Laplace transforms:-

$f(t)$ be the function of 't' defined for $t>0$, then the laplace transform of $f(t)$ defined by.

$$L[f(t)] = \int_0^\infty e^{-st} f(t)\,dt = F(s)$$

condition for existence of laplace transforms:-

1st condition : $f(t)$ should be continuous or continuous in the closed integral $[a,b]$.

→ $f(t)$ should be exponential of order that is (i.e.,) $\lim_{s\to\infty} e^{-st} f(t) = 0$

Inverse Laplace transforms:-

If laplace transforms of $f(t)$ is $F(s)$ i.e, $L[f(t)] = F(s)$, then $f(t)$ is called and the inverse laplace transforms of $F(s)$ and is written by

$$[f(t)] = L^{-1}[F(s)]$$

Laplace formula table:-

$L[f(t)] = F(s)$	$L^{-1}[F(s)] = f(t)$
$L[1] = 1/s$	$L^{-1}[1/s] = 1$
$L[t] = 1/s^2$	$L^{-1}[1/s^2] = t$
$L[t^n] = \frac{n!}{s^{n+1}}$	$L^{-1}\left[\frac{n!}{s^{n+1}}\right] = t^n$
$L[e^{at}] = \frac{1}{s-a}$	$L^{-1}\left[\frac{1}{s-a}\right] = e^{at}$
$L[e^{-at}] = \frac{1}{s+a}$	$L^{-1}\left[\frac{1}{s+a}\right] = e^{-at}$
$L[\sin at] = \frac{a}{s^2+a^2}$	$L^{-1}\left[\frac{a}{s^2+a^2}\right] = \sin at$

$L\left[\frac{\sin at}{a}\right] = \frac{1}{s^2+a^2}$ $\quad$ $L^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{\sin at}{a}$

$L[\cos at] = \frac{s}{s^2+a^2}$ $\quad$ $L^{-1}\left[\frac{s}{s^2+a^2}\right] = \cos at$

$L[K] = \frac{K}{s}$

Partial fraction:

1) find $L^{-1}\left[\frac{1}{s(s+1)(s+2)}\right]$

By PF,

$$\frac{1}{s(s+1)(s+2)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2} \longrightarrow (1)$$

$$\frac{1}{s(s+1)(s+2)} = \frac{A(s+1)(s+2) + Bs(s+2) + Cs(s+1)}{s(s+1)(s+2)}$$

$$1 = A(s+1)(s+2) + Bs(s+2) + Cs(s+1)$$

Put $s = 0$,

$$1 = A(1)(2) + 0 + 0$$

$$2A = 1 \Rightarrow \boxed{A = 1/2}$$

Put $s = -1$

$$1 = 0 + B(-1)(-1+2) + 0$$

$$1 = -B \Rightarrow \boxed{B = -1}$$

Put $s = -2$

$$1 = 0 + 0 + C(-2)(-2+1)$$

$$1 = 2C \Rightarrow \boxed{C = 1/2}$$

$$(1) \Rightarrow \frac{1}{s(s+1)(s+2)} = \frac{\frac{1}{2}}{s} - \frac{1}{s+1} + \frac{1/2}{s+2}$$

$$L^{-1}\left[\frac{1}{s(s+1)(s+2)}\right] = 1/2\, L^{-1}[1/s] - L^{-1}\left[\frac{1}{s+1}\right] + 1/2\, L^{-1}\left[1/s+2\right]$$

$= \frac{1}{2}(1) - e^{-t} + \frac{1}{2} e^{-2t}$

2) Find $L^{-1}\left[\frac{s^2}{(s+1)(s^2+4)}\right]$

By PF,

$$\frac{s^2}{(s+1)(s^2+4)} = \frac{A}{s+1} + \frac{Bs+C}{s^2+4} \rightarrow (1)$$

$$= \frac{A(s^2+4) + (Bs+C)(s+1)}{(s+1)(s^2+4)}$$

$$s^2 = A(s^2+4) + (Bs+C)(s+1)$$

put $s = -1$,

$$(-1)^2 = A((-1)^2+4) + 0$$

$$1 = 5A \Rightarrow \boxed{A = 1/5}$$

put $s = 0$

$$0 = A(0+4) + (0+C)(0+1)$$

$$0 = 4A + C$$

$$4(1/5) + C = 0$$

$$\boxed{C = -4/5}$$

equating the coefficient of s^2 in.

$$s^2 = As^2 + 4A + Bs^2 + Bs + Cs + C$$

$$1 = A + B$$

$$B = 1 - 1/5 = 4/5$$

$$(1) \Rightarrow \frac{s^2}{(s+1)(s^2+4)} = \frac{\frac{1}{5}}{s+1} + \frac{\frac{4}{5}s - \frac{4}{5}}{s^2+4}$$

$$L^{-1}\left[\frac{s^2}{(s+1)(s^2+4)}\right] = \frac{1}{5}L^{-1}\left[\frac{1}{s+1}\right] + \frac{4}{5}L^{-1}\left[\frac{s}{s^2+2^2}\right] - 4/5\, L^{-1}\left[\frac{1}{s^2+2^2}\right]$$

$$= \frac{1}{5} e^{-t} + \frac{4}{5} \cos 2t - \frac{4}{5} \frac{\sin 2t}{2}.$$

$$= \frac{1}{5} e^{-t} + \frac{4}{5} \cos 2t - \frac{2}{5} \sin 2t$$