Even function:-

A real function $f(x)$ is said to be even

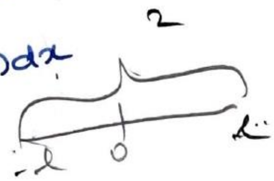
if $\quad f(x) = f(-x)$.

$f(x) = x^2 \quad f(-x) = (-x)^2 = x^2$

$f(x) = f(-x)$

If $f(x)$ is an even function then

$$\int_{-l}^{l} f(x)\,dx = 2\int_{0}^{l} f(x)\,dx$$



Odd function:

A real function $f(x)$ is said to be odd

if $\quad f(-x) = -f(x)$

$f(x) = x \quad f(-x) = -x = (-1)x = -f(x)$

If $f(x)$ is an odd function then

$$\int_{-l}^{l} f(x)\,dx = 0.$$

Note: If $f(x)$ does not satisfies even and odd functions then it is called neither even nor odd function.

$$\int_{-\pi}^{\pi} x\,dx = \left[\frac{x^2}{2}\right]_{-\pi}^{\pi} = \frac{\pi^2}{2} - \frac{(-\pi)^2}{2} = 0.$$

$$\int_{-\pi}^{\pi} x^2\,dx = \left[\frac{x^3}{3}\right]_{-\pi}^{\pi} = \frac{\pi^3}{3} - \frac{(-\pi)^3}{3} = \frac{\pi^3}{3} + \frac{\pi^3}{3} = \frac{2\pi^3}{3}$$

$$2\int_{0}^{\pi} f(x)\,dx$$

Example:-

1. $f(x) = x^2$

$f(-x) = (-x^2)$

$= x^2 = f(x)$

It is Even function

2. $f(x) = x\cos x$

$\cos x =$
$\sin x =$

$f(-x) = (-x)\cos(-x)$

$= -x\cos x$

$= -f(x)$

It is odd function

$\cos x = \cos x$
$\cos(-x) = \cos x$
$\sin(x) = \sin x$
$\sin(-x) = -\sin x$

3. $f(x) = x\sin x \Rightarrow$ Even function

4. $f(x) = |x| \Rightarrow$ Even function

5. $f(x) = x + x^2 \Rightarrow$ Neither Even nor odd.

Note:

1. Even function × Even fn = Even fn
2. odd Fn × odd fn = Even fn
3. Even fn × odd fn = odd fn
4. odd fn × Even fn = odd fn

* For even function, $b_n = 0$
* For odd function $a_0 = 0$ and $a_n = 0$

(1) Find the fourier series for the function $f(x) = |x|$ in $-\pi \le x \le \pi$

$f(x) = |x| = x$
$f(-x) = |-x| = x$

$f(x) = |x| = x$

$f(-x) = |-x| = x$

$f(x) = f(-x)$

$\therefore$ $f(x)$ is even function

$\therefore b_n = 0.$

The fourier series is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

To find a_0:

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{2}{\pi}\int_{0}^{\pi} f(x)\,dx$$

Since for even fn, $\int_{-l}^{l} f(x)\,dx = 2\int_{0}^{l} f(x)\,dx$

$$= \frac{2}{\pi}\int_{0}^{\pi} x\,dx$$

$$= \frac{2}{\pi}\left[\frac{x^2}{2}\right]_0^{\pi} = \frac{1}{\pi}\left[\pi^2\right]$$

$$\boxed{a_0 = \pi}$$

To find a_n:

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x\cos nx\,dx = \frac{2}{\pi}\int_{0}^{\pi} x\cos nx\,dx$$

$u = x \qquad v = \cos nx$

$u' = 1 \qquad v_1 = \frac{\sin nx}{n}$

$u'' = 0 \qquad v_2 = -\frac{\cos nx}{n^2}$

$$= \frac{2}{\pi}\left[x\frac{\sin nx}{n} - (1)\left(-\frac{\cos nx}{n^2}\right)\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[0 + \frac{\cos n\pi}{n^2} - \frac{\cos 0}{n^2}\right] = \frac{2}{\pi}\left[\frac{(-1)^n - 1}{n^2}\right]$$

$$= \frac{2}{\pi n^2}\left[(-1)^n - 1\right]$$

$\therefore$ The fourier series is

$$f(x) = \frac{\pi}{2} + \sum_{n=1}^{\infty}\frac{2}{n^2\pi}\left[(-1)^n - 1\right]\cos nx .$$