



DEPARTMENT OF MATHEMATICS

UNIT-V LAPLACE TRANSFORM

PERIODIC FUNCTIONS :

A funct. $f(t)$ is said to be periodic if $f(t+T) = f(t)$ for all values of t and for certain values of T . The smallest value of T for which $f(t+T) = f(t)$ for all t is called the period of the func.

Ex (1) The funct. $\sin t$ & $\cos t$ are periodic functions, both having period 2π .

$$\sin t = \sin(t+2\pi) = \sin(t+4\pi) = \dots$$

Consider the func. $f(t) = \begin{cases} t & \text{if } 0 < t < 2 \\ 4-t & \text{if } 2 < t < 4 \end{cases}$ and $f(t+4) = f(t)$
 $\therefore f(t)$ is a periodic func. with period 4.

LT of periodic functions :

Let $f(t)$ be a periodic function with period T . Then

$$\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t) dt.$$



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① Find LT of $f(t) = \left(\frac{2t}{3}\right)$, $0 < t < 3$ & $f(t+3) = f(t)$

Soln: $f(t)$ is a periodic func. with period 3.

(a) $T=3$

$$\begin{aligned} \text{WRT } L[f(t)] &= \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t) dt \\ &= \frac{1}{1-e^{-3s}} \int_0^3 e^{-st} \left(\frac{2t}{3}\right) dt = \frac{1}{1-e^{-3s}} \left(\frac{2}{3}\right) \int_0^3 e^{-st} t dt \\ &= \frac{1}{1-e^{-3s}} \left(\frac{2}{3}\right) \left[\frac{te^{-st}}{-s} - \frac{e^{-st}}{s^2} \right]_0^3 \\ &= \frac{1}{1-e^{-3s}} \left(\frac{2}{3}\right) \left[\frac{3e^{-3s}}{-s} - \frac{e^{-3s}}{s^2} + \frac{1}{s^2} \right] \\ &= \frac{1}{1-e^{-3s}} \left(\frac{2}{3}\right) \left[\frac{1-e^{-3s}}{s^2} - \frac{3e^{-3s}}{s} \right] \end{aligned}$$

② Find the LT of $f(t)$ if $f(t) = e^t$, $0 < t < 2\pi$ and $f(t) = f(t+2\pi)$

Soln: $f(t)$ is a periodic function with period 2π (a) $T=2\pi$

$$\begin{aligned} L[f(t)] &= \frac{1}{1-e^{-2\pi s}} \int_0^{2\pi} e^{-st} e^t dt \\ &= \frac{1}{1-e^{-2\pi s}} \int_0^{2\pi} e^{(1-s)t} dt = \frac{1}{1-e^{-2\pi s}} \left[\frac{e^{(1-s)t}}{(1-s)} \right]_0^{2\pi} \\ &= \frac{e^{2\pi(1-s)} - 1}{(1-s)(1-e^{-2\pi s})} \end{aligned}$$



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3) Find LT of $f(t) = \begin{cases} t, & 0 < t < 1 \\ 2-t, & 1 < t < 2 \end{cases}$ such that $f(t+2) = f(t)$

Soln: $f(t)$ is a periodic func. with period 2 @ $\omega = \tau = 2$,

$$L[f(t)] = \frac{1}{1-e^{-2s}} \int_0^2 e^{-st} f(t) dt$$

$$= \frac{1}{1-e^{-2s}} \left\{ \int_0^1 e^{-st} t dt + \int_1^2 e^{-st} (2-t) dt \right\}$$

$$= \frac{1}{1-e^{-2s}} \left\{ \frac{e^{-s}}{-s} - \frac{e^{-s}}{s^2} + \frac{1}{s^2} + \frac{e^{-2s}}{s^2} + \frac{e^{-s}}{s} - \frac{e^{-s}}{s^2} \right\}$$

$$= \frac{1}{1-e^{-2s}} \left\{ \frac{1-e^{-s}}{s^2} - \frac{e^{-s}}{s} + \frac{e^{-2s}-e^{-s}}{s^2} + \frac{e^{-s}}{s} \right\}$$

$$= \frac{1}{1-e^{-2s}} \left\{ \frac{1}{s^2} [e^{-2s} - 2e^{-s} + 1] \right\}$$

$$= \frac{(1-e^{-s})^2}{s^2(1-e^{-2s})} = \frac{(1-e^{-s})^2}{s(1-e^{-s})(1+e^{-s})} = \frac{(1-e^{-s})}{s(1+e^{-s})}$$

) Find LT of the periodic func. $f(t) = \begin{cases} 1, & 0 < t < a \\ -1, & a < t < 2a \end{cases}$ & $f(t+2a) = f(t)$.

Soln: $\frac{(1-e^{-sa})^2}{s(1-e^{-2as})}$



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