$$

$$= 157.07 \text{ cm}^2$$

$$1000000 = \frac{Q}{2} \times 157.07 q_1 + \frac{Q}{2} \times 320 q_2 + \frac{Q}{2} \times 200 q_3$$

$$1000000 = 314.14 q_1 + 640 q_2 + 400 q_3 \rightarrow (1)$$

$$\beta_1 = \frac{1}{2 A_1 G_1 t} \oint q ds$$

$$\beta_1 = \frac{1}{\frac{Q}{2} \times 157.07 G_1 \times 0.3} \left[ \frac{\pi r}{2} q_1 + 200(q_1 - q_2) \right]$$

$$= \frac{31.4}{181.57 q_1 + 200 q_1 - 200 q_2}$$

$$\beta_1 = 0.0106/G_1 [51.4 q_1 - 20 q_2] \rightarrow (1) \checkmark$$

$$\beta_2 = \frac{1}{2 A_2 G_1 t} \oint q ds$$

$$= \frac{1}{\frac{Q}{2} \times 320 G_1 \times 0.3} \left[ 16 q_2 + 16 q_2 + 16 q_2 + 20(q_2 - q_3) \right]$$

$$= \frac{1}{5.08 \times 10^{-3}/G_1} [72 q_2 - 20 q_1 - 20 q_3] \rightarrow (2)$$

$$\beta_3 = \frac{1}{2 A_3 G_1 t} \oint q ds$$

$$= \frac{1}{\frac{Q}{2} \times 200 G_1 \times 0.3} \left[ 10 q_3 + 20 q_3 + 10 q_3 + 20(q_3 - q_2) \right]$$

$$= \frac{8.333 \times 10^{-3}}{G_1} [60 q_3 - 20 q_2] \rightarrow (3)$$

$$\beta_2 = \beta_3 \rightarrow (4) \quad \& \quad \beta_1 = \beta_3 \rightarrow (5)$$

$$5.08 \times 10^{-3}/G_1 [72 q_2 - 20 q_1 - 20 q_3] = 8.333 \times 10^{-3}/G_1 [60 q_3 - 20 q_2]$$

$$374.4 q_2 - 104 q_1 - 104 q_3 = 499.98 q_3 - 166.66 q_2$$

$$+ 104.16 q_1 - 541.06 q_2 - 604.14 q_3 = 0$$

$$374.49_2 + 166.669_2 - 1049_1 = 499.989_3 + 1049_3$$

= 0

$$-541.069_2 + 1049_1 + 603.989_3 = 0 \rightarrow \textcircled{B}$$

$$\beta_1 = \beta_3$$

$$-603.989_3$$

$$0.01069_1 [51.49_1 - 209_2] = 8.333 \times 10^3 G_1 [609_3 - 209_2]$$

$$0.54489_1 - 0.2129_2 = 0.4999_3 - 0.16669_2 + 0.212$$

$$0.54489_1 = 0.4999_3 + 0.015249_2 \rightarrow \textcircled{C}$$

$$0.54489_1 - 0.04549_2 = 0.4999_3 = 0 \rightarrow \textcircled{C}$$

$$q_1 = \frac{541.06}{104} q_2 \quad q_1 = 5.202 q_2 \quad q_1 = 65.61 \quad \text{sub(A)(B)(C) in}$$

$$q_2 = 84.04$$

$$q_3 = 63.99$$

Solve:-

calc

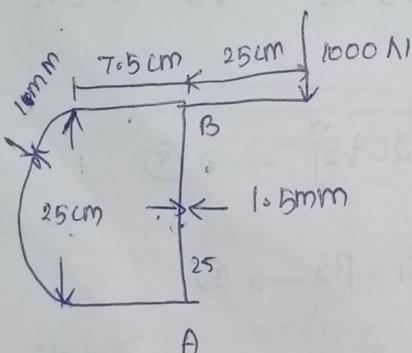
$$\beta_1 = 17.949$$

$$\beta_3 = 17.949$$

$$H.W? \Rightarrow \beta_2 = 17.949$$

10. Determine the shear flow and shear centre for the section shown in Fig.

X



By force of equilibrium

$$q_1 \times 25 - q_2 \times 25 = 1000$$

$$25q_1 - 25q_2 = 1000$$

$$2q_2 \left( \frac{7.5 \times 25}{2} \right) + 2q_2 \left[ \frac{\pi r^2}{2} + \frac{7.5 \times 25}{2} \right] = 1000 \times 25$$

