unit-V - Numerical Solution For ODE

Fourth order Range Kutta - Method or R-K Method :-

$\frac{dy}{dx} = f(x,y)$, $y(x_0) = y_0$

$h = x_1 - x_0$

$K_1 = hf(x_0, y_0)$

$K_2 = hf(x_0 + h/2, y_0 + \frac{K_1}{2})$

$K_3 = hf(x_0 + h/2, y_0 + K_2/2)$

$K_4 = hf(x_0 + h, y_0 + K_3)$

$\Delta y = 1/6 (K_1 + 2K_2 + 2K_3 + K_4)$

$y_1 = y_0 + \Delta y$

1) Use R-K Method solve

$\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$, $y(0) = 1$

Find y at $x = 0.2$

(4) digit after point

Given:

$f(x,y) = \frac{y^2 - x^2}{y^2 + x^2}$

$x_0 = 0$, $y_0 = 1$ } $y(0) = 1$

$x_1 = 0.2$ $y_1 = ?$

$h = x_1 - x_0 = 0.2 - 0 = 0.2$.

$h = 0.2$

$K_1 = hf(x_0, y_0) = hf(0,1)$

$= 0.2\left(\frac{1^2 - 0^2}{1^2 + 0^2}\right)$

$= 0.2\,(1)$

$K_1 = 0.2$

$K_2 = 0.2\, f(0 + 0.2/2,\ 1 + 0.2/2)$

$= 0.2\, f(0.1, 1.1)$

$= 0.2\left[\frac{1.1^2 - 0.1^2}{1.1^2 + 0.1^2}\right]$

$= 0.2\left[\frac{1.21 - 0.01}{1.21 + 0.01}\right]$

$= 0.2\left[\frac{1.2}{1.22}\right]$

$K_2 = 0.1967$

$K_3 = 0.2\, f\left(0 + 0.2/2,\ 1 + \frac{0.1967}{2}\right)$

$= 0.2\, f(0.1,\ 1.09835)$

$= 0.2\left[\frac{1.09835^2 - 0.1^2}{1.09835^2 + 0.1^2}\right]$

$= 0.2\left[\frac{1.20655 - 0.01}{1.20655 + 0.01}\right]$

$= 0.2\left[\frac{1.1965}{1.2165}\right] = 0.2 \times 0.9835$

$= 0.1967$

$k_4 = 0.2 f(0+0.2, 1+0.1967)$

$= 0.2 f(0.2, 1.1967)$

$= 0.2 \left[\frac{1.1967^2 - 0.2^2}{1.1967^2 + 0.2^2}\right]$

$= 0.2 \left[\frac{1.4321 - 0.04}{1.4321 + 0.04}\right]$

$= 0.2 \left[\frac{1.3921}{1.4721}\right]$

$= 0.2 (0.9457)$

$k_4 = 0.1892$

$\Delta y = \frac{1}{6}(0.2 + 2 \times 0.1967 + 2(0.1967) + 0.189$

$= 0.1959.$

$y_1 = y_0 + \Delta y$

$= 1 + 0.1959$

$\boxed{y_1 = 1.1959}$

$x_1 = 0.2 \quad , \quad y_1 = 1.1959$

2) Given $f(x,y) = x^3 + y \quad y(0) = 2$

Find $y(0.2)$ & $y(0.4)$.

Given:

$f(x,y) = x^3 + y$

$x_0 = 0$

$y_0 = 2$

$x_1 = 0.2 \quad y_1 = ?$

$y_2 = ?$ $\quad x_2 = 0.4$.

$h = x_1 - x_0 = 0.2 - 0 = 0.2$.

$K_1 = 0.2 f(0, 2)$

$= 0.2 [0 + 2]$

$K_1 = 0.4$ //

$K_2 = 0.2 f\left(0 + \frac{0.2}{2}, 2 + \frac{0.4}{2}\right)$

$= 0.2 f(0.1, 2.2)$

$= 0.2 [0.1^3 + 2.2]$

$= 0.2 [2.201]$

$= 0.4402$

$K_3 = 0.2 f\left(0 + 0.2/2, 2 + \frac{0.4402}{2}\right)$

$= 0.2 f(0.1, 2.2201)$

$= 0.2 [0.1^3 + 2.2201]$

$= 0.2 [2.2211)$

$= 0.4443$

$K_4 = 0.2 f(0 + 0.2, 2 + 0.4443)$

$= 0.2 f(0.2, 2.4443)$

$= 0.2 [0.2^3 + 2.4443]$

$= 0.2 [2.4523]$

$= 0.4905$.

$$\Delta y = \frac{1}{6}\left[0.4 + 2(0.4402) + 2(0.4443) + 0.4905\right]$$

$$= \frac{1}{6}\left[2.6595\right]$$

$$\Delta y = 0.4433 \quad //$$

$$y_1 = 2 + 0.4433$$

$$y_1 = 2.4433 \quad //$$

$$h = x_2 - x_1 = 0.4 - 0.2 = 0.2$$

$$k_1 = 0.2 f(0.2, 2.4432)$$

$$= 0.2\left[0.2^3 + 2.4432\right)$$

$$= 0.2\left[2.4512\right]$$

$$= 0.4902$$

$$k_2 = 0.2 f\left(0.2 + \frac{0.2}{2}, 2.4432 + \frac{0.4902}{2}\right]$$

$$= 0.2 f(0.3, 2.6883)$$

$$= 0.2\left[0.3^3 + 2.6883\right)$$

$$= 0.2\left[2.7153\right]$$

$$= 0.5431$$

$$k_3 = 0.2 f\left(0.2 + \frac{0.2}{2}, 2.4432 + \frac{0.5431}{2}\right)$$

$$= 0.2\left[0.3, 2.7148\right]$$

$$= 0.2\left[0.3^3 + 2.7148\right]$$

$$= 0.2\left[2.7418\right]$$