



UNIT-V LAPLACE TRANSFORM

Applications of Laplace transforms to

Differential Equations:

If $L[f(t)] = F(s)$ then

$$L[y'(t)] = sL(y) - y(0)$$

$$L[y''(t)] = s^2L(y) - sy(0) - y'(0)$$

① Solve the differential equations using LT

$$y'' + 4y' + 4y = e^{-t} \text{ given that } y(0) = 0 \text{ and } y'(0) = 0$$

Soln: $y'' + 4y' + 4y = e^{-t}$

Taking LT on both sides,

$$L(y'' + 4y' + 4y) = L(e^{-t})$$

$$L(y'') + 4L(y') + 4L(y) = \frac{1}{s+1}$$

$$[s^2L(y) - sy(0) - y'(0)] + 4[sL(y) - y(0)]$$

$$+ 4L(y) = \frac{1}{s+1}$$



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Given: $y(0) = 0, y'(0) = 0$

$$\Rightarrow [s^2 L(y) - s \cdot 0 - 0] + 4 [s L(y) - 0] + 4 L(y) = \frac{1}{s+1}$$
$$\Rightarrow s^2 L(y) + 4s L(y) + 4 L(y) = \frac{1}{s+1}$$
$$\Rightarrow (s^2 + 4s + 4) L(y) = \frac{1}{s+1}$$
$$\Rightarrow L(y) (s+2)^2 = \frac{1}{s+1}$$
$$\Rightarrow L(y) = \frac{1}{(s+1)(s+2)^2}$$
$$y = L^{-1} \left[\frac{1}{(s+1)(s+2)^2} \right]$$
$$\frac{1}{(s+1)(s+2)^2} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$$
$$1 = A(s+2)^2 + B(s+2)(s+1) + C(s+1)$$

Put $s = -2 \Rightarrow C = -1$

Put $s = -1 \Rightarrow A = 1$

Put $s = 0 \Rightarrow B = -1$

$$\frac{1}{(s+1)(s+2)^2} = \frac{1}{s+1} - \frac{1}{s+2} - \frac{1}{(s+2)^2}$$
$$y = L^{-1} \left(\frac{1}{(s+1)(s+2)^2} \right)$$
$$= L^{-1} \left(\frac{1}{s+1} \right) - L^{-1} \left(\frac{1}{s+2} \right) - L^{-1} \left(\frac{1}{(s+2)^2} \right)$$
$$= e^{-t} - e^{-2t} - t e^{-2t}$$

$L^{-1} \left(\frac{1}{(s+2)^2} \right)$
$= e^{-2t} \cdot L^{-1} \left(\frac{1}{s^2} \right)$
$= e^{-2t} \cdot t$