(2) Find the fourier series for the function

$f(x) = \frac{(\pi - x)^2}{2}$ in $0 \le x \le 2\pi$

$$f(x) = \frac{(\pi - x)^2}{2}$$

Fourier series for the function $f(x)$ in the interval $[0, 2\pi]$ is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

To find a_0:

$$a_0 = \frac{1}{\pi} \int_0^{2\pi} f(x)\,dx = \frac{1}{\pi} \int_0^{2\pi} \frac{(\pi - x)^2}{2}\,dx$$

$$= \frac{1}{2\pi} \int_0^{2\pi} (\pi - x)^2\,dx = \frac{1}{2\pi} \left[\frac{(\pi - x)^3}{-3} \right]_0^{2\pi}$$

$$= \frac{-1}{6\pi} \left[(\pi - 2\pi)^3 - \pi^3 \right] = \frac{-1}{6\pi} \left[(-\pi)^3 - \pi^3 \right]$$

$$= \frac{2\pi^3}{6\pi} = \frac{\pi^2}{3}$$

To find a_n:

$$a_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos nx\,dx = \frac{1}{\pi} \int_0^{2\pi} \frac{(\pi - x)^2}{2} \cos nx\,dx$$

$$= \frac{1}{2\pi} \int_0^{2\pi} (\pi - x)^2 \cos nx\,dx$$

$u = (\pi - x)^2$ $v = \cos nx$

$u' = 2(\pi - x)(-1)$ $v_1 = \frac{\sin nx}{n}$

$= -2(\pi - x)$ $v_2 = \frac{-\cos nx}{n^2}$

$u'' = -2(-1) = 2$, $u''' = 0$ $v_3 = \frac{-\sin nx}{n^3}$

$$a_n = \frac{1}{2\pi}\left[(\pi-x)^2\frac{\sin nx}{n} - \left[-2(\pi-x)\right]\left[\frac{-\cos nx}{n^2}\right] + 2\left[\frac{-\sin nx}{n^3}\right]\right]_0^{2\pi}$$

$$= \frac{1}{2\pi}\left[0 - 2(-\pi)\frac{\cos n(2\pi)}{n^2} - 0 - 0 + 2(\pi)\frac{\cos 0}{n^2} + 0\right]$$

$$= \frac{1}{2\pi}\left[\frac{2\pi}{n^2} + \frac{2\pi}{n^2}\right] = \frac{1}{2\pi}\left[\frac{4\pi}{n^2}\right]$$

$$\boxed{a_n = \frac{2}{n^2}}$$

TO find b_n:

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\int_0^{2\pi}\frac{(\pi-x)^2}{2}\sin nx\,dx$$

$$= \frac{1}{2\pi}\int_0^{2\pi}(\pi-x)^2\sin nx\,dx$$

$u = (\pi-x)^2$ $\qquad$ $v = \sin nx$

$u' = 2(\pi-x)(-1)$ $\qquad$ $v_1 = \frac{-\cos nx}{n}$

$= -2(\pi-x)$ $\qquad$ $v_2 = \frac{-\sin nx}{n^2}$

$u'' = -2(-1) = 2$ $\qquad$ $v_3 = \frac{\cos nx}{n^3}.$

$u''' = 0$

$$b_n = \frac{1}{2\pi}\left[(\pi-x)^2\left(\frac{-\cos nx}{n}\right) - \left[-2(\pi-x)\left(-\frac{\sin nx}{n^2}\right)\right] + 2\left[\frac{\cos nx}{n^3}\right]\right]_0^{2\pi}$$

$$= \frac{1}{2\pi}\left[-(\pi-2\pi)^2\frac{\cos n(2\pi)}{n} - 0 + \frac{2\cos n2\pi}{n^3} + \frac{\pi^2\cos 0}{n} + 0 - \frac{2\cos 0}{n^3}\right]$$

$$= \frac{1}{2\pi}\left[\frac{-\pi^2}{n} + \frac{2}{n^3} + \frac{\pi^2}{n} - \frac{2}{n^3}\right] \qquad \boxed{b_n = 0}$$

The fourier series is

$$f(x) = \frac{\pi^2/3}{2} + \sum_{n=1}^{\infty} \frac{2}{n^2} \cos nx + 0$$

$$= \frac{\pi^2}{6} + \sum_{n=1}^{\infty} \frac{2}{n^2} \cos nx$$

Interval : $[0, 2l]$

③ Find the fourier series for the function $f(x) = x^2$ in $(0, 2l)$

$$f(x) = x^2 \text{ in } (0, 2l)$$

The fourier series is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

To find a_0:

$$a_0 = \frac{1}{l}\int_0^{2l} f(x)\,dx = \frac{1}{l}\int_0^{2l} x^2\,dx$$

$$= \frac{1}{l}\left[\frac{x^3}{3}\right]_0^{2l} = \frac{1}{3l}\left[8l^3 - 0\right]$$

$$a_0 = \frac{8l^2}{3}$$

$$a_n = \frac{1}{l}\int_0^{2l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$= \frac{1}{l}\int_0^{2l} x^2 \cos\left(\frac{n\pi x}{l}\right)dx$$