$$2 q_2 \left[ \frac{7.5 \times 25}{2} \right] + 2 q_2 \left[ \frac{\pi \times (12.5)^2}{2} + \frac{7.5 \times 25}{2} \right] = 25000$$

$$187.5 q_2 + 2(245.43 q_2 + 16.25) = 25000$$

$$q_2 = 28.87 \text{ N/cm}$$

$$25 q_1 - 25(28.87) = 1000$$

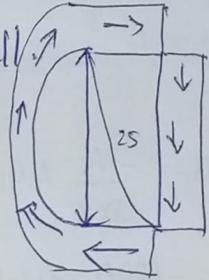
$$\begin{aligned} & 25 q_1 = 1000 + 721.75 \\ & q_1 = 68.875 \end{aligned}$$

$$q_1 = \frac{1721.75}{25}$$

$$q_1 = 68.875 \text{ N/cm}$$

to locate shear centre  $\rightarrow$  the value of  $q_t$

from  $q_1$  &  $q_2$  so that the all to produce zero twist for the shell.



$$\beta = \frac{1}{2AG_I} \int \frac{q ds}{t} = 0$$

$$(7.5 \times 2) + (\pi \times 12.5)$$

$$\frac{1}{2AG_I} \int_0^{25} \left( \frac{28.87 - q_t}{0.1} \right) ds + \int_{25}^0 \left( \frac{68.875 - q_t}{0.15} \right) ds = 0$$

$$(7.5 \times 2) + (\pi \times 12.5)$$

$$\int_0^{25} \left( \frac{28.875 - q_t}{0.1} \right) ds + \int_0^{25} \left( \frac{68.875 - q_t}{0.15} \right) ds = 0$$

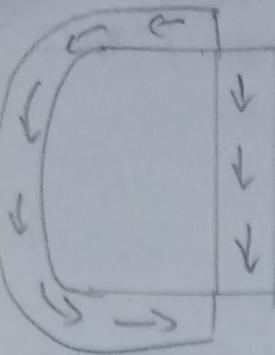
$$\boxed{\int ds = S} \text{ sub limit}$$

$$\left( \frac{28.875 - q_t}{0.1} \right)_{54.26} q_1 + \left( \frac{68.875 - q_t}{0.15} \right)_{25} = 0$$

$$15669.076 - 54.26 q_1 + 11479.16 - 166.66 q_t = 0$$

$$709.365 q_t = 27148.076 \Rightarrow q_t = \frac{27148.076}{709.365} = [38.87 \text{ N/cm}]$$

