PARTIAL FRACTION :

Find $L^{-1}\left[\frac{1}{s(s+1)(s+2)}\right]$

Soln: $\frac{1}{s(s+1)(s+2)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2}$

$1 = A(s+1)(s+2) + B(s+2)s + C\,s(s+1)$

put $s=-1$

$1 = A(0) + B(-1)(1) + C(0)$

$\Rightarrow B = -1$

put $s=-2$,

$1 = A(0) + B(0) + C(-2)(-1)$

$1 = 2C \Rightarrow C = 1/2$

put $s=0$

$1 = A(1)(2) + B(0) + C(0)$

$\Rightarrow 1 = 2A \Rightarrow A = 1/2$

$$\frac{1}{s(s+1)(s+2)} = \left[\frac{1/2}{s} - \frac{1}{s+1} + \frac{1/2}{s+2}\right]$$

$$L^{-1}\left[\frac{1}{s(s+1)(s+2)}\right] = L^{-1}\left[\frac{1/2}{s}\right] - L^{-1}\left[\frac{1}{s+1}\right] + L^{-1}\left[\frac{1/2}{s+2}\right]$$

$$= \frac{1}{2}L^{-1}\left[\frac{1}{s}\right] - L^{-1}\left[\frac{1}{s+1}\right] + \frac{1}{2}L^{-1}\left[\frac{1}{s+2}\right]$$

$$= \frac{1}{2} - e^{-t} + \frac{1}{2}e^{-2t}$$

$$= \frac{1}{2}\left[1 - 2e^{-t} + e^{-2t}\right]$$

2) Find $L^{-1}\left[\frac{2s^2+4s+5}{(s+1)(s-3)^2}\right]$

Soln:

$$\left[\frac{2s^2+4s+5}{(s+1)(s-3)^2}\right] = \frac{A}{s+1} + \frac{B}{s-3} + \frac{C}{(s-3)^2}$$

$$2s^2+4s+5 = A(s-3)^2 + B(s-3)(s+1) + C(s+1)$$

Put $s=3$,

$$18+12+5 = A(0)+B(0)+C(4)$$

$$\Rightarrow C = 35/4$$

put $s=-1$,

$$2-4+5 = A(-4)^2 + B(-4)(0) + C(0)$$

$$\Rightarrow A = 3/16$$

put $s=0$,

$$5 = 9A - 3B + C$$

$$\Rightarrow B = \frac{29}{16}$$

$$\therefore L^{-1}\left[\frac{2s^2+4s+5}{(s+1)(s-3)^2}\right] = \frac{3}{16}L^{-1}\left[\frac{1}{s+1}\right] + \frac{29}{16}L^{-1}\left[\frac{1}{s-3}\right] + \frac{35}{4}L^{-1}\left[\frac{1}{(s-3)^2}\right]$$

$$= \frac{3}{16}e^{-t} + \frac{29}{16}e^{3t} + \frac{35}{4}te^{3t}$$

3) Find $L^{-1}\left[\frac{s^2}{(s+1)(s^2+4)}\right]$

Soln:

$$\frac{s^2}{(s+1)(s^2+4)} = \frac{A}{s+1} + \frac{Bs+C}{s^2+4}$$

$$s^2 = A(s^2+4) + Bs + C(s+1)$$

put $s = -1$

$$\Rightarrow 1 = A(5) + B(-1)(0) + C(0)$$

$$\Rightarrow A = 1/5$$

put $s = 0$

$$\Rightarrow 0 = 4A + C \Rightarrow C = -4/5$$

put $s = -4$

$$16 = 20A + 12B - 3C$$

$$\Rightarrow B = 4/5$$

$$\therefore L^{-1}\left[\frac{s^2}{(s+1)(s^2+4)}\right] = L^{-1}\left[\frac{1/5}{(s+1)} + \frac{4/5\,s - 4/5}{(s^2+4)}\right]$$

$$= \frac{1}{5}L^{-1}\left[\frac{1}{s+1}\right] + \frac{4}{5}L^{-1}\left[\frac{s}{s^2+4}\right] - \frac{4}{5}L^{-1}\left[\frac{1}{s^2+4}\right]$$

$$= 1/5\, e^{-t} + 4/5 \cos 2t - 4/5 \frac{\sin 2t}{2}$$