1

problems on $(-l, l)$

II. Find the fourier series of $f(x) = \begin{cases} l+x, & -l \le x \le 0 \\ l-x, & 0 \le x \le l \end{cases}$

Soln:

Now, $f(x) = \begin{cases} \phi_1(x), & -l \le x \le 0 \\ \phi_2(x), & 0 \le x \le l \end{cases}$

$\phi_1(x) = l + x \;;\; \phi_2(x) = l - x$

Now $\phi_1(-x) = l - x = \phi_2(x)$

$\Rightarrow f(x)$ is even $\Rightarrow b_n = 0$

$$a_0 = \frac{2}{l}\int_0^l f(x)\,dx = \frac{2}{l}\int_0^l (l-x)\,dx$$

$$= \frac{2}{l}\left[\frac{(l-x)^2}{-2}\right]_0^l$$

$$= \frac{-1}{l}\left[0 - l^2\right] = l$$

$a_0 = l$

$$a_n = \frac{2}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}\,dx$$

$$= \frac{2}{l}\int_0^l (l-x)\cos\frac{n\pi x}{l}\,dx \qquad \int u v\,dx = u v_1 - u' v_2 + \cdots$$

$$= \frac{2}{l}\left[(l-x)\sin\frac{n\pi x}{l}\left(\frac{l}{n\pi}\right) - (-1)\frac{l^2}{n^2\pi^2}\left(-\cos\frac{n\pi x}{l}\right)\right]_0^l$$

$$= \frac{2}{l}\left[(l-x)\frac{l}{n\pi}\sin\frac{n\pi x}{l} - \frac{l^2}{n^2\pi^2}\cos\frac{n\pi x}{l}\right]_0^l$$

$$= \frac{2}{l}\left[\frac{-l^2}{n^2\pi^2}(-1)^n + \frac{l^2}{n^2\pi^2}\right] = \frac{2}{l}\,\frac{l^2}{n^2\pi^2}\left[1 - (-1)^n\right]$$

$$= \frac{2l}{n^2\pi^2}\left[1 - (-1)^n\right]$$

$$a_n = \begin{cases} 4l/n^2\pi^2, & \text{if } n \text{ is odd} \\ 0, & \text{if } n \text{ is even} \end{cases}$$

$$\therefore f(x) = \frac{l}{2} + \sum_{n=odd}^{\infty} \frac{4l}{n^2\pi^2} \cos nx$$

$$= \frac{l}{2} + \frac{4l}{\pi^2} \sum_{n=odd}^{\infty} \frac{1}{n^2} \cos nx$$