$$28.8795 - 9t = -9.39206$$



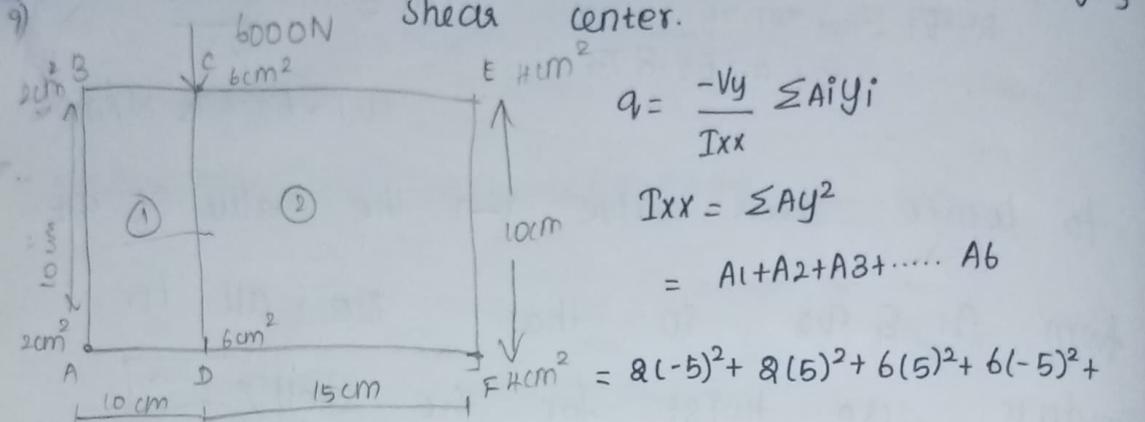
Taking moment about A

$$1000 \times 8 = 9.39806 \times 2 \left[ \frac{\pi (12.5)^2}{2} + \frac{7.5(25)}{2} \right] + 9.398 [7.5 \times 25]$$

$$= 30.6725 \text{ N/cm}$$

$$\theta = 8.1875 \text{ rad}$$

ii) Find the shear flow and twist per shear length, 9)



$$q = \frac{-V_y}{I_{xx}} \sum A_i y_i$$

$$I_{xx} = \sum A_i y^2$$

$$= A_1 + A_2 + A_3 + \dots A_6$$

$$= 8(-5)^2 + 8(5)^2 + 6(5)^2 + 6(-5)^2 + 4(-5)^2 = 50 + 50 + 150 + 150 + 100 + 100 = 600$$

$$q_{AB} = \frac{-V_y}{I_{xx}} A_1 y_1$$

section - II

$$= -\frac{6000}{600} \times 2(-5)$$

$$= -100 \text{ N/mm}$$

$$q_{CD} = \frac{-V_y}{I_{xx}} A_3 y_3$$

$$= \frac{6000}{600} \times 6 \times 5 = 300$$

$$q_{BC} = q_{AB} - \frac{V_y}{I_{xx}} A_2 y_2$$

$$q_{DF} = q_{CD} - \frac{V_y}{I_{xx}} A_4 y_4$$

$$= -100 + \frac{6000}{600} \times 2 \times 5$$

$$= 300 + \frac{6000}{600} \times 6(-5) = 0$$

$$= 0$$

$$q_{EF} = q_{DE} + \frac{6000}{600} A_6 y_6 \xrightarrow{0+6000 \text{ N/mm}^2} = -\frac{6000}{600} = -100$$

$$q_{CD} = q_{BC} + \frac{V_y}{I_{xx}} A_3 y_3$$

$$q_{EC} = q_{FE} + \frac{6000}{600} + A_5 y_5$$

$$= 0 + \frac{6000}{600} \times 6 \times 5$$

$$= -200 + 10 \times 4 \times 5$$

$$= 300$$

$$q_{DA} = q_{CD} + \frac{V_y}{I_{xx}} A_4 y_4$$

$$= -800 + 200$$

$$= 300 + \frac{6000}{600} \times 6(-5)$$

$$= 0$$

$$\text{I} \\ q_{AB} = q_0 - 100$$

$$q_{BC} = q_0 + 0$$

let  $q_0$ , be the indeterminate shear flow,

$$q_{AB} = q_0 + q_{VAB}$$

$$q_{CD} = q_0 + 300$$

$$q_{DA} = q_0 + 0$$

$$\text{II} \\ q_{CD} = q_0 + 300$$

$$q_{DF} = q_0' + 0$$

$$q_{FE} = q_0' - 800$$

$$q_{EC} = q_0' + 0$$

0/10/23

$$\beta_1 = 0$$

$$\beta_1 = \frac{1}{2A_1 G t} \int q ds$$

$$0 = \frac{1}{2A_1 G t} \left[ -100 + q_0 \right] 10 + (q_0(10)) + q_0 \times 10 + \left( q_0 - q_0' + \frac{300}{10} \right)$$

$$0 = -1000 + 10q_0 + 10q_0 + 10q_0 + 10q_0 - 10q_0' + 3000$$

$$0 = 2000 + 40q_0 - 10q_0' \rightarrow ①$$

$$\beta_2 = \frac{1}{2A_2 G t} \int q ds$$

$$\beta_2 = 0$$

$$0 = \frac{1}{2A_2 G t} \left[ (300 + q_0' - q_0) 10 + (q_0' \times 15) + (q_0' - 200) 10 \right. \\ \left. + (q_0'(15)) \right]$$

