Taylor's series:

$$y_{n+1} = y_n + \frac{h}{1!}y_n' + \frac{h^2}{2!}y_n'' + \frac{h^3}{3!}y_n''' + \frac{h^4}{4!}$$

1) Find y at $x = 0.1$ & $x = 0.2$ $\quad y' = x + y$

$y(0) = 1$

Given:

$x_0 = 0 \quad y_0 = 1$

$y' = x + y \qquad y'''' = y'''$

$y'' = 0 + 1 + y'$

$y''' = 0 + y''$

$h = x_1 - x_0 = 0.1$

$x_1 = 0.1 \quad y_1 = ?$

$x_2 = 0.2 \quad y_2 = ?$

By Taylor's series:

Put $n = 0$

$$y_1 = y_0 + 0.1[0+1] + \frac{(0.1)^2}{2}[1+0+1] + \frac{(0.1)^3}{6}[1+0+1] + \frac{(0.1)^4}{24}[2]$$

$$= 1 + 0.1 + 0.01 + 0.00033 + 0.000008 33$$

$$y_1 = 1.11033$$

Put $n = 1$

$$y_2 = y_1 + \frac{0.1}{1}[y_1'] + \frac{(0.1)^2}{2}[y_1''] + \frac{(0.1)^3}{6}[y_1'''] + \frac{(0.1)^4}{24}[y_1'''']$$

$y_1' = x_1 + y_1 = 0.1 + 1.11033 = 1.21033$

$y_1'' = 1 + y_1' = 1 + 1.21033 = 2.21033$

$y_1''' = y_1'' = 2.21033$

$y^{iv} = y''' = 2.21033$

$y_2 = 1.11033 + 0.1[1.21033] + \frac{(0.1)^2}{2}[2.21033]$

$+ \frac{(0.1)^3}{6}[2.21033]$

$+ (0.1)^4/24 [2.21033]$

$= 1.11033 + 0.121033 + 0.00110516$

$+ 0.0003684$

$+ 0.0000092098$

$\boxed{y_2 = 1.233}$

2) Find $y(0.1)$, $y' = x^2 + y$ $y(0) = 1$

Given:

$y_0' = x^2 + y$ $x_0 = 0$ $y_0 = 1$

$y_0'' = 2x + y'$ $x_1 = 0.1$ $y_1 = 0$

$y_0''' = 2 + y''$

$y_0^{iv} = -y'''$ $h = 0.1 - 0$

$-(2 - (2x - (x^2 + y)) = 0.1$

$= 2 - (2x - (x^2 + y))$

$= 2 - (+1)$