$u = x^2$ $\quad$ $v = \cos\left(\frac{n\pi x}{l}\right)$

$u' = 2x$ $\quad$ $v_1 = \frac{\sin\left(\frac{n\pi x}{l}\right)}{(n\pi/l)}$

$u'' = 2$ $\quad$ $v_2 = \frac{-\cos\left(\frac{n\pi x}{l}\right)}{(n\pi/l)^2}$

$u''' = 0$ $\quad$ $v_3 = \frac{-\sin(n\pi x/l)}{(n\pi/l)^3}$

$$a_n = \frac{1}{l}\left[x^2\left(\frac{\sin\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)}\right) - 2x\left[\frac{-\cos\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^2}\right] + 2\left[\frac{-\sin\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^3}\right]\right]_0^{2l}$$

$$= \frac{1}{l}\left[0 + 2(2l)\frac{\cos\left(\frac{n\pi 2l}{l}\right)}{\left(\frac{n\pi}{l}\right)^2} - 0 - 0 - 0 + 0\right]$$

$$= \frac{1}{l}\left[4l \cos(n2\pi)\frac{l^2}{n^2\pi^2}\right]$$

$$= \frac{4l^2}{n^2\pi^2}(1)$$

$$a_n = \frac{4l^2}{n^2\pi^2}$$

To find b_n:

$$b_n = \frac{1}{l}\int_0^{2l} f(x)\sin\left(\frac{n\pi x}{l}\right)dx = \frac{1}{l}\int_0^{2l} x^2 \sin\left(\frac{n\pi x}{l}\right)dx$$

$u = x^2 \qquad v = \sin\left(\frac{n\pi x}{l}\right)$

$u' = 2x \qquad v_1 = \dfrac{-\cos\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)}$

$u'' = 2 \qquad v_2 = \dfrac{-\sin\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^2}$

$u''' = 0 \qquad v_3 = \dfrac{\cos\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^3}$

$$b_n = \frac{1}{l}\left[\left[-(x^2)\frac{\cos\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)}\right] - (2x)\left(\frac{-\sin\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^2}\right) + 2\left(\frac{\cos\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^3}\right)\right]_0^{2l}$$

$$= \frac{1}{l}\left[-4l^2\frac{\cos\left(\frac{n\pi 2l}{l}\right)}{\left(\frac{n\pi}{l}\right)} + 2(2l)\left[\frac{\sin\left(\frac{n\pi 2l}{l}\right)}{\left(\frac{n\pi}{l}\right)^2}\right] + 2\left[\frac{\cos\left(\frac{n\pi 2l}{l}\right)}{\left(\frac{n\pi}{l}\right)^3}\right]\right.$$

$$\left. + 0 - 0 - 2\frac{\cos 0}{\left(\frac{n\pi}{l}\right)^3}\right]$$

$$= \frac{1}{l}\left[\frac{-4l^3}{n\pi}(\cos n2\pi) + \frac{2l^3}{n^3\pi^3}(\cos n2\pi) - \frac{2l^3}{n^3\pi^3}(1)\right]$$

$$= \frac{1}{l}\left[\frac{-4l^3}{n\pi} + \cancel{\frac{2l^3}{n^3\pi^3}} - \cancel{\frac{2l^3}{n^3\pi^3}}\right]$$

$$\boxed{b_n = \frac{-4l^2}{n\pi}}$$

∴ The fourier series is

$$f(x) = \frac{\frac{8l^2}{3}}{2} + \sum_{n=1}^{\infty} \left(\frac{4l^2}{n^2\pi^2}\right) \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} \left(\frac{-4l^2}{n\pi}\right) \sin\left(\frac{n\pi x}{l}\right)$$

$$= \frac{4l^2}{3} + \sum_{n=1}^{\infty} \left(\frac{4l^2}{n^2\pi^2}\right) \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} \left(\frac{-4l^2}{n\pi}\right) \sin\left(\frac{n\pi x}{l}\right)$$

Even function:-

A real function $f(x)$ is said to be even

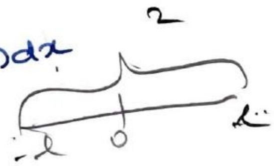
if $f(x) = f(-x)$.

$f(x) = x^2$, $f(-x) = (-x)^2 = x^2$.

$f(x) = f(-x)$

If $f(x)$ is an even function then

$$\int_{-l}^{l} f(x)\,dx = 2\int_{0}^{l} f(x)\,dx$$



odd function:

A real function $f(x)$ is said to be odd

if $f(-x) = -f(x)$

$f(x) = x$, $f(-x) = -x = (-1)x = -f(x)$

If $f(x)$ is an odd function then

$$\int_{-l}^{l} f(x)\,dx = 0.$$