$$0 = [(q_0' - q_0 + 300) 10 + 15q_0' + (-200 + q_0') 10 + 15q_0']$$

$$0 = 10q_0' - 10q_0 + 3000 + 15q_0' - 2000 + 10q_0' + 15q_0'$$

$$0 = 50q_0' - 10q_0 + 1000 \rightarrow ②$$

(x)

$$q_{v0} = -57.89$$

112.5

56.28

$$18948 = 200q_1 + 300q_2$$

$$(q_{v0})_y = -31.57$$

$$q_{VAB} = 57.89 - 100 = 42.11$$

$$q_{VDF} = -31.57$$

$$q_{VBC} = -57.89$$

$$q_{FCE} = 31.57 - 200 = 168.43$$

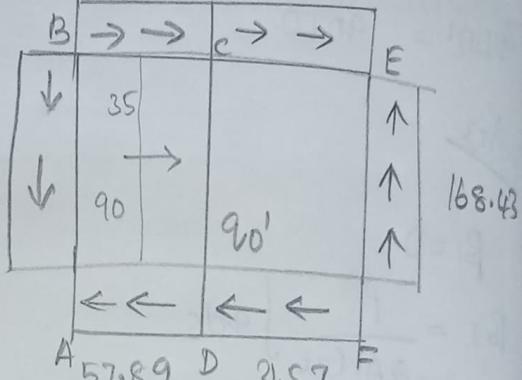
$$q_{VCD} = -57.89 + 300$$

$$q_{VCE} = -31.57$$

$$= 357.89$$

57.89 31.57

$$q_{VDA} = -57.89$$



$$q_{VCD} = -31.57 + 300 = 268.43 \quad 42.11$$

$$q_{VFE}$$

Mx ve

$$(42.11 \times 10 \times 10) - (57.89 \times 10 \times 10) - (31.57 \times 15 \times 10) + (168.43 \times 10 \times 15)$$

$$421 - 5789 - 4735.5 + 25264.5 = -6000e \quad = -6000e$$

$$18951 = -6000e$$

$$e = -\frac{18951}{6000} = -3.158$$

$$B_2 = \frac{1}{2}AGt \int q_{vds}$$

$$= \frac{1}{2}AGt [(q_2 - q_1)10 + (q_2 \times 15)]$$

$$= \frac{1}{2}AGt [50q_2 - 10q_1]$$

$$T = 2Aq_v t$$

$$q_v t = T/2A; T = Vx e$$

$$= -6000 \times -3.158 = 18948$$

$$T = 2A_1 q_1 + 2A_2 q_2$$

$$= 2 \times 1000 q_1 + 2 \times 150 q_2$$

$$18948 = 200q_1 + 300q_2 \rightarrow ⑤$$

$$B_1 = \frac{1}{2}AGt \int q_{vds}$$

$$= \frac{1}{2}AGt [(q_1 \times 10) + (q_1 \times 10) + (q_1 \times 10) + (q_1 - q_2)10]$$

$$= \frac{1}{2}AGt [10q_1 + 10q_1 + 10q_1 - 10q_2]$$

$$= \frac{1}{2}AGt [40q_1 - 10q_2]$$

$$\frac{1}{2(200)} (40q_1 - 10q_2) = \frac{1}{2(150)} (50q_2 - 10q_1)$$

$$1.5 (40q_1 - q_2) = 50 (q_2 - 10q_1)$$

$$60q_1 - 15q_2 = 50q_2 - 10q_1$$

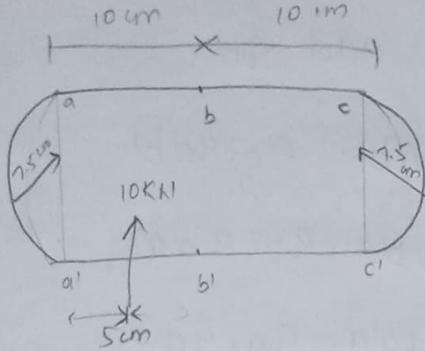
$$70q_1 - 65q_2 = 0$$

$$q_1 = 36.31$$

$$q_2 = 39.11$$

$$q_{VAB} = 36.31 - 100 = -63.69; q_{VBC} = 36.31$$

10/10/23  
19)



