



DEPARTMENT OF MATHEMATICS

UNIT -V LAPLACE TRANSFORM

CONVOLUTION :

Defn.

If $f(t)$ and $g(t)$ are two functions defined for $t \geq 0$ then the convolution of $f(t)$ & $g(t)$ is defined as $f(t) * g(t) = (f * g)(t) = \int_0^t f(u)g(t-u) du$

NOTE: $f(t) * g(t) = g(t) * f(t)$

CONVOLUTION THEOREM :

If $f(t)$ & $g(t)$ are two Laplace transformable functions defined for $t \geq 0$ then

$L[f(t) * g(t)]$ is given by

$$L[f(t) * g(t)] = L[f(t)] * L[g(t)]$$

$$L^{-1}[F(s) \cdot G(s)] = L^{-1}[F(s)] * L^{-1}[G(s)]$$



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① Using convolution theorem find $L^{-1} \left[\frac{s}{(s^2+a^2)^2} \right]$

Soln: $L^{-1} \left[\frac{s}{(s^2+a^2)^2} \right] = L^{-1} \left[\frac{s}{s^2+a^2} \cdot \frac{1}{s^2+a^2} \right]$

$$= L^{-1} \left[\frac{s}{s^2+a^2} \right] \cdot L^{-1} \left[\frac{1}{s^2+a^2} \right] \quad [\text{By Conv. thm}]$$

$$= \cos at \cdot \frac{\sin at}{a}$$

$5A \sin B = \frac{1}{2} [\sin(A+B) - \sin(A-B)]$
 $5A \cos B = \frac{1}{2} [\cos(A+B) + \cos(A-B)]$
 $5A \sin B = -\frac{1}{2} [\cos(A+B) - \cos(A-B)]$

$$= \frac{1}{a} \cos at \cdot \sin at$$

$$= \frac{1}{a} \int_0^t \cos au \sin a(t-u) du \quad [\text{By Conv. defn}]$$

$$= \frac{1}{2a} \int_0^t [\sin at - \sin(2au-at)] du$$

$$= \frac{1}{2a} \left[u \sin at + \frac{\cos(2au-at)}{2a} \right]_0^t$$

$$= \frac{1}{2a} \left[t \sin at + \frac{\cos at}{2a} - 0 - \frac{\cos at}{2a} \right]$$

$$= \frac{1}{2a} t \sin at$$



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Find the inverse LT by convolution thm. $\frac{1}{s^2(s+5)}$

$$\begin{aligned}
 \mathcal{L}^{-1} \left[\frac{1}{s^2(s+5)} \right] &= \mathcal{L}^{-1} \left[\frac{1}{s^2} \right] * \mathcal{L}^{-1} \left[\frac{1}{s+5} \right] \\
 &= t * e^{-5t} \\
 &= \int_0^t u e^{-5(t-u)} du \\
 &= e^{-5t} \int_0^t u e^{5u} du \\
 &= e^{-5t} \left[\frac{u e^{5u}}{5} - \frac{e^{5u}}{25} \right]_0^t \\
 &= e^{-5t} \left[\frac{t e^{5t}}{5} - \frac{e^{5t}}{25} - \left(-\frac{1}{25} \right) \right] \\
 &= \frac{t}{5} - \frac{1}{25} + \frac{e^{-5t}}{25} \\
 &= \frac{1}{25} [5t + e^{-5t} - 1]
 \end{aligned}$$

3) using convolution theorem find

i) $\frac{1}{(s^2+a^2)^2}$ soln: $\frac{1}{2a^2} \left[\frac{\sin at}{a} - t \cos at \right]$

ii) $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$ soln: $\frac{a \sin at - b \sin bt}{a^2 - b^2}$