$$b = b' = 2 \text{ cm}$$

$$a = a' = c = c' = 1 \text{ cm}$$

$$q_v = -\frac{V}{I_{xx}} y A$$

$$I_{xx} = \sum A_i y_i^2$$

$$\begin{aligned} &= A_a y_a^2 + A_b y_b^2 + A_c y_c^2 + A_{a'} y_{a'}^2 + A_{b'} y_{b'}^2 + A_{c'} y_{c'}^2 \\ &= 1(7.5)^2 + 2(7.5)^2 + 1(7.5)^2 + 1(-7.5)^2 + 2(-7.5)^2 + 1(-7.5)^2 \end{aligned}$$

$$I_{xx} = 450$$

$$q_{ab} = -\frac{10 \times 10^3}{450} 1 \times (7.5)$$

$$q_{c'b'} = q_{cc'} - \frac{V_y}{I_{xx}} A_{c'} y_{c'}$$

$$q_{ab} = -166.66$$

$$= -666.65 - \frac{10 \times 10^3}{450} \times (1 \times (-7.5))^2$$

$$q_{bc} = q_{ab} - \frac{V_y}{I_{xx}} A_b y_b$$

$$= -499.99.$$

$$= -166.66 - \frac{10 \times 10^3}{450} \times 2(7.5)$$

$$q_{ba'} = q_{c'b'} - \frac{V_y}{I_{xx}} A_{b'} y_{b'}$$

$$q_{bc} = -499.99$$

$$= -499.99 - \frac{10 \times 10^3}{450} \times 2(-7.5)^2$$

$$q_{bc'} = q_{bc} - \frac{V_y}{I_{xx}} A_c y_c$$

$$q_{ba'} = -166.65$$

$$= -499.99 - \frac{10 \times 10^3}{450} \times 1(7.5)$$

$$q_{cc'} = -666.65$$

$$q_{a'a} = -166.65 - \frac{10 \times 10^3}{450} (1 \times 7.5)$$

$$= -166.65 + 166.65 \\ = 0$$

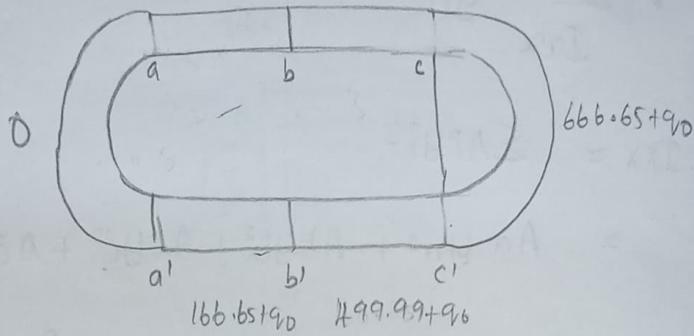
Intermediate shear flow add  $q_0$

$$q_{ab} = q_0 + q_{ab} \quad q_{c'b'} = q_0 + q_{c'b'}$$

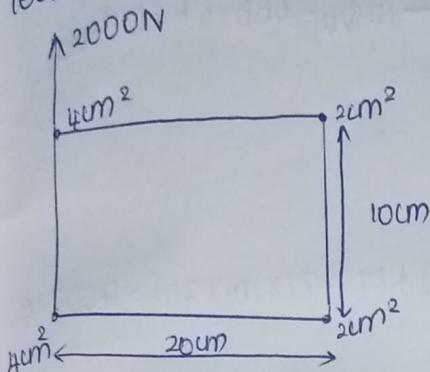
$$q_{bc} = q_0 + q_{bc} \quad q_{ba'} = q_0 + q_{ba'}$$

$$q_{cc'} = q_0 + q_{cc'} \quad q_{a'a} = q_0 + q_{a'a}$$

$$= 166.66 + q_0 \quad 499.99 + q_0$$



7/10/23  
 Q3) The figure shows a single cell beam with 4 flange. find the internal shear flow force system when the beam came the external load of 2000N at shown



$$q_v = -\frac{V_y}{I_{xx}} \sum A_i y_i$$

$$I_{xx} = \sum A_i y_i^2$$

$$= A_1 y_1^2 + A_2 y_2^2 + A_3 y_3^2 + A_4 y_4^2$$

$$= 4(-5)^2 + 4(5)^2 + 2(5)^2 + 2(-5)^2$$

$$= 100 + 100 + 50 + 50 = 300$$

$$q_{AB} = -\frac{V_y}{I_{xx}} \cdot A_1 y_1$$

$$q_{AB} = -\frac{2000}{300} \times 4 \times (-5)$$

$$q_{AB} = 133.33$$

$$\begin{aligned} q_{BC} &= q_{AB} - \frac{V_y}{I_{xx}} A_2 y_2 \\ &= 133.33 - \frac{2000}{300} \times 4 \times 5 \\ &= 0 \end{aligned}$$

$$\begin{aligned} q_{CD} &= q_{BC} - \frac{V_y}{I_{xx}} A_3 y_3 \\ &= 0 - \frac{2000}{300} \times 2 \times 5 \\ &= -66.66 \end{aligned}$$

$$q_{DA} = q_{CD} - \frac{V_y}{I_{xx}} A_4 y_4$$

$$= -66.66 - \frac{2000}{300} \times 2 \times (-5)$$

$$= 0$$

let  $q_0$  be the intermediate shear flow existing at the ? for the section

$$q_{AB} = q_0 + 133.33$$

$$q_{BC} = q_0$$

$$q_{CD} = q_0 - 66.66$$

$$q_{DA} = q_0$$

$$\beta = 0$$

$$B = \frac{1}{2Agt} \int q ds$$

$$0 = \frac{1}{2Agt} \left[ (q_0 + 133.33)10 + q_{v0} \times 20 + (q_{v0} - 66.66) \times 10 + 20q_{v0} \right]$$

$$0 = \frac{1}{2Agt} \left[ 10q_0 + 1333.33 + 20q_{v0} - 10q_{v0} - 666.66 + 20q_{v0} \right]$$

$$0 = 60q_0 + 666.7$$

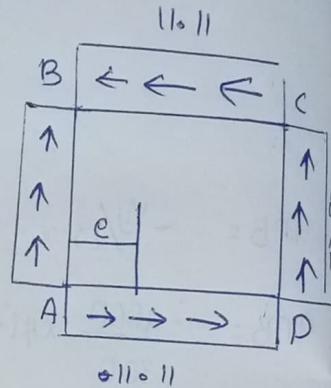
$$M = V \times e$$

$$(11.11 \times 20 \times 10) + (77.77 \times 10 \times 20) = 20000e$$

$$2222 + 1554 = 2000e$$

$$17776 = 2000e$$

$$e = \frac{17776}{2000} = 8.88$$



$$T = V \times e$$

$$= 2000 \times 8.88 = 17760 \text{ Ncm}$$

$$T = 2Aq_t$$

$$q_t = T / 2A = \frac{17760}{2 \times 200}$$

$$q_t = 44.4 \text{ N/cm}$$

$$q_{AB}^{II} = q_0 + 133.33 + q_t$$

$$= 11.11 + 133.33 + 44.4$$

$$= 166.62$$

$$\sigma_{VBC}^{II} = \sigma_0 + \sigma_t$$

$$= -11 \cdot 11 + 44 \cdot 4$$

$$= 33.29$$

$$\sigma_{VCD}^{II} = \sigma_{VB} - \sigma_{b6} - \sigma_{b6} + \sigma_t$$

$$= -11 \cdot 11 - 66 \cdot 66 + 44 \cdot 4$$

$$= -33.37$$

$$\sigma_{DVA}^{II} = \sigma_0 + \sigma_t$$

$$= 71 \cdot 11 + 44 \cdot 4$$

$$= 33.